Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Tasks

TASK LIST

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

Department of Education

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Year 7 Mathematics

Multi-Strand Activity

Australian Infographic

Produced by a Department of Education - MAWA partnership project

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TASK 215: AUSTRALIAN INFOGRAPHIC

Overview

This task is designed to give students an opportunity to collect data, convert the data to percentages and display the data in an appropriate Infographic.

Students will need

- calculators
- access to the internet

Relevant content descriptions from the Western Australian Curriculum

- Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)
- Round decimals to a specified number of decimal places (AMNA156)

Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
  - express one quantity as a percentage of another
  - represent the common unit fractions as decimals as in Activity 1
- *understanding* when they
  - choose appropriate data displays to represent their data.
- *reasoning* when they
  - formulate and solve authentic problems using measurements (Question 8).

Watch the video at https://www.youtube.com/watch?v=nHhNNozJRLE

If we could shrink Australia's population to a village of precisely 100 people, with all the existing human ratios remaining the same, what would it look like?

Activity 1

1. You are going to create an infographic of what the Australian population would look like if it was *shrunk down to exactly 100 people.*

   Before you get started, you will need to use the Australian Bureau of Statistics (ABS) website to find the answers to the following questions. You can find the ABS website here: http://www.censusdata.abs.gov.au/census_services/getproduct/census/2011/quickstat/0

   Note: This census data was obtained from the 2011 Census. It is best to use more up-to-date information if possible.

   a. What was the total population of Australia in 2011?
   21 507 717

   b. How many Australians were male? How many were female?

   Male: 10 634 013
   Female: 10 873 704

   c. How many Australians were under 20? How many were between 20 and 64? Over 64?

   Under 20: 5 549 823
   20 to 64: 12 945 607
   Over 64: 3 012 289

2. Write down at least 3 more questions of your own about the Australian population.

   Check the various questions posed

3. Use the ABS website to answer your questions as well.

   Various relevant answers

4. Convert each of your answers in Questions 2 and 4 to percentages of the total population. You must show how you got your answer in the calculation column. Remember to round your answer to the nearest whole number.

Category	Number	Calculation	Percentage
2011 Aust. Population	21 507 717	n/a	100%
Males	10 634,013	10 634 013 ÷ 21 507 717 = 0.4944	49%
Females	10 873 704	10 873 704 ÷ 21 507 717 = 0.5055	51%
Under 20 years	5 549 823	5 549 823 ÷ 21 507 717 = 0.2580	26%
20 – 64 years	12 945 607	12 945 607 ÷ 21 507 717 = 0.6019	60%
Over 64 years	3 012 289	3 012 289 ÷ 21 507 717 = 0.1400	14%

5. Why did we round to the nearest whole number in Question 5?

   We are trying to find what the population would look like if it was scaled down to 100 people, so each person represents 1% of the population and you can't have a part of a person!

6. From your information, can you tell how many people are males that are under 18? Why or why not?

   No, because they are two separate measures and we don't have enough information.

7. What would be your best estimate of the number of males who are under 18? State your assumptions

   Any reasonable answer; e.g., approx. 12% of 5 549 823 or 666 000.

8. Add all your percentages. Do you get a number less or greater than 100%? How is this possible?

   If so, then this is due to rounding, and is a normal occurrence.

9. You are going to create a poster to showing what the Australian population would look like if it was shrunk to a village of exactly 100 people.
   a. Before you start your poster, perform an internet image search for "Infographics people" and collect some ideas. . Decide which sorts of graphs you will to use on your poster (e.g., bar chart, pie graph, pictogram, etc.).
   b. Draft any charts you are going to use.
   c. Design your poster
   d. Check it with your teacher and make any amendments necessary.
   e. Create your final, colourful copy of your poster.

Watch the video at https://www.youtube.com/watch?v=nHhNNozJRLE

If we could shrink Australia's population to a village of precisely 100 people, with all the existing human ratios remaining the same, what would it look like?

Activity 1

1. You are going to create an infographic of what the Australian population would look like if it was *shrunk down to exactly 100 people.*

   Before you get started, you will need to use the Australian Bureau of Statistics (ABS) website to find the answers to the following questions. You can find the ABS website here:
   http://www.censusdata.abs.gov.au/census_services/getproduct/census/2011/quickstat/0

   a. What was the total population of Australia in 2011?

   b. How many Australians were male? How many were female?

   c. How many Australians were under 20? How many were between 20 and 64? Over 64?

