Department of
Education

AUSTRALIAN CURRICULUM

MATHEMATICS
YEAR 7

Integers

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YEAR 7

Integers

Student's name: ________________________________

Teacher's name: ________________________________

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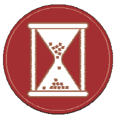
Department of
Education

Contents

Signposts

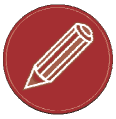
Each symbol is a sign to help you.

Here is what each one means:



The recommended time you should take to complete this section.

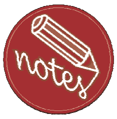
An explanation of key terms, concepts or processes.



A written response.
Write your answer or response in your journal.

Correct this task using the answers at the end of the resource.

Calculators may not be used here.



Make notes describing how you attempted to solve the problem. Keep these notes to refer to when completing the Self-evaluation task. Your teacher may wish you to forward these notes.

Introduction

This resource should take you approximately two weeks to complete. It comprises seven learning sections, a summary section and a review task section.

The learning sections have the following headings:

- **Key words**
  These are the main words that you need to understand and use fluently to explain your thinking.

- **Warm-up**
  Warm-up tasks should take you no longer than 10 minutes to complete. These are skills from previous work you are expected to recall from memory, or mental calculations that you are expected to perform quickly and accurately. If you have any difficulties in answering these questions, please discuss them with your teacher.

- **Review**
  Some sections have reviews immediately after the warm-up. The skills in these reviews are from previous work and are essential for that section. You will use these to develop new skills in mathematics. Please speak to your teacher immediately if you are having any trouble in completing these activities.

- **Focus problem**
  Focus problems are designed to introduce new concepts. They provide examples of the types of problems you will be able to solve by learning the new concepts in this resource. Do not spend too long on these but do check and read the solutions thoroughly.

- **Skills development**
  These help you consolidate new work and concepts. Most sections include skills development activities which provide opportunities for you to become skilled at using new procedures, apply your learning to solve problems and justify your ideas. Please mark your work after completing each part.

Correcting your work

Please mark and correct your work as you go. Worked solutions are provided to show how you should set out your work. If you are having any difficulty in understanding them, or are getting the majority of the questions wrong, please speak to your teacher immediately.

Journal

Please keep an exercise book to record your notes and to summarise your learning. At the end of each section, write definitions for the key words that were introduced for that section.

Curriculum details

Content Descriptions

This resource provides learning and teaching to deliver the Australian Curriculum: Mathematics for the following Year 7 Content Descriptions.

Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

Investigate and use square roots of perfect square numbers (ACMNA150)

Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)

Compare, order, add and subtract integers (ACMNA280)

Content Descriptions	1	2	3	4	5	6	7	R
ACMNA149		■	■					■
ACMNA150	■							■
ACMNA151				■	■			■
ACMNA280						■	■	■

■ Indicates the content description is explicitly covered in that section of the resource.

Previous relevant Content Descriptions

The following Content Descriptions should be considered as prior learning for students using this resource.

At Year 6 level

Identify and describe properties of prime, composite, square and triangular numbers (ACMNA122)

Investigate everyday situations that use integers. Locate and represent these numbers on a number line (ACMNA124)

Proficiency strand statements at Year 7 level

At this year level:

Understanding includes describing patterns in uses of indices with whole numbers, recognising equivalences between fractions, decimals, percentages and ratios, plotting points on the Cartesian plane, identifying angles formed by a transversal crossing a pair of lines, and connecting the laws and properties of numbers to algebraic terms and expressions

Fluency includes calculating accurately with integers, representing fractions and decimals in various ways, investigating best buys, finding measures of central tendency and calculating areas of shapes and volumes of prisms

Problem Solving includes formulating and solving authentic problems using numbers and measurements,working with transformations and identifying symmetry, calculating angles and interpreting sets of data collected through chance experiments

Reasoning includes applying the number laws to calculations, applying known geometric facts to draw conclusions about shapes, applying an understanding of ratio and interpreting data displays

General capabilities

General capabilities	1	2	3	4	5	6	7	R
Literacy	■	■	■	■	■			■
Numeracy	■	■	■	■	■	■	■	■
Information and communication technology (ICT) capability								
Critical and creative thinking	■	■	■	■	■	■	■	■
Personal and social capability						■		
Ethical behaviour								
Intercultural understanding								

■ Indicates general capabilities are explicitly covered in that section of the resource.

Cross-curriculum priorities

Cross-curriculum priorities	1	2	3	4	5	6	7	R
Aboriginal and Torres Strait Islander histories and cultures								
Asia and Australia's engagement with Asia								
Sustainability							■	

■ Indicates cross-curriculum priorities are explicitly covered in that section of the resource.

1. Square numbers and square roots

When you complete this section you should be able to:

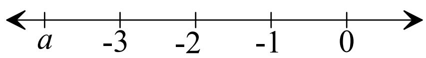
- define square numbers, perfect squares and square roots
- use the notation for square numbers and square roots
- calculate the square root for a perfect square
- estimate the square root of any number using the perfect square it falls between.

Key words

- square number
- perfect square
- square root

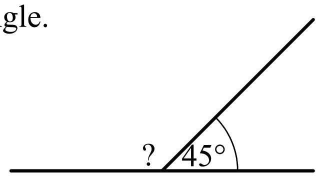
Warm-up 1

1. Is 3 a factor of 9? __________
2. 3 + 8 = __________
3. What is the missing number?



   a = __________

4. Circle the greater fraction. $\frac{1}{3}$ or $\frac{1}{4}$
5. What is a half of 20? ________
6. 2.3 + 5.3 = __________
7. 2.4 × 2 = __________
8. Write 0.5 as a fraction. __________
9. Complete: 2 , 4, 6, _________
10. Determine the size of the missing angle.



____

Review 1.1

Example
Which of the following numbers are **square numbers**?

1, 2, 4, 9, 10, 44, 49, 94, 100, 1000

Solution
The square numbers in this list are:

$1\ (1\times1=1^2),\ 4\ (2\times2=2^2),\ 9\ (3\times3=3^2),\ 49\ (7\times7=7^2),\ 100\ (10\times10=10^2)$.

1. Make a list of the first 20 square numbers.

2. Use your calculator to find a square number between 1300 and 1400.

Review 1.2

Example
Square the following numbers.

3, 7, 12, 20, 25, 100

Solution
$3^2=9$, $7^2=49$, $12^2=144$, $20^2=400$, $25^2=625$, $100^2=10\ 000$

1. Complete the following squaring of numbers:

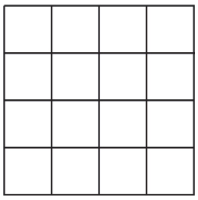
(a) $23^2=$ ______ (b) $18^2=$ ______

(c) $33^2=$ ______ (d) $111^2=$ ______

(e) $1^2=$ ______ (f) $0.5^2=$ ______

Check your work before continuing.

Focus problem 1



The square drawn here represents an area of 16 square units.

What are the length and width of this square?

What are the length and width of a square of area 1369 square units?

Check your work before continuing.

Perfect numbers

Perfect squares are not to be confused with perfect numbers. A perfect number is a number that is equal to the sum of its factors, not including itself.

The smallest perfect number is 6.
Its factors are 1, 2, 3 and 6 so the sum of its factors, excluding itself, is 1 + 2 + 3 which equals 6.
Hence it's a perfect number.

The next perfect number is between 20 and 30. See if you can find what it is.

Skills development 1

Square roots

- The square of 7 is 49.
- This can be written as $7^2 = 49$.
- The number 49 is known as a **square number** or **perfect square**.
- The **square root** of 49 is 7.
- This can be written as $\sqrt{49}$.
- Most calculators have a function for finding square roots of numbers.

Example

Find $\sqrt{441}$

Solution

$21 \times 21 = 441$

Therefore

$\sqrt{441} = 21$

1. Find the following square roots.

   First, see if you know them from memory already. If you don't know them, look at your list from Review 1.1, question one. If the square root is not on the list, use your calculator to find it.

   (a) $\sqrt{9}$ ________ (b) $\sqrt{64}$ ________

   (c) $\sqrt{36}$ ________ (d) $\sqrt{400}$ ________

   (e) $\sqrt{144}$ ________ (f) $\sqrt{289}$ ________

   (g) $\sqrt{1369}$ ________ (h) $\sqrt{9801}$ ________

Check your work before continuing.

Investigation 1

$$\sqrt{700}$$

We can work out that the **square root** of 676 is 26.

$26 \times 26 = 676$ therefore $\sqrt{676} = 26$

And that the square root of 729 is 27.

$27 \times 27 = 729$ therefore $\sqrt{729} = 27$

But what would the square root of 700 be?

1. Write the numbers 676, 729 and 700 in order from smallest to largest.

   ________ ________ ________

   ________ ________ ________

2. Write the square root of 676 underneath the 676.
3. Write the square root of 729 underneath the 729.
4. Looking at the numbers you have written, what can you say about the type of number the square root of 700 would be?