2. Write down at least 3 more questions of your own about the Australian population.

3. Use the ABS website to answer your questions as well.

4. Convert each of your answers in Questions 2 and 4 to percentages of the total population. You must show how you got your answer in the calculation column. Remember to round your answer to the nearest whole number.

Category	Number	Calculation	Percentage
2011 Aust. Population	21 507 717	n/a	100%

5. Why did we round to the nearest whole number in Question 4?

6. From your information, can you tell how many people are males that are under 18? Why or why not?

7. What would be your best estimate of the number of males who are under 18? State your assumptions.

8. Add all your percentages. Do you get a number less or greater than 100%? How is this possible?

9. You are to create a poster to show what the Australian population would look like if it was shrunk to a village of exactly 100 people.

   a. Before you start your poster, perform an internet image search for “Infographics people” and collect some ideas. Decide which sorts of graphs you will to use on your poster (e.g., bar chart, pie graph, pictogram, etc.)
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Year 7 Mathematics

Multi-Strand Activity

Shirley's Shapes

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 238: SHIRLEY'S SHAPES

Overview

This task is an investigation into the best piece of material to buy in order to make several different kites. Students work out areas, tessellate shapes and find the amount of wasted material before deciding on which piece of material is the best for the given situation.

Students will need

- grid paper
- calculators (optional)

Relevant content descriptions from the Western Australian Curriculum

- Establish the areas of rectangles, triangles and parallelograms and use these in problem solving (AMMG159)
- Describe translations, reflections in an axis, and rotations of multiples of 90 on the Cartesian plane using coordinates. Identify line and rotational symmetries. (ACMMG181)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)
- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

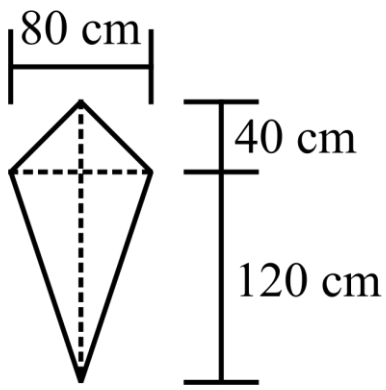
Students can demonstrate

- *fluency* when they
  - calculate accurately with integers
  - investigate best buys
- *reasoning* when they
  - apply known geometric facts to draw conclusions about shapes
- *problem solving* when they
  - solve authentic problems using numbers and measurements
  - work with transformations and symmetry

Activity 1

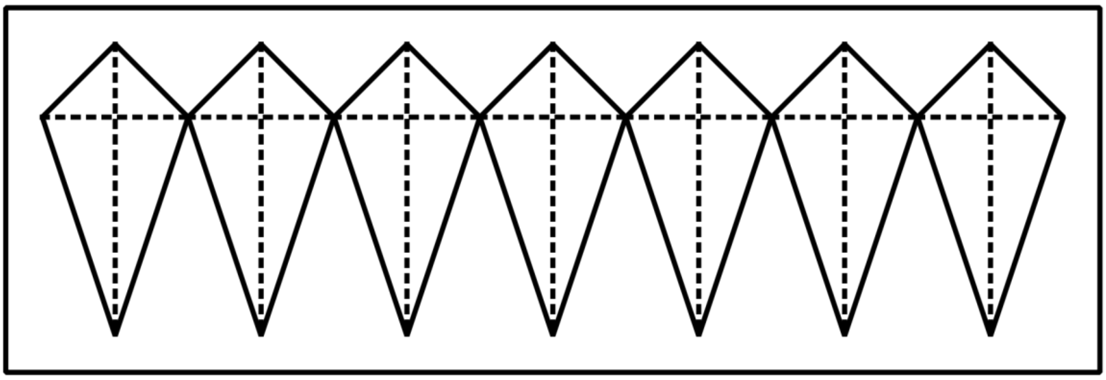
Shirley has decided to make kites as Christmas presents for all her friends.

To make a kite she is going to stretch a piece of material over two pieces of wooden dowel tied together in a cross shape. An illustration of the finished kite is shown below.



Shirley has one piece of material that is 2 m by 6 m but she is not sure which is the best way to fit the kite shapes onto her material.

One option that Shirley has come up with is shown below…

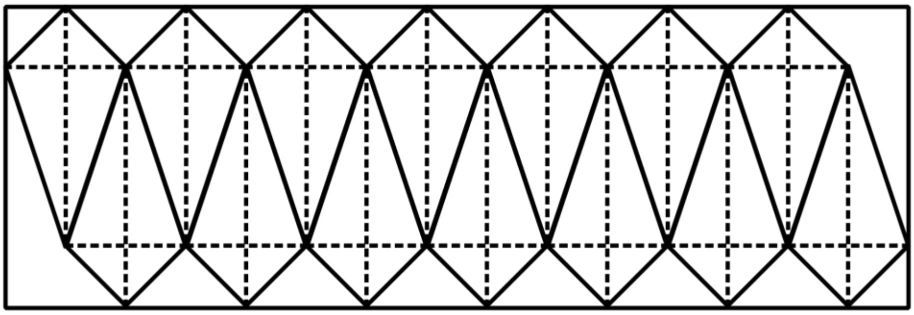


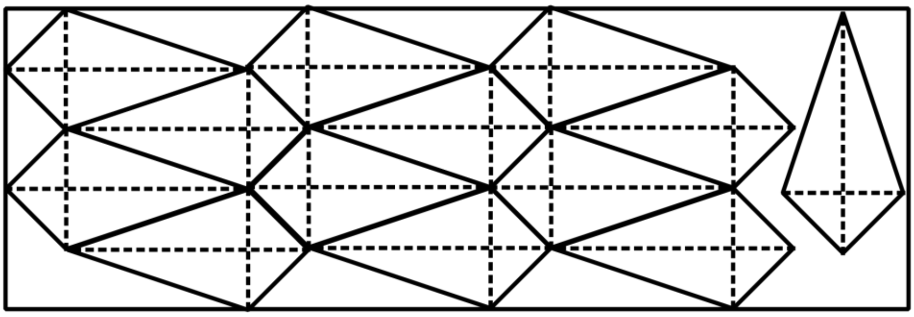
… but she thinks that there may be a better way to mark out the kites on her material.

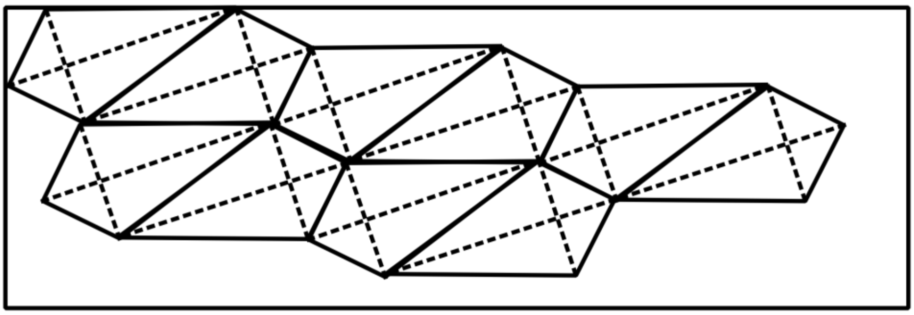
1. What is the total area of Shirley's piece of material?
   12 m^2

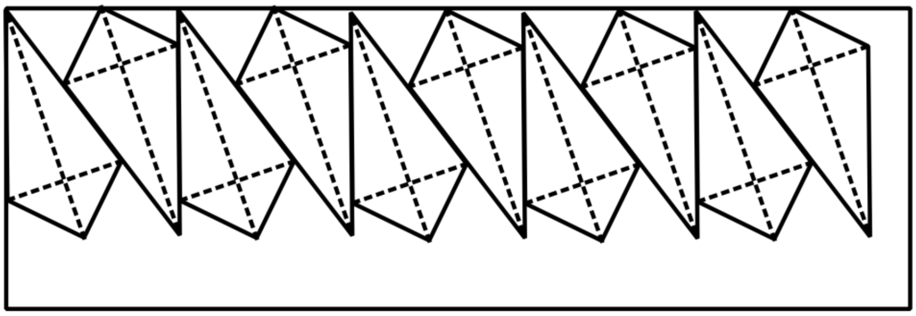
2. What is the total area of each kite in square metres?
   0.64 m^2