   ____

5. Write down a guess of what you think the square root of 700 might be. ________
6. Using your calculator, check if your guess was correct.

If a guess of 26.5 was made we can calculate that $26.5 \times 26.5 = 702.25$. This is close, but a guess of 26.4 might be closer.

7. Continue making improved guesses and checking with your calculator. Stop when the square of your guess has 700 in the whole number part.
8. Find the two whole numbers between which these square roots lie:

   (a) $\sqrt{10}$ _____ & _____ (b) $\sqrt{2}$ _____ & _____ (c) $\sqrt{500}$ _____ & _____

9. Use the same method of guess and check to refine your square roots for (a), (b) and (c) of question 8 until the whole number part is correct.

   (a) $\sqrt{10}$ = ________ (b) $\sqrt{2}$ = ________ (c) $\sqrt{500}$ = ________

Check your work before continuing.

2. Whole numbers as products of prime numbers

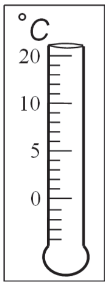
When you complete this section you should be able to:

- define prime and composite numbers
- find prime factors of whole numbers
- represent a whole number as a product of primes and powers of primes
- use index notation to represent powers of primes.

Key words

- prime number
- composite number
- factor
- divisible
- index notation

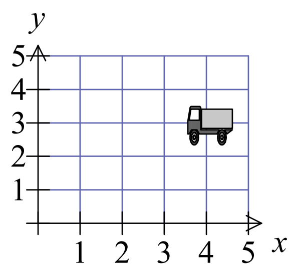
Warm-up 2



1. Is 5 a common factor of 10 and 15?
2. 17 – 9 = __________
3. The temperature was 0 degrees but it dropped 4 degrees.

   What is the new temperature? __________
4. Insert <, > or = to make the following sentence true. $\frac{1}{2} \square \frac{1}{3}$
5. $\frac{1}{2} \times 8 =$ __________
6. Round 8.2 to a whole number. __________
7. $3\overline{)9.6}$ __________
8. Write 10% as a decimal. __________
9. Complete: 0.2, 0.4, 0.6, __________
10. Circle the correct point of the truck.

    A. (3, 4) B. (3, 1) C. (4, 2) D. (4, 3)



Review 2.1

A **factor** is a whole number that divides exactly into a given whole number.

The factors of 12 are 1, 2, 3, 4, 6 and 12. We can also say that 12 is **divisible** by 1, 2, 3, 4, 6 and 12.

Example
What are the factors of 30?

Solution
1, 2, 3, 5, 6, 10, 15 and 30

The word factor is also used to describe the pair of numbers whose product is a given number.

In the example $2 \times 6 = 12$ the 2 and 6 are known as factors and 12 is known as the product.

Example
Write the number 40 as a product of as many factor pairs as you can.

Solution
$1 \times 40 = 40$, $2 \times 20 = 40$, $4 \times 10 = 40$, $5 \times 8 = 40$

1. List the factors of the following numbers.

   (a) 10 ____________________ (b) 18 ____________________

   (c) 50 ____________________ (d) 36 ____________________

2. What type of numbers will have an odd number of factors?

3. Which of these numbers is 80 divisible by?

   (a) 1 ____________________ (b) 5 ____________________

   (c) 16 ____________________ (d) 25 ____________________

4. Write the following numbers as a product of as many different factor pairs as possible.

   (a) 24 ____

   (b) 100 ____

Review 2.2

A **prime number** is one which has two **factors**. These factors are the number one, and the number itself. Numbers with more than two factors are known as **composite numbers**. The number one is neither prime nor composite. Hence 5 will be a prime number (factors 1 and 5) but 6 will be a composite number (factors 1, 2, 3, and 6).

Example

List the numbers from 1 to 20 as prime, composite or neither.

Solution

Prime: 2, 3, 5, 7, 11, 13, 17, 19

Composite: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

Neither: 1

1. Write out all the factors for these numbers and hence label them as prime or composite.

   (a) 21 ______________________ __________

   (b) 23 ______________________ __________

   (c) 77 ______________________ __________

   (d) 49 ______________________ __________

2. Suggest why ‘1’ is neither a prime nor composite number.

   ____

   ____

Check your work before continuing.

Focus problem 2

The number 7200 can be written as $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$. Check this with your calculator. This is known as writing 7200 as a product of **prime numbers**.

Using **index notation** this product of prime numbers can be written as $2^5 \times 3^2 \times 5^2$.

This says that 7200 has five **factors** of 2, two factors of 3 and two factors of 5.

1. Which of these sets of factors are also equal to 7200?

   (a) $32 \times 9 \times 25$ ________

   (b) $7 \times 20 \times 10$ ________

   (c) $8 \times 9 \times 100$ ________

2. Write 7200 as a product of 4 other sets of factors.

   ____

   ____

   ____

   ____

Check your work before continuing.

Prime numbers

Prime numbers have been studied since the times of the ancient Greeks.

In recent years, finding new prime numbers has been a task set for supercomputers.

As it has been proven that there are infinitely many prime numbers, the search for a new biggest prime number continues every day.

As of 2011 the biggest prime number found had nearly thirteen million digits in the number.

Skills development 2

Being able to write a **composite number** as a product of **prime numbers** is an important step in finding the lowest common denominator or highest common divisor of two numbers.

The number 24 when written as product of primes is $2 \times 2 \times 2 \times 3$. This can also be written as $2^3 \times 3$. You can see here that the three **factors** of 2 have been written using a power, as 2 to the power 3. This is also known as **index notation**.

Example

Write 40 as a product of prime numbers.

Solution

There are many paths to finding the **prime factors**. The following shows one person's steps.

$40 = 4 \times 10$

$4 = 2 \times 2$

$10 = 2 \times 5$

$\therefore 40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

1. Write the following numbers as a product of primes, using index notation where appropriate.

(a) 10 ____________ (b) 30 ____________

(c) 56 ____________ (d) 63 ____________

(e) 150 ____________ (f) 99 ____________

(g) 900 ____________ (h) 1 000 000 ____________

Check your work before continuing.

3. Lowest common multiple and greatest common divisor

When you complete this section you should be able to:

- find the lowest common multiple of a pair of whole numbers
- find the greatest common divisor of a pair of whole numbers.

Key words

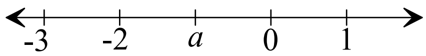
- lowest common multiple
- greatest common divisor
- lowest common denominator

Warm-up 3

1. Circle the prime number. 4, 7, 9, 12.

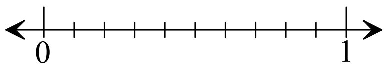
2. $4 \times 8 =$ ____________

3. What is the missing number?



$a =$ ____________

4. Locate $\frac{1}{2}$ on the number line.



5. What is a third of 15? ____________

6. Estimate the sum by first rounding to whole numbers. $6.9 + 3.2 \approx$ ____________

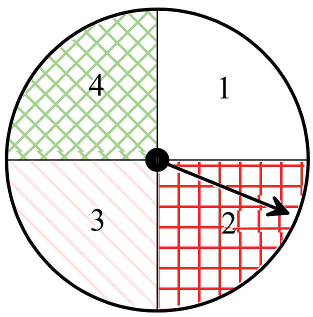
7.
$$\begin{array}{r} 0.1 \\ \times \quad 8 \\ \hline \end{array}$$

8. Write $\frac{50}{100}$ as a percentage. ____________%

9. Complete: $\frac{1}{10}, \frac{2}{10}, \frac{3}{10},$ ____________

10. Determine the probability the spinner will land on a 2.

Express your answer as a fraction. ____________



Review 3

Example
Make a list of the first 10 multiples of 8.

Solution
8, 16, 24, 32, 40, 48, 56, 64, 72, 80

1. Write the first ten multiples of the following numbers.

   (a) 5 ____________________

   (b) 7 ____________________

   (c) 12 ____________________

   (d) 15 ____________________

   (e) 20 ____________________

   (f) 30 ____________________

Check your work before continuing.

Focus problem 3

To add together the fractions $\frac{17}{56}$ and $\frac{25}{63}$ requires writing both as equivalent fractions with a common denominator of 504.

1. Write the first 10 multiples of 56.

2. Write the first 10 multiples of 63.

3. Rewrite the fractions $\frac{17}{56}$ and $\frac{25}{63}$ as equivalent fractions with a denominator of 504.

4. Add together the two fractions.

5. Explain why 504 was used as a common denominator.

Check your work before continuing.

Common denominators

Pairs of fractions have many common denominators, although it is the **lowest common denominator** that is often used to add or subtract fractions.

The lowest common denominator for two fractions, whose denominators are prime numbers, will always simply be the product of the two primes.

For example, the two fractions $\frac{7}{13}$ and $\frac{8}{19}$ which have prime number denominators have a lowest common denominator of 247 which is the product of the two primes.

Skills development 3.1

Example
Find the **greatest common divisor** of 20 and 30 from their prime factorisation.

Solution
Prime factorisation of 20: $20 = 2 \times 2 \times 5$

Prime factorisation of 30: $30 = 2 \times 3 \times 5$

From the above, common factors of 2 and 5 can be seen in the two prime factorisations.

These two common factors, are shown below as $\langle 2 \rangle$ and $[5]$.

$$20 = \langle 2 \rangle \times 2 \times [5]$$

$$30 = \langle 2 \rangle \times 3 \times [5]$$

Now the greatest common divisor 10 can be found as the product of these two common factors: $\langle 2 \rangle \times [5] = 10$. (Note that the other factors of 2 and 3 are not used.)

1. Find the greatest common divisor for these pairs of numbers using prime factorisation.

   (a) 8 and 12

   Prime factorisation of 8: ______________________

   Prime factorisation of 12: ______________________

   Greatest common divisor: ______________________

   (b) 15 and 20

   ____

   ____

   ____

   (c) 28 and 42

   ____

   ____

   ____

Skills development 3.2

Example

Find the **lowest common multiple** of 20 and 30 from their prime factorisation.

Solution

Prime factorisation of 20: $20 = 2 \times 2 \times 5$

Prime factorisation of 30: $30 = 2 \times 3 \times 5$

From the above, common factors of 2 and 5 can be seen in the two prime factorisations.

These two common factors, are shown below as $\langle 2 \rangle$ and $[5]$.

$20 = \langle 2 \rangle \times 2 \times [5]$

$30 = \langle 2 \rangle \times 3 \times [5]$

Now the lowest common multiple 60 can be found as the product of the common factors and any other factors: $\langle 2 \rangle \times [5] \times 2 \times 3 = 60$. (Note how all the other factors are used.)

1. Find the lowest common multiple for these pairs of numbers using prime factorisation.

(a) 8 and 12

Prime factorisation of 8: ______________________

Prime factorisation of 12: ______________________

Lowest common multiple: ______________________

(b) 15 and 20

____

____

(c) 18 and 26

____

____

Check your work before continuing.

4. Associative and commutative laws

When you complete this section you should be able to:

- apply the associative law to aid computations
- apply the commutative law to aid computations.

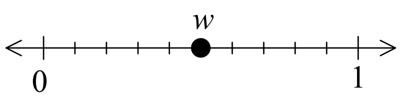
Keywords

- associative property or associative law
- commutative property or commutative law

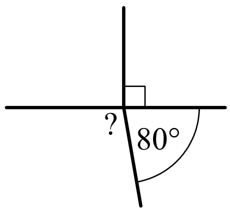
Warm-up 4

1. Complete the pattern showing square numbers. 1, 4, 9, 16, ________
2. $24 \div 6 =$ ________
3. The temperature was minus 6 degrees but it went up 2 degrees.

   What is the new temperature? ________
4. Express the value of *w* as a fraction.



5. $\frac{1}{4} \times 16 =$ ________
6. $5 \times 100 =$ ________
7. $5.5 \div 5 =$ ________
8. Write 100% as fraction. ________
9. Complete: 9, 7, 5, ________
10. Determine the size of the missing angle. ________



Review 4

Examples

Use the rule for order of operations to evaluate the following.

1. $5 + 3 \times 2 =$
2. $(5 + 3) \times 2 =$
3. $3 + 5 \times 2 =$
4. $5 + (3 \times 2) =$

Solutions

1. Multiplication of 3 times 2 is completed before the addition of 5.

   $5+3\times2=5+6=11$

2. The bracketed operation of addition is completed before multiplication.

   $(5+3)\times2=8\times2=16$

3. Multiplication first, but the changed order creates a different answer.

   $3+5\times2=3+10=13$

4. The same result as question 1 as the brackets do not change the order.

   $5+(3\times2)=5+6=11$

Evaluate:

(a) $8+4\times3=$ ____________ (b) $7\times2+5=$ ____________

(c) $8+5-3=$ ____________ (d) $(11-5)\times5=$ ____________

(e) $6+6\div2=$ ____________ (f) $8\div(4+4)=$ ____________

(g) $3+4\times5+6=$ ____________ (h) $6+4\times5\div2=$ ____________

Check your work before continuing.

Focus problem 4

1. Add together these numbers without using a calculator.

   37 + 51 + 56 + 23 + 92 + 63 + 77 + 8 + 49 + 44

2. Next add together these numbers mentally.

   37 + 63 + 92 + 8 + 49 + 51 + 56 + 44 + 77 + 23

3. Now add together these numbers mentally.

   (37 + 63) + (92 + 8) + (49 + 51) + (56 + 44) + (77 + 23)

4. Finally, see if you can find an easy way to add together these numbers mentally.

   17 + 57 + 86 + 74 + 43 + 9 + 91 + 83 + 14 + 26

5. Explain how the first and fourth problems can be made much easier than they first appear.

Check your work before continuing.

Savants

Some people are born with a condition known as Savant syndrome. While they have developmental disorders, they are often also brilliant in some special area.

One of these areas of brilliance can be mental calculations. Some savants have shown an unbelievable capacity to remember numbers and perform huge mental calculations in seconds.

This ability was the subject of the 1988 movie *Rain Man* that was based on the life of savant Kim Peek.

Rain Man 1988, film, United Artists, Los Angeles.

Skills development 4.1

One property of numbers is that the order in which they are added or multiplied makes no difference to the result. This is known as a **commutative property** or **commutative law**.

Example
Evaluate these to compare the result of different orders of operation.

1. (a) $5 + 9$ (b) $9 + 5$
2. (a) 10×7 (b) 7×10
3. (a) $8 - 5$ (b) $5 - 8$
4. (a) $12 \div 3$ (b) $3 \div 12$

Solution
1. (a) $5 + 9 = 14$ (b) $9 + 5 = 14$ $\therefore$ Addition is commutative.
2. (a) $10 \times 7 = 70$ (b) $7 \times 10 = 70$ $\therefore$ Multiplication is commutative.
3. (a) $8 - 5 = 3$ (b) $5 - 8 = (-3)$ $\therefore$ Subtraction is NOT commutative.
4. (a) $12 \div 3 = 4$ (b) $3 \div 12 = 0.25$ $\therefore$ Division is NOT commutative.

1. For each of these equations, make up another equation using the same numbers, to show whether the operation is commutative.
   (a) $7 + 15 = 22$ ______ (b) $12 - 9 = 3$ ______
   (c) $9 \times 7 = 63$ ______ (d) $8 \div 40 = 0.2$ ______

2. In some cases mental calculations are easier if the order is changed. Write down whether you think each of these calculations is easier when the order is changed.
   (a) $17 + 90 + 23$ or $17 + 23 + 90$ ______
   (b) $5 \times 19 \times 20$ or $5 \times 20 \times 19$ ______
   (c) $19 + 81 + 37$ or $19 + 37 + 81$ ______

3. Evaluate these, without using a calculator, reordering the operations to make your calculating easier.
   (a) $5 \times 13 \times 2 =$ ______
   (b) $27 + 37 + 13 =$ ______
   (c) $4 \times 14 \times 5 =$ ______
   (d) $19 + 33 + 51 + 37 =$ ______

Check your work before continuing.

Skills development 4.2

Another property of numbers is that the grouping of numbers that are added or multiplied makes no difference to the result. This is known as an **associative property** or **associative law**.

Example

Evaluate these to compare the result of different orders of operation.

1.	(a)	$(3 + 7) + 9$	(b)	$3 + (7 + 9)$
2.	(a)	$(4 \times 2) \times 5$	(b)	$4 \times (2 \times 5)$
3.	(a)	$(10 - 5) - 3$	(b)	$10 - (5 - 3)$
4.	(a)	$(12 \div 4) \div 2$	(b)	$12 \div (4 \div 2)$

Solution

1.	(a)	$(3 + 7) + 9 = 19$	(b)	$3 + (7 + 9) = 19$	∴ Addition is associative.
2.	(a)	$(4 \times 2) \times 5 = 40$	(b)	$4 \times (2 \times 5) = 40$	∴ Multiplication is associative.
3.	(a)	$(10 - 5) - 3 = 2$	(b)	$10 - (5 - 3) = 8$	∴ Subtraction is NOT associative.
4.	(a)	$(12 \div 4) \div 2 = 1.5$	(b)	$12 \div (4 \div 2) = 6$	∴ Division is NOT associative.

1. For each of these equations, make up another equation using the same numbers, to show whether the operation is associative.

   (a) $(3 + 11) + 9 = 23$ ______________ (b) $(12 - 9) - 2 = 1$ ______________

   (c) $(3 \times 5) \times 4 = 60$ ______________ (d) $(50 \div 10) \div 5 = 1$ ______________

2. In some cases mental calculations are easier if the grouping is changed. Write down which of these calculations you think is easier.