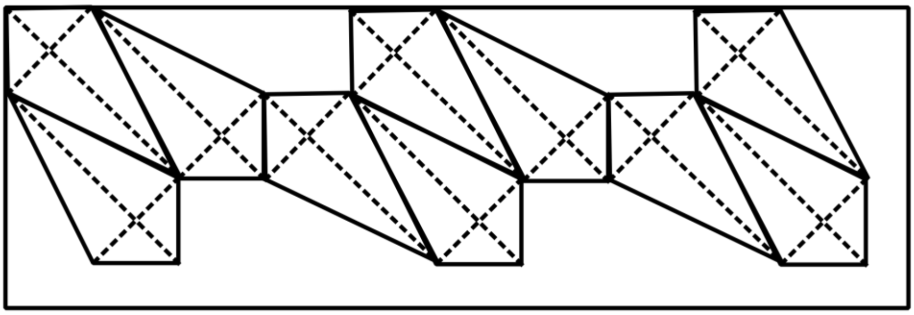
3. Using grid paper, and a scale of 1:10, draw as many different ways of laying out the kites as possible.

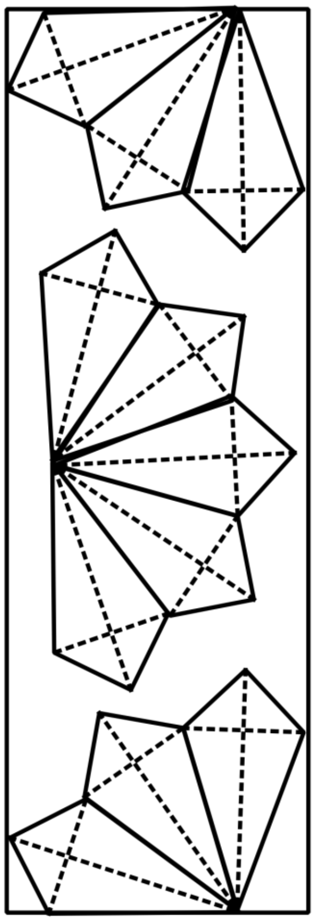












4. For each design, including Shirley's original one, complete the following table.

Design #	Total Number of Kites Made	Total Area of Kites (m^2)	Total Area of Waste	Waste as a % of Total Material
Shirley's Design	7	4.48	7.52	63%
1	14	8.96	3.04	25%
2	13	8.32	3.68	31%
3	10	6.40	5.60	47%
4	10	6.40	5.60	47%
5	10	6.40	5.60	47%
6	11	7.04	4.94	41%

5. Which design would you recommend Shirley use? Why?
   Answers will vary. Here, Design Number 2 has the least wastage.

6. Shirley's first set of kites were so popular, she has decided to make some more. She has enough money to buy 12 m^2 and she can buy the material in 1.2 m, 2.4 m or 3 m widths.
   What would the dimensions be if Shirley bought -
   a. The 1.2 m fabric?
      1.2 m x 10 m

   b. The 2.4 m fabric?
      2.4 m x 5 m

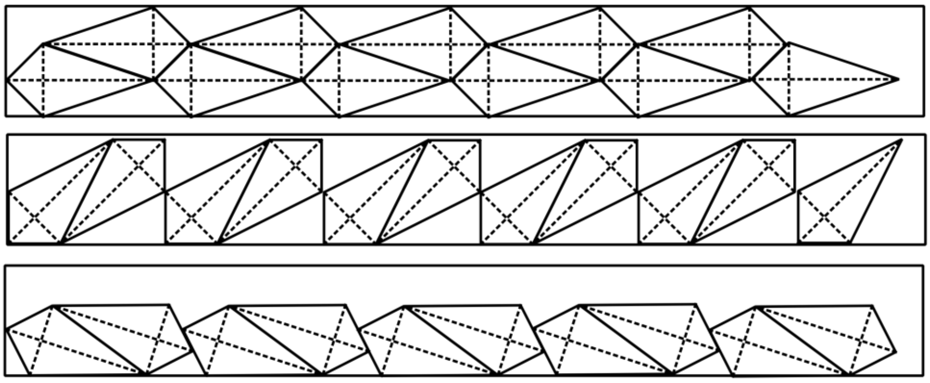
   c. The 3 m fabric?
      3 m x 4 m

7. Which piece of fabric should Shirley buy? For each piece of fabric identified in Question 6 -
   a. Use grid paper and a scale of 1:40, and draw as many different ways of laying out the kites as possible.

   b. For each of the designs in part (a), complete a table with the following headings.