   (a) $(29 + 25) + 75$ or $29 + (25 + 75)$ ______________

   (b) $(17 \times 5) \times 20$ or $17 \times (5 \times 20)$ ______________

   (c) $(2.1 + 7.9) + 4.3$ or $2.1 + (7.9 + 4.3)$ ______________

3. Evaluate these, without using a calculator, regrouping the operations to make your calculating easier.

   (a) $(9 \times 8) \times 5 =$ ______________

   (b) $(39 + 37) + 13 =$ ______________

   (c) $(9 \times 25) \times 4 =$ ______________

   (d) $(19 + 37) + 63 =$ ______________

Check your work before continuing.

5. Distributive law

When you complete this section you should be able to:

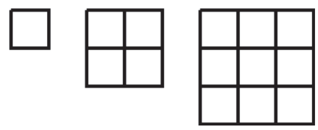
- apply the distributive law to aid computations.

Keywords

- distributive property or distributive law

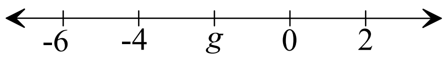
Warm-up 5

1. Draw the next shape in the pattern to represent ‘square numbers’.



2. 17 + 5 = ___________

3. What is the missing number?



g = ___________

4. $\frac{1}{3} + \frac{1}{3} =$ ___________

5. What is a fifth of 10? ___________

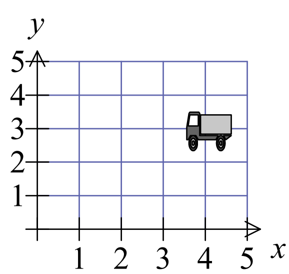
6. 4 cm = ___________ mm

7. $3 + 2 \times 4 =$ ___________

8. Write $\frac{1}{2}$ as decimal. ___________

9. Complete: 0.8, 0.6, 0.4, ___________

10.



The truck is at (4, 3).
If the truck moves 2 units down, where will it then be?

____

Review 5

Examples

Use the rule for order of operations to evaluate these:

1. $5 \times (4 + 2)$
2. $5 \times 4 + 5 \times 2$
3. $5 \times 4 + 2$
4. $(5 \times 4) + 2$

Solutions

1.	$5 \times (4 + 2) = 30$	Brackets first.
2.	$5 \times 4 + 5 \times 2 = 30$	Note how this result is the same as question 1.
3.	$5 \times 4 + 2 = 22$	Without the brackets, the result is different to question 1.
4.	$(5 \times 4) + 2 = 22$	The brackets here don't alter the order from question 3.

1. Evaluate these without using a calculator.

   (a) $8 \times (5 + 3) =$ ______________ (b) $8 \times 5 + 8 \times 3 =$ ______________

   (c) $4 \times (9 + 2) =$ ______________ (d) $4 \times 9 + 4 \times 2 =$ ______________

   (e) $7 \times (6 - 3) =$ ______________ (f) $7 \times 6 - 7 \times 3 =$ ______________

   (g) $9 \times (7 + 3) =$ ______________ (h) $9 \times 7 + 3 =$ ______________

   (i) $12 \times (15 - 5) =$ ______________ (j) $12 \times 15 - 12 \times 5 =$ ______________

2. Give a reason why in one pair of questions, the answers differ.

   ____

3. Which was easier to do, part (i) or part (j)? Why?

   ____

Check your work before continuing

Focus problem 5

Over one working week Marina works the following hours:

Monday:	5 hours
Tuesday:	6 hours
Wednesday:	8 hours
Thursday:	8 hours
Friday:	7 hours
Saturday:	3 hours

Marina's pay rate is $14.75 per hour.

1. Calculate how much Marina will be paid each day.

2. Calculate how many hours Marina works in the week.

3. Calculate how much Marina will be paid for the week's work.

Marina's pay for the week can be calculated by working out each day's pay then adding these up to find a total for the week. It can also be calculated by adding up the hours for the week first, then calculating the total pay for this many hours.

4. Write out these two sets of calculations and comment on which one you think is easier to do.

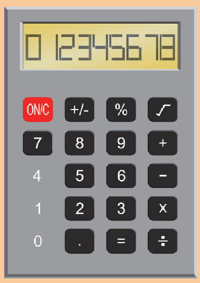
Check your work before continuing.

Vintage calculators

The first desktop electronic calculator was made in 1963.

It weighed about 15 kg.

Around 1970 these became hand held size, and were powered by batteries rather than mains electricity.



Skills development 5.1

The **distributive property** (or **distributive law**) allows multiplication to be distributed over addition (or subtraction) without altering the result of a calculation. It can be used in some calculations to make them easier to do mentally.

Examples

Use the distributive property to evaluate these without a calculator.

1. $8 \times (7 + 9)$
2. $5 \times (21 - 17)$
3. 8×59
4. 5×127
5. 6×97

Solutions

1. $8 \times (7 + 9) = 8 \times 7 + 8 \times 9 = 56 + 72 = 128$

   Does distributing the multiplication before adding make it easier by avoiding 8×16?

2. $5 \times (21 - 17) = 5 \times 21 - 5 \times 17 = 105 - 85 = 20$

   In this case it looks like it would be easier to do the subtraction first.

3. $8 \times 59 = 8 \times (50 + 9) = 8 \times 50 + 8 \times 9 = 400 + 72 = 472$

   Is that easier than 8×59?

4. $5 \times 127 = 5 \times 100 + 5 \times 20 + 5 \times 7 = 500 + 100 + 35 = 635$

   Did distributing the multiplication make it easier to do mentally?

5. $6 \times 97 = 6 \times 100 - 6 \times 3 = 600 - 18 = 582$

   Did this technique make the mental calculation easier to do than 6×97?

1. Evaluate these without a calculator using the distributive property if it makes it easier.

   (a) $6 \times (8 + 5) =$ ____________

   (b) $7 \times (13 - 8) =$ ____________

   (c) $5 \times 63 =$ ____________

   (d) $8 \times 117 =$ ____________

   (e) $4 \times 99 =$ ____________

Skills development 5.2

The **distributive property** (or **distributive law**) also allows division to be distributed over addition (or subtraction) without altering the result of a calculation.

Examples

Use the distributive property to evaluate these without a calculator.

1. $(34 + 17) \div 17$
2. $279 \div 9$

Solutions

1. $(34 + 17) \div 17 = 34 \div 17 + 17 \div 17 = 2 + 1$

   Does this make it easier by avoiding $34 + 17$ and $51 \div 17$?

2. $279 \div 9 = 270 \div 9 + 9 \div 9 = 30 + 1 = 31$

   Does this make the division easier?

1. Evaluate these without a calculator using the distributive property if it makes it easier.

   (a) $(48 + 56) \div 8 =$ ______________

   (b) $65 \div 13 - 52 \div 13 =$ ______________

   (c) $147 \div 7 =$ ______________

Skills development 5.3

The distributive property is also used in addition of fractions.

Example

$\frac{8}{13} + \frac{4}{13}$ i.e. $8 \div 13 + 4 \div 13$

Solution

$\frac{8}{13} + \frac{4}{13} = \frac{8+4}{13} = \frac{12}{13}$ i.e. $(8 + 4) \div 13 = 12 \div 13$

1. Explain how the distributive property is used in the addition of fractions.

Check your work before continuing.

6. Adding integers

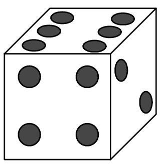
When you complete this section you should be able to:

- compare and order integers
- add integers without using a calculator.

Keywords

- integer
- negative
- positive

Warm-up 6

1. $3 \times 10 =$ ____________
2. $26 - 8 =$ ____________
3. The temperature is minus 3 degrees. How much will it need to increase to get to zero degrees? ____________
4. $\frac{3}{4} - \frac{2}{4} =$ ____________
5. $\frac{1}{4} \times 20 =$ ____________
6. 2 kg = ____________ g
7. $2 \times 5 + 2 =$ ____________
8. Write 0.1 as a percentage. ______________
9. $\frac{4}{5}, \frac{3}{5}, \frac{2}{5},$ ____________
10. 
A six-sided die is rolled. Express, as a fraction, the probability that it lands on a 6. ______________

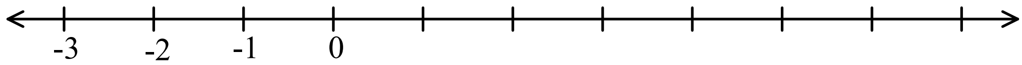
Review 6

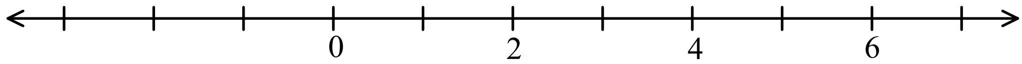
Example
Write the **integers** in this list in order from smallest to largest. Leave out the numbers that are not integers.