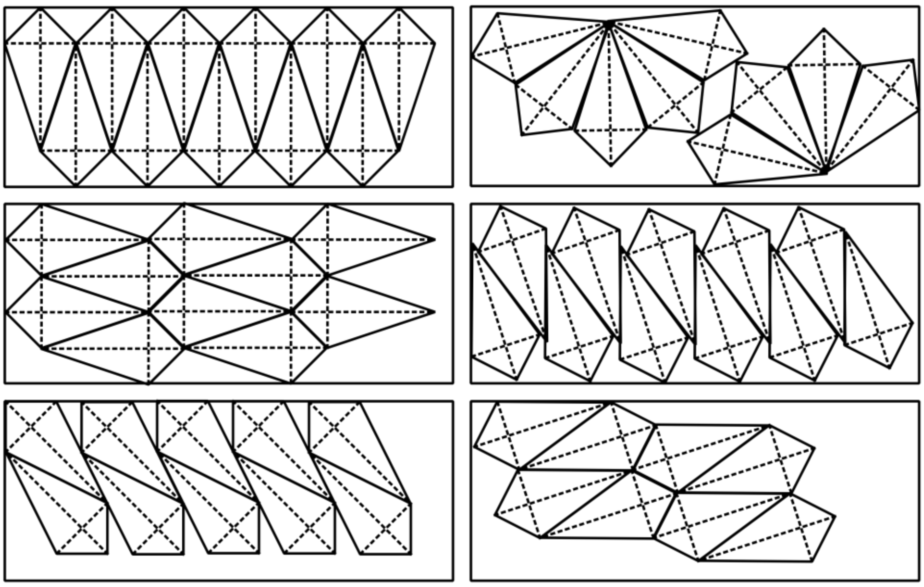
Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material

1.2 m material



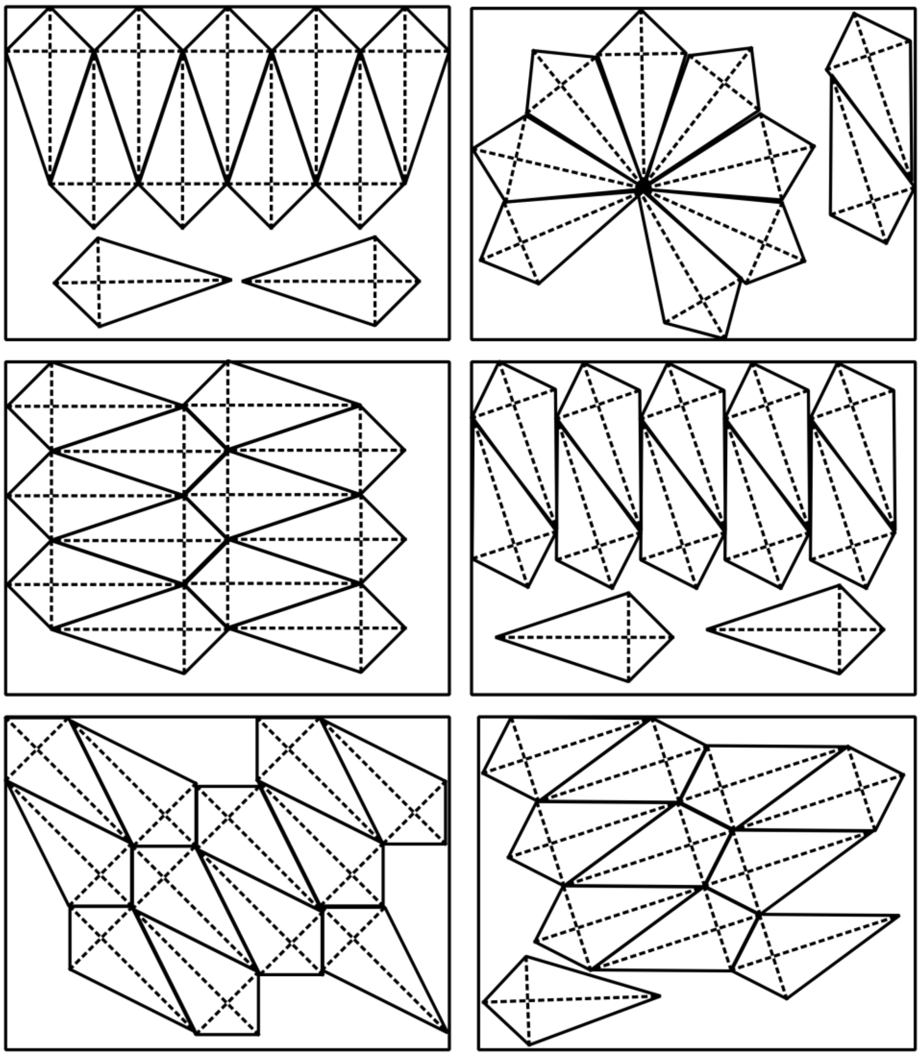
Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material
1	11	7.04	4.94	41%
2	11	7.04	4.94	41%
3	10	6.4	5.6	47%

2.4 m material



Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material
4	11	7.04	4.94	41%
5	10	6.4	5.6	47%
6	10	6.4	5.6	47%
7	11	7.04	4.94	41%
8	10	6.4	5.6	47%
9	8	5.12	6.88	57%

3 m material



Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material
10	11	7.04	4.94	41%
11	10	6.4	5.6	47%
12	12	7.68	4.32	36%
13	12	7.68	4.32	36%
14	12	7.68	4.32	36%
15	11	7.04	4.94	41%

c. Based on your research, make a recommendation to Shirley about which piece of material she should buy.

Shirley should buy the 3 x 4 m piece of material, as she can make the most kites/have the least wastage with this piece.

However, if she can find another store that supplies 2 m wide material, 2 m by 6 m is even better.

Activity 1

Shirley has decided to make kites as Christmas presents for all her friends.

To make a kite she is going to stretch a piece of material over two pieces of wooden dowel tied together in a cross shape. An illustration of the finished kite is shown below.

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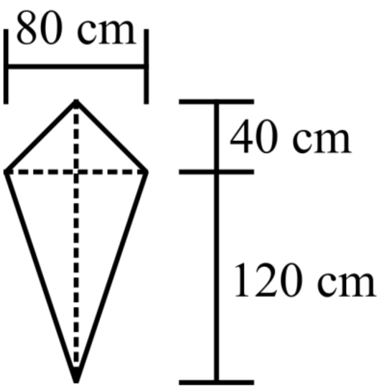
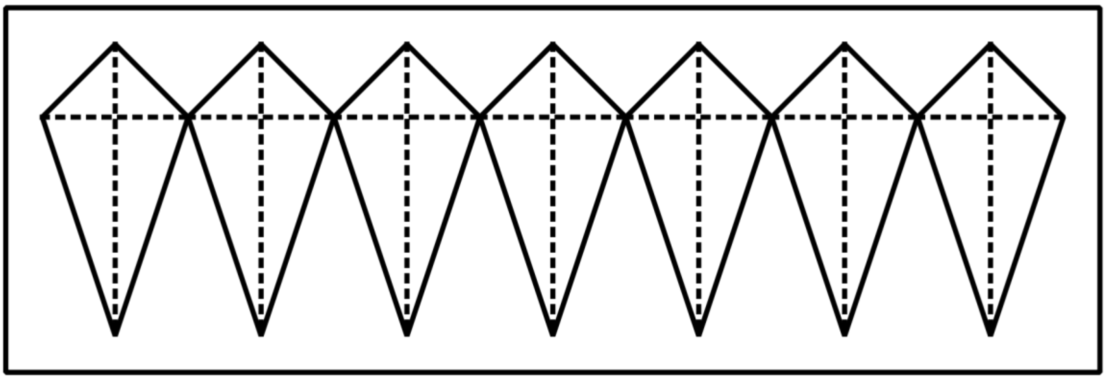


Shirley has one piece of material that is 2m by 6m but she is not sure which is the best way to fit the kite shapes onto her material.

One option that Shirley has come up with is shown below…



… but she thinks that there may be a better way to mark out the kites on her material.

1. What is the total area of Shirley's piece of material?

2. What is the total area of each kite in square metres?

3. Using grid paper, and a scale of 1:40, draw as many different ways of laying out the kites as possible.

4. For each of your designs, complete the following table.

Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material

5. Which design would you recommend Shirley use? Why?

6. Shirley's first set of kites was so popular, she has decided to make some more. She has enough money to buy 12 m^2 and she can buy the material in 1.2 m, 2.4 m or 3 m widths.

What would the dimensions be if Shirley bought -

a. The 1.2 m fabric?

b. The 2.4 m fabric?

c. The 3 m fabric?