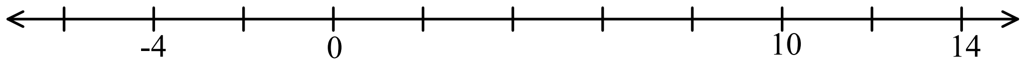
12, −5, 3, 0, −3, 7, 2.5, 1, −11, $\frac{3}{5}$, 8, −1.

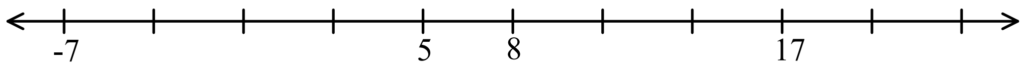
Solution
−11, −5, −3, −1, 0, 1, 3, 7, 8, 12

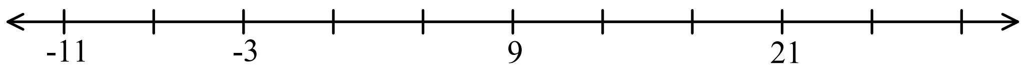
1. Complete these number lines by writing in all the numbers not shown. Make sure you follow the scale correctly.

(a)


(b)


(c)


(d)


(e)


Check your work before continuing.

Focus problem 6

In a bank account, credits are where money is added to an account and debits are where money is deducted from an account. Credits can be represented as **positive** numbers (eg $60), and debits as **negative** numbers (eg -$35). Each time a credit or debit is added to an account (transactions), the balance will change. The balance is how much is in the account.

This table shows the record of an account.

Date	Transaction	Balance
January 15th		$425
January 20th	$60	$485
February 1st	(-$35)	$450
February 18th	$60	
February 29th	(-$110)	
March 3rd	$72	
March 17th	(-$63)	
March 31st	(-$27)	
April 14th	$122	$504

Complete the balance column of the account record.

Check your work before continuing.

Negative numbers

The representation of what were positive numbers and negative numbers first appeared at around 200 BCE (Before Common Era) in China. Their use even long ago was based around making records of the balance of accounts.

The Indian Brahmagupta wrote rules for dealing with negatives around 600 CE (Common Era), but it wasn't until a few hundred years ago that British mathematicians started writing down rules for using negative numbers.

Over the many years that negatives were being considered they were, at various times, thought to be illogical, non-existent and absurd.

Skills development 6

Examples

Complete the following additions.

1. 8 + (-5)
2. (-6) + 4
3. (-5) + (-7)

Solutions

1. 8 + (-5) = 3

This can be represented as the following.

8: + + + + +   + + +

(-5): − − − − −

It can be seen from this diagram that the five **negatives** will cancel with five **positives**, resulting in three positives being left. Hence the result is 3.

2. (-6) + 4 = (-2)

This can be represented as the following.

(-6): − − − −   − −

4: + + + +

It can be seen from this diagram that the four positives will cancel with four negatives, resulting in two negatives being left. Hence the result is (-2).

3. (-5) + (-7) = (-12)

This can be represented as the following.

(-5): − − − − −

(-7): − − − − − − −

It can be seen from this diagram that nothing will cancel out. Combining the two numbers will result in there being 12 negatives. Hence the result is (-12).

1. Find the results of these additions. Include diagrams to support your results.

(a) 6 + (-2) ______________

____

(b) (-9) + 4 ______________

____

(c) (-4) + (-2) ______________

____

(d) (-3) + 8 ______________

____

(e) 5 + 3 ______________

____

2. Evaluate these.

(a) (-6) + 2 ________

(b) (-5) + (-5) ________

(c) 8 + (-5) ________

(d) (-5) + 9 ________

(e) (-7) + 6 ________

(f) (-9) + (-6) ________

(g) 12 + (-4) ________

(h) (-13) + 13 ________

(i) (-20) + 9 ________

(j) (-8) + (-9) ________

(k) 17 + (-6) ________

(l) (-15) + 8 ________

(m) (-12) + 8 ________

(n) (-6) + (-6) ________

(o) 18 + (-11) ________

(p) (-15) + 20 ________

Check your work before continuing.

7. Subtracting integers

When you complete this section you should be able to:

- subtract integers without using a calculator.

Keywords

- opposite

Warm-up 7

1. $80 \div 10 =$ ______

2. $12 \times 7 =$ ______

3. The temperature is 2 degrees.

   How much will it need to decrease to get to minus 2 degrees? ______

4. $\frac{2}{4} + \frac{1}{2} =$ ______

5. $\frac{1}{5} \times 30 =$ ______

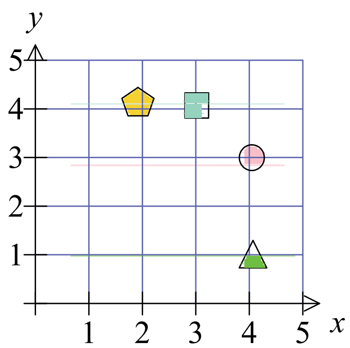
6. 4000 mL = ______ L

7. $10 \div (3 + 2) =$ ______

8. Find 10% of $20. ______

9. Describe the rule for the following pattern.

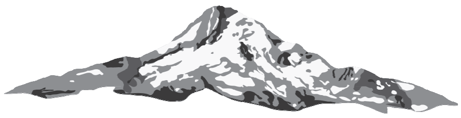
   2, 4, 6, 8, 10, … ______

10. 

    Which shape is at (4, 3)? ______

Focus problem 7

In Australia the highest point is Mount Kosciuszko which has an altitude of 2228 m above sea level. The lowest point is at Lake Eyre which is 15 m below sea level.



The difference between these two can be found, like any difference, by using subtraction.

Work out the difference between these two altitudes.

Check your work before continuing.

Mount Kosciuszko

Every year more than 30 000 people walk to the summit of Australia's highest mountain. These large numbers of people put an environmental strain on the area.

To accommodate these tourists, Australia's highest toilet was built at an altitude of 2100 metres. This stone construction toilet is at Rawson's Pass and is at the intersection of two walk trails. It has been built using the most modern ideas in sustainability and sensitivity to the environment.

As unpleasant as the topic may be seen to be, human waste management in our sensitive tourist areas is very important and improvements are rapidly being made to old practices that were not sustainable.

Skills development 7.1

The **opposite** of an integer is the number which added to it gives a result of zero. It can be found an equal distance from, but on the other side of, zero. A minus sign is used to show 'the opposite'.

Examples
What are the opposites of these numbers?

(-10) 23 (-1) 0

Solutions
1. − (-10) = 10
2. − 23 = (-23)
3. − (-1) = 1
4. − 0 = 0

1. Write down the opposite of the following numbers.

(a) (-15) ________ (b) 8 ________
(c) (-12) ________ (d) (-32) ________
(e) 7 ________ (f) 0 ________

Skills development 7.2

Subtraction gives the same result as adding the opposite.

Examples
Rewrite these subtractions so that the opposite is being added, then complete the addition.

1. 7 − (-5)
2. 8 − (-10)
3. (-4) − (-5)
4. (-9) − 7
5. 10 − 4

Solutions
1. 7 − (-5) = 7 + 5 = 12
2. 8 − (-10) = 8 + 10 = 18
3. (-4) − (-5) = (-4) + 5 = 1
4. (-9) − 7 = (-9) + (-7) = (-16)
5. 10 − 4 = 10 + (-4) = 6 Note that this question was easier if you just subtracted!

1. Rewrite these subtractions as adding the **opposite**, and then evaluate.

(a) 6 − (-2) = 6 + ______________
= 8

(b) (-9) − 4 = ______ + ______
= ______________

(c) (-4) − (-2) ______________________
____

(d) (-3) − 8 ______________________
____

(e) 5 − 3 ______________________
____

2. Evaluate these.

(a) (-6) − 2 ________
(b) (-5) − (-5) ________
(c) 8 − (-5) ________
(d) (-5) − 9 ________
(e) (-7) − 6 ________
(f) (-9) − (-6) ________
(g) 12 − (-4) ________
(h) (-13) − 13 ________
(i) (-20) − 9 ________
(j) (-8) − (-9) ________

3. Evaluate these.

(a) 17 − (-6) ________
(b) 8 + (-6) ________
(c) 11 + (-15) ________
(d) (-15) − 8 ________
(e) (-3) − 20 ________
(f) (-12) − 8 ________
(g) (-10) + (-7) ________
(h) (-8) + 20 ________
(i) (-6) − (-6) ________
(j) (-10) + 7 ________
(k) 18 − (-11) ________
(l) (-15) − 20 ________

Check your work before continuing.

8. Summary

- A square number, or perfect square, is the result of multiplying a number by itself. For example: 81 is the square of 9 as $9 \times 9 = 81$.
- The square root of 81 is represented as $\sqrt{81}$ and is equal to 9 as $9 \times 9 = 81$.
- All composite numbers can be written as a product of prime numbers. This is known as prime factorisation, and may involve the use of powers (indices). The prime factorisation of 600 is $600 = 2^3 \times 3 \times 5^2$.
- The lowest common multiple (LCM) of two numbers is the smallest integer that is a multiple of both numbers.
- The lowest common multiple can be found by listing multiples or by using the prime factorisation of both numbers.
- The greatest common divisor (GCD) of two numbers is the largest integer that is a factor of both numbers.
- The greatest common divisor can be found by listing factors or by using the prime factorisation of both numbers.
- The commutative property (or commutative law) allows for the order of numbers in addition or multiplication to be changed without changing the result.
- The associative property (or associative law) allows for the grouping of numbers in addition or multiplication to be changed without changing the result.
- The distributive property (or distributive law) allows for distributing multiplication (or division) over addition (or subtraction) without changing the result.
- When adding integers we pair off the positives and negatives. These cancel each other out. The balance remaining is the sum.
- When subtracting integers it is often easiest to change the subtraction to adding the opposite.

9. Review tasks

The following tasks will assist you to consolidate your learning and understanding of the concepts introduced in this resource, and assist you to prepare for assessments.

Task A

Name: ______________________________ Suggested time: 40 minutes

Actual time taken: __________

Instructions:

Complete this work on your own.

You may use a calculator, but show how you got your answer.

Attempt every question. Take as long as you need and record the time in the space provided above after you have finished.

1. (a) $\sqrt{36}$ ________ (b) $\sqrt{400}$ ________

2. (a) Which of these numbers is a perfect square? 10, 100, 1000 ____________________

   (b) Explain your choice from part (a). ____

3. Explain how we know that the square root of 800 lies between 28 and 29.

   ____

   ____

   ____

4. Write the following numbers as a product of primes, using index notation where appropriate.

   (a) 30 ______________________

   (b) 140 ______________________

5. Find the greatest common divisor for 24 and 40 using prime factorisation.

6. Find the lowest common multiple for 12 and 15 using prime factorisation.

7. For each of these calculations, reorder, regroup or redistribute to write a calculation that would be easier to complete mentally. Then explain what you changed and how it made the calculation easier.

(a) $(39 + 79) + 21 =$ ______

Explain: ______

(b) $9 \times 19 + 9 \times 21 =$ ______

Explain: ______

(c) $4 \times 37 \times 25 =$ ______

Explain: ______

(d) $13 \times 99 =$ ______

Explain: ______

8. Evaluate these.

(a) $15 - (-4)$ ______

(b) $7 + (-11)$ ______

(c) $(-5) + 9$ ______

(d) $(-6) - (-4)$ ______

(e) $(-8) + (-6)$ ______

(f) $(-3) - 8$ ______

Task B

Name: ______________________________ Suggested time: 40 minutes

Actual time taken: ____________

Instructions:

Complete this work on your own.

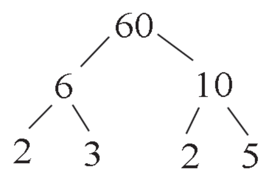
You may use a calculator, but show how you got your answer.

Attempt every question. Take as long as you need and record the time in the space provided above after you have finished.

Factor Trees

Prime factorisation can be completed using factor trees. A factor tree uses pairs of factors to break down composite numbers until only prime numbers remain.

This diagram shows 60 broken down using a factor tree.



First 60 is written with factors 6 and 10.

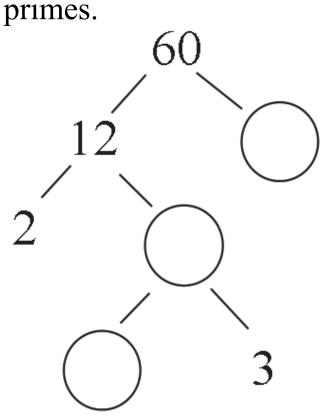
Then 6 is written with factors 2 and 3, and 10 is written with factors 2 and 5.

The result is that 60 in primes is 2 × 2 × 3 × 5.

Here is another factor tree used to write 60 as a product of primes.

This tree starts with a different pair of factors.

1. Complete the missing parts.



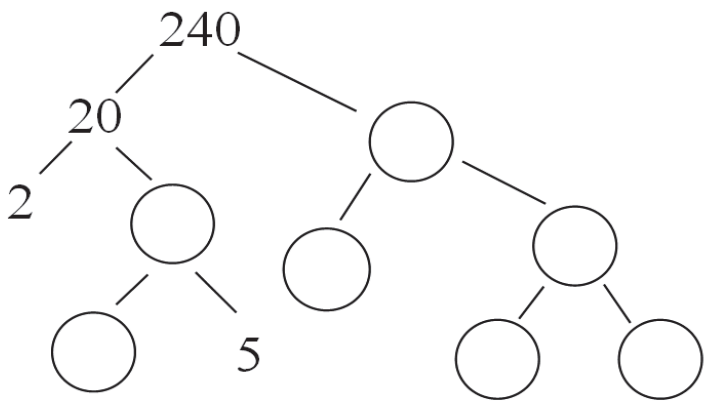
Note that the ends of the branches still contain the prime factors of 60.

It is also possible to break down 60 starting with the factors 2 and 30.

2. Draw a factor tree for 60 by starting with 2 and 30.

3. Check that your tree still ends at the four prime factors, 2, 2, 3 and 5.

4. Complete the missing parts in this factor tree.



5. Use the tree to write the prime factorisation for 240. Remember to only write the primes at the end of branches of the tree.

____

6. Check that the product of your prime factors is 240.

7. Use factor trees to write the prime factorisations for the following.

   (a) 100 ____________________

   (b) 800 ____________________

Self-evaluation task

Please complete the following.

How well did you manage your own learning using this resource?

	Always	Usually	Rarely	Not sure
Each section took approximately 45 minutes to complete.	☐	☐	☐	☐
I needed extra help.	☐	☐	☐	☐
I marked and corrected my work at the end of each section.	☐	☐	☐	☐
I made the journal entries and summaries when asked.	☐	☐	☐	☐
I have kept to my work schedule.	☐	☐	☐	☐

How much mathematics have you learnt using this resource?

	Always	Usually	Rarely	Not sure
Understanding I understand the connection between the laws of numbers and mental computation methods.	☐	☐	☐	☐
Fluency I can calculate accurately with integers.	☐	☐	☐	☐
Problem Solving I solved problems using numbers and roots.	☐	☐	☐	☐
Reasoning I can apply the properties of numbers to calculations.	☐	☐	☐	☐

Write a list of topics for which you need additional assistance. Discuss these with your teacher.

Solutions

1. Square numbers and square roots

Solutions to Warm-up 1

1. Yes
2. 11
3. (-4)
4. $\frac{1}{3}$
5. 10
6. 7.6
7. 4.8
8. $\frac{1}{2}$
9. 8
10. 135

Solutions to Review 1.1

1. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400
2. 1369 (37 squared).

Solutions to Review 1.2

1. (a) 529 (b) 324
   (c) 1089 (d) 12321
   (e) 1 (f) 0.25

Solution to Focus problem 1

What are the length and width of this square?

Length is four units, width is four units.

What are the length and width of a square of area 1369 square units?

$\sqrt{1369} = 37$ ie the length and width would be 37 units.

Solutions to Skills development 1

1. (a) 3 (b) 8
   (c) 6 (d) 20
   (e) 12 (f) 17
   (g) 37 (h) 99

Solutions to Investigation 1

1. 676, 700, 729
2. & 3. 26 27
4. It would be a decimal number between 26 and 27.
5. Any guess between 26 and 27 is appropriate such as 26.5.
6. For example: $26.5^2 = 702.25$
7. 26.46 squared will give 700.1316 so that would be as close as suggested.
8. (a) Between 3 and 4
   (b) Between 1 and 2
   (c) Between 22 and 23
9. (a) 3.2 would be satisfactory as it squares to 10.24.
   (b) 1.5 would be satisfactory as it squares to 2.25.
   (c) 22.37 would be satisfactory as it squares to 500.4169.

2. Whole numbers as products of prime numbers

Solutions to Warm-up 2

1. Yes
2. 8
3. (-4º)
4. >
5. 4
6. 8
7. 3.2
8. 0.1
9. 0.8
10. D. (4, 3)

Solutions to Review 2.1

1. (a) 1, 2, 5, 10 (b) 1, 2, 3, 6, 9, 18
   (c) 1, 2, 5, 10, 25, 50 (d) 1, 2, 3, 4, 6, 9, 12, 18, 36
2. Square numbers
3. (a) 1 Yes (b) 5 Yes
   (c) 16 Yes (d) 25 No
4. (a) $24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6$
   (b) $100 = 1 \times 100 = 2 \times 50 = 4 \times 25 = 5 \times 20 = 10 \times 10$

Solutions to Review 2.2

1. (a) 21: 1, 3, 7, 21 – composite
   (b) 23: 1, 23 – prime
   (c) 77: 1, 7, 11, 77 – composite
   (d) 49: 1, 7, 49 – composite
2. Answers may vary.
   For example, it has only one factor.

Solution to Focus problem 2

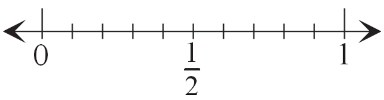
1. Which of these sets of factors are also equal to 7200?
   (a) $32 \times 9 \times 25$ Yes
   (b) $7 \times 20 \times 10$ No
   (c) $8 \times 9 \times 100$ Yes
2. Write 7200 as a product of 4 other sets of factors.
   Many sets are possible such as; $2 \times 9 \times 400$.
   You can check your sets with a calculator.

Solutions to Skills development 2

1. (a) 2×5 (b) $2 \times 3 \times 5$
   (c) $2^3 \times 7$ (d) $3^2 \times 7$
   (e) $2 \times 3 \times 5^2$ (f) $3^2 \times 11$
   (g) $2^2 \times 3^2 \times 5^2$ (h) $2^6 \times 5^6$

3. Lowest common multiple and greatest common divisor

Solutions to Warm-up 3

1. 7
2. 32
3. (-1)
4. 

5. 5
6. 10
7. 0.8
8. 50%
9. $\frac{4}{10}$
10. $\frac{1}{4}$

Solutions to Review 3

1. (a) 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
   (b) 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
   (c) 12, 24, 36, 48, 60, 72, 84, 96, 108, 120
   (d) 15, 30, 45, 60, 75, 90, 105, 120, 135, 150
   (e) 20, 40, 60, 80, 100, 120, 140, 160, 180, 200
   (f) 30, 60, 90, 120, 150, 180, 210, 240, 270, 300

Solution to Focus problem 3

1. Write out the first 10 multiples of 56.
   56, 112, 168, 224, 280, 336, 392, 448, 504, 560
2. Write out the first 10 multiples of 63
   63, 126, 189, 252, 315, 378, 441, 504, 567, 630
3. Rewrite the fractions $\frac{17}{56}$ and $\frac{25}{63}$ as equivalent fractions with a denominator of 504.
   $\frac{17}{56} = \frac{9}{9} \times \frac{17}{56} = \frac{153}{504}$ and $\frac{25}{63} = \frac{8}{8} \times \frac{25}{63} = \frac{200}{504}$
4. Add together the two fractions.
   $\frac{17}{56} + \frac{25}{63} = \frac{153}{504} + \frac{200}{504} = \frac{353}{504}$
5. Explain why 504 was used as a common denominator.
   It was the smallest common denominator and hence both fractions could be represented as equivalent fractions that could then be added together.

Solutions to Skills development 3.1

1. (a) $8 = 2 \times 2 \times 2$
   $12 = 2 \times 2 \times 3$
   Hence the GCD is $2 \times 2 = 4$.
   (b) $15 = 3 \times 5$
   $20 = 2 \times 2 \times 5$
   Hence the GCD is 5.
   (c) $28 = 2 \times 2 \times 7$
   $42 = 2 \times 3 \times 7$
   Hence the GCD is $2 \times 7 = 14$.

Solutions to Skills development 3.2

1. (a) $8 = 2 \times 2 \times 2$
   $12 = 2 \times 2 \times 3$
   Hence the LCM is $2 \times 2 \times 2 \times 3 = 24$.
   (b) $15 = 3 \times 5$
   $20 = 2 \times 2 \times 5$
   Hence the LCM is $2 \times 2 \times 3 \times 5 = 60$.
   (c) $18 = 2 \times 3 \times 3$
   $26 = 2 \times 13$
   Hence the LCM is $2 \times 3 \times 3 \times 13 = 234$.

4. Associative and commutative laws

Solutions to Warm-up 4

1. 25
2. 4
3. (-4°)
4. $\frac{1}{2}$
5. 4
6. 500
7. 1.1
8. $\frac{1}{1} = \frac{100}{100}$
9. 3
10. 100°

Solutions to Review 4

1. (a) $8+4\times 3 = 8+12 = 20$
   (b) $7\times 2+5 = 14+5 = 19$
   (c) $8+5-3 = 13-3 = 10$
   (d) $(11-5)\times 5 = 6\times 5 = 30$
   (e) $6+6\div 2 = 6+3 = 9$
   (f) $8\div(4+4) = 8\div 8 = 1$
   (g) $3+4\times 5+6 = 3+20+6 = 29$
   (h) $6+4\times 5\div 2 = 6+20\div 2 = 6+10 = 16$

Solution to Focus problem 4

1. Add together these numbers without using a calculator.

   37 + 51 + 56 + 23 + 92 + 63 + 77 + 8 + 49 + 44

   = 500

2. Next add together these numbers mentally.

   37 + 63 + 92 + 8 + 49 + 51 + 56 + 44 + 77 + 23

   = 500

3. Now add together these numbers mentally.

   (37 + 63) + (92 + 8) + (49 + 51) + (56 + 44) + (77 + 23)

   = 100 + 100 + 100 + 100 + 100

   = 500

4. Finally, see if you can find an easy way to add together these numbers mentally.

   $17 + 57 + 86 + 74 + 43 + 9 + 91 + 83 + 14 + 26$

   $= (17 + 83) + (57 + 43) + (86 + 14) + (74 + 26) + (9 + 91)$

   $= 500$

5. Explain how the first and fourth problems can be made much easier than they first appear. The additions can be made easier by changing the order and grouping of the numbers.

Solutions to Skills development 4.1

1. (a) $15 + 7 = 22$ so commutative (b) $9 - 12 = (-3)$ so NOT commutative
   (c) $7 \times 9 = 63$ so commutative (d) $40 \div 8 = 5$ so NOT commutative

2. (a) Second order is easier.
   (b) Second order is easier.
   (c) Second order is more difficult.

3. (a) $5 \times 2 \times 13 = 10 \times 13 = 130$
   (b) $27 + 13 + 37 = 40 + 37 = 77$
   (c) $4 \times 5 \times 14 = 20 \times 14 = 280$
   (d) $19 + 51 + 33 + 37 = 70 + 70 = 140$

Solutions to Skills development 4.2

1. (a) $3 + (11 + 9) = 23$ (associative) (b) $12 - (9 - 2) = 5$ (NOT associative)

   (c) $3 \times (5 \times 4) = 60$ (associative) (d) $50 \div (10 \div 5) = 25$ (NOT associative)

2. (a) $29 + (25 + 75)$

   (b) $17 \times (5 \times 20)$

   (c) $(2.1 + 7.9) + 4.3$

3. (a) $(9 \times 8) \times 5 = 9 \times (8 \times 5) = 9 \times 40 = 360$

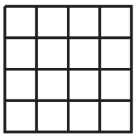
   (b) $(39 + 37) + 13 = 39 + (37 + 13) = 39 + 50 = 89$

   (c) $(9 \times 25) \times 4 = 9 \times (25 \times 4) = 9 \times 100 = 900$

   (d) $(19 + 37) + 63 = 19 + (37 + 63) = 19 + 100 = 119$

5. Distributive law

Solutions to Warm-up 5

1. 
2. 22
3. (-2)
4. $\frac{2}{3}$
5. 2
6. 40 mm
7. 11
8. 0.5
9. 0.2
10. (4, 1)

Solutions to Review 5

1. (a) 64 (b) 64
   (c) 44 (d) 44
   (e) 21 (f) 21
   (g) 90 (h) 66
   (i) 120 (j) 120
2. In the pair (g) and (h), the part (h) is not expanded as in the other examples.
3. Part (i) as it wasn't necessary to multiply 12 by 15.

Solution to Focus problem 5

1. Monday: \$73.75, Tuesday: \$88.50, Wednesday: \$118, Thursday: \$118, Friday: \$103.25, Saturday: \$44.25
2. 37 hours
3. \$545.75
4. Calculation 1:

$$\$14.75 \times 5 + \$14.75 \times 6 + \$14.75 \times 8 + \$14.75 \times 8 + \$14.75 \times 7 + \$14.75 \times 3$$
$$= \$73.75 + \$88.50 + \$118 + \$118 + \$103.25 + \$44.25$$
$$= \$545.75$$

Calculation 2:

$(5 + 6 + 8 + 8 + 7 + 3) \times \$14.75 = 37 \times \$14.75 = \$545.75$

Calculation 2 is easier to do as there are fewer operations.

Solutions to Skills development 5.1

1. Students may choose to do these differently but should get the same result.

   (a) $6 \times (8 + 5) = 6 \times 8 + 6 \times 5 = 48 + 30 = 78$

   (b) $7 \times (13 - 8) = 7 \times 5 = 35$

   (c) $5 \times 63 = 5 \times 60 + 5 \times 3 = 300 + 15 = 315$

   (d) $8 \times 117 = 8 \times 100 + 8 \times 10 + 8 \times 7 = 800 + 80 + 56 = 936$

   (e) $4 \times 99 = 4 \times 100 - 4 \times 1 = 400 - 4 = 396$

Solutions to Skills development 5.2

1. Students may choose to do these differently but should get the same result.

   (a) $(48 + 56) \div 8 = 48 \div 8 + 56 \div 8 = 6 + 7 = 13$

   (b) $65 \div 13 - 52 \div 13 = (65 - 52) \div 13 = 13 \div 13 = 1$

   (c) $147 \div 7 = 140 \div 7 + 7 \div 7 = 20 + 1 = 21$

Solutions to Skills development 5.3

1. The distributive property allows the addition of the numerators before the division by the denominator.

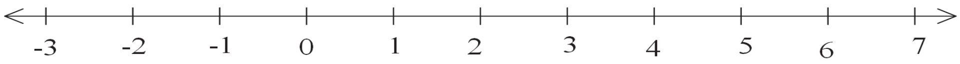
6. Adding integers

Solutions to Warm-up 6

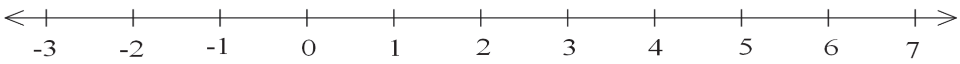
1. 30
2. 18
3. 3°
4. $\frac{1}{4}$
5. 5
6. 2000 g
7. 12
8. 10%
9. $\frac{1}{5}$
10. $\frac{1}{6}$

Solutions to Review 6

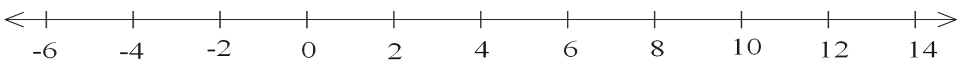
1. (a)



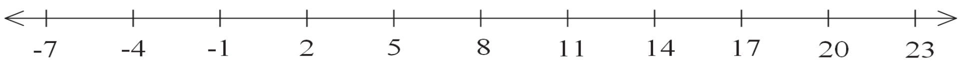
(b)



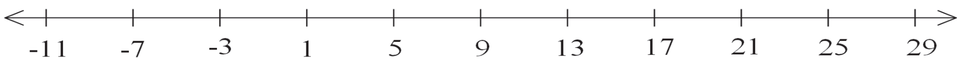
(c)



(d)



(e)



Solution to Focus problem 6

Date	Transaction	Balance
January 15th		$425
January 20th	$60	$485
February 1st	(-$35)	$450
February 18th	$60	$510
February 29th	(-$110)	$400
March 3rd	$72	$472
March 17th	(-$63)	$409
March 31st	(-$27)	$382
April 14th	$122	$504

Solutions to Skills development 6

1. (a) 6: + + + + + +
   (-2): − −

   6 + (-2) = 4

   (b) (-9): − − − − − − − − −
   4: + + + +

   (-9) + 4 = (-5)

   (c) (-4): − − − −
   (-2): − −

   (-4) + (-2) = (-6)

   (d) (-3): − − −
   8: + + + + + + + +

   (-3) + 8 = 5

   (e) 5: + + + + + + +
   3: + + +

   5 + 3 = 8

2. (a) (-4) (b) (-10)
   (c) 3 (d) 4
   (e) (-1) (f) (-15)
   (g) 8 (h) 0
   (i) (-11) (j) (-17)
   (k) 11 (l) (-7)
   (m) (-4) (n) (-12)
   (o) 7 (p) 5

7. Subtracting integers

Solutions to Warm-up 7

1. 8
2. 84
3. 4°
4. 1
5. 6
6. 4 L
7. 2
8. $2
9. Increasing by 2
10. Circle

Solution to Focus problem 7

To find the difference between the altitudes 2228 m above sea level and 15 m below sea level we represent 2228 m above sea level as +2228 or just 2228 and 15 m below sea level as (-15).

To find the difference we need to complete the subtraction: 2228 − (-15).

To complete this subtraction we use the rule that subtracting gives the same result as adding the opposite. The opposite of a number is the number which when added results in zero.

The opposite of (-15) is 15.

So 2228 − (-15) becomes 2228 + 15 which equals 2243.

Hence the difference in altitudes is 2243 metres.

Any subtraction can be changed to an equivalent addition by adding the opposite. This idea is used where it is difficult to mentally complete the subtraction, as it often is when subtracting with negative numbers.

Solutions to Skills development 7.1

1. (a) 15 (b) (-8)
   (c) 12 (d) 32
   (e) (-7) (f) 0

Solutions to Skills development 7.2

1. (a) 6 − (-2) = 6 + 2 = 8
   (b) (-9) − 4 = (-9) + (-4) = (-13)
   (c) (-4) − (-2) = (-4) + 2 = (-2)
   (d) (-3) − 8 = (-3) + (-8) = (-11)
   (e) 5 − 3 = 5 + (-3) = 2 Note: You would not normally do part (e) like this as the original subtraction is easier to do, but notice how both methods give the same result.

2. (a) (-8) (b) 0
   (c) 13 (d) (-14)
   (e) (-13) (f) (-3)
   (g) 16 (h) (-26)
   (i) (-29) (j) 1

3. (a) 23 (b) 2
   (c) (-4) (d) (-23)
   (e) (-23) (f) (-20)
   (g) (-17) (h) 12
   (i) 0 (j) (-3)
   (k) 29 (l) (-35)

Solutions to Review tasks

Solutions to Task A

1. (a) 6

   (b) 20

2. (a) Which of these numbers is a perfect square? 100

   (b) To be a perfect square the square root needs to be a whole number.

3. 28 × 28 = 784 and 29 × 29 = 841. As 800 is between these two perfect squares, its square root must lie between 28 and 29.

4. (a) 2 × 3 × 5

   (b) $2^2 \times 5 \times 7$

5. 24 = 2 × 2 × 2 × 3

   40 = 2 × 2 × 2 × 5

   Hence GCD = 2 × 2 × 2 = 8

6. 12 = 2 × 2 × 3

   15 = 3 × 5

   Hence GCM = 2 × 2 × 3 × 5 = 60

7. (a) 39 + (79 + 21)

   Regrouping the 79 + 21 made that part easy resulting in 39 + 100.

   (b) 9 × (19 + 21)

   Taking out the factor of 9 gives the easier addition of 19 + 21 and then 9 × 40.

   (c) 4 × 25 × 37

   Reordering the multiplication gives the easy 4 × 25 and then 37 × 100.

   (d) 13 × 100 − 13 × 1

   Expanding into two easier multiplications before working gives an easy subtraction of 1300 − 13.

8. (a) 19

   (b) (-4)

   (c) 4

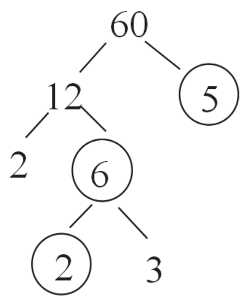
   (d) (-2)

   (e) (-14)

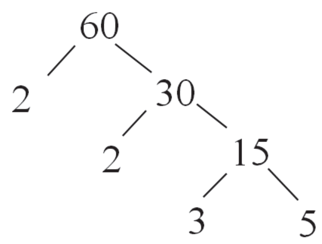
   (f) (-11)

Solutions to Task B

1. Complete the missing parts.

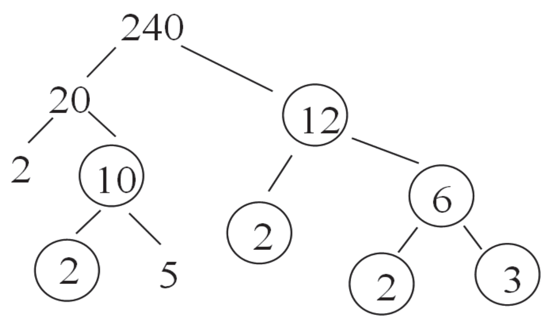


2. Draw a factor tree for 60 by starting with 2 and 30.



Others possible.

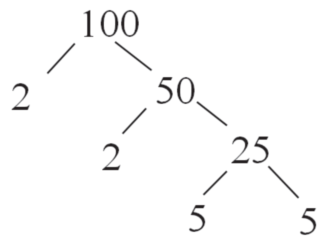
4. Complete the missing parts in this factor tree.



5. $240 = 2^4 \times 3 \times 5$

7. Use factor trees to write the prime factorisations for:

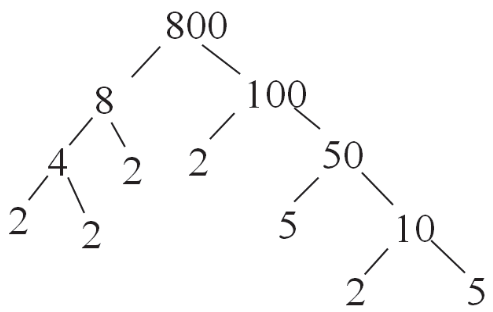
(a) 100



Others possible.

Hence: $100 = 2^2 \times 5^2$

(b) 800



Others possible.

Hence: $800 = 2^5 \times 5^2$

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