7. Which piece of fabric should Shirley buy? For each piece of fabric identified in Question 6 -

a. Use grid paper and a scale of 1:40, and draw as many different ways of laying out the kites as possible.

b. For each of the designs in part (a), complete a table with the following headings.

Design #	Total Number of Kites Made	Total Area of Kites	Total Area of Waste	Waste as a % of Total Material

c. Based on your research, make a recommendation to Shirley about which piece of material she should buy.

Department of Education

The Mathematical Association of Western Australia Inc.

MAΣWA

Year 7 Mathematics

Multi-Strand Activity

Bank Notes

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 3: BANK NOTES

Overview

In this task students are asked to consider the following question:

If the Australian government was to introduce a $200 note, what might its dimensions be?

Using an investigative process, students are directed to obtain information about Australian bank notes currently in circulation. Information can be obtained from the website for the Reserve Bank of Australia.

http://banknotes.rba.gov.au/banknote-features/#!note/5

Details are also provided in the solutions.

Students will need

- rulers
- access to the internet

Relevant content descriptors from the Western Australian Curriculum

- Investigate, interpret and analyse graphs from authentic data (ACMNA180)
- Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)

Students can demonstrate

- reasoning when they
  - explain the pattern of widths and lengths of bank notes
  - interpret the displays of plotted values
- problem solving when they
  - can use the measurements given to identify the pattern
  - can apply the data collected to solve the problem posed

Use the activities provided to assist you to answer the following question:

If the Australian government was to introduce a $200 note, what might its dimensions be?

<u>Activity 1</u>

Using Australian banknotes or the internet, locate the information needed to complete the table.

Width and length of the Australian bank notes

Note	*$5*	*$10*	*$20*	*$50*	*$100*
Width (mm)	*130*	*137*	*144*	*151*	*158*
Length (mm)	*65*	*65*	*65*	*65*	*65*

<u>Activity 2</u>

1. Write a sentence to describe how the lengths of banknotes change as their value increases.

   As the value of the note increases, the length stays the same (65 mm).

   Each note has the same length regardless of its value.

2. Write a sentence to describe how the widths of banknotes change as their value increases.

   As the value increases the width increases by 7 mm.

3. Complete the table below. On the first row, enter the amounts by which the notes increase in value from left to right. On the second row, record the number of times the second note increases in value from the one before it.

Note	*$5*	*$10*	*$20*	*$50*	*$100*	
Amount of increase		*$5*	*$10*	*$30*	*$50*	
Multiple of increase		*twice*	*twice*	*2.5 times*	*twice*	

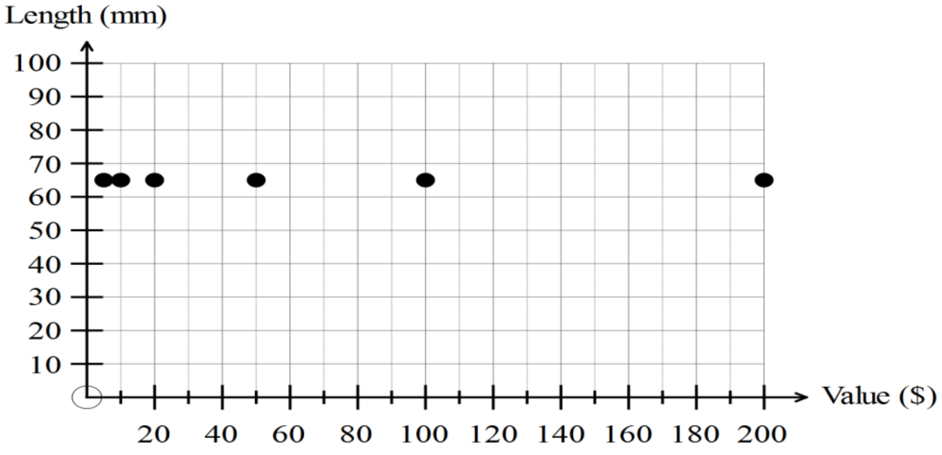
4. Identify and describe any patterns in the increases in the table in Question 3.

   The increase in value is not constant. There is no pattern of increase.

   Mostly, but not always, each note is twice the value of the one before.

Activity 3

1. Using the data from Activity 1, plot the points showing the relationship between length and the value of the note.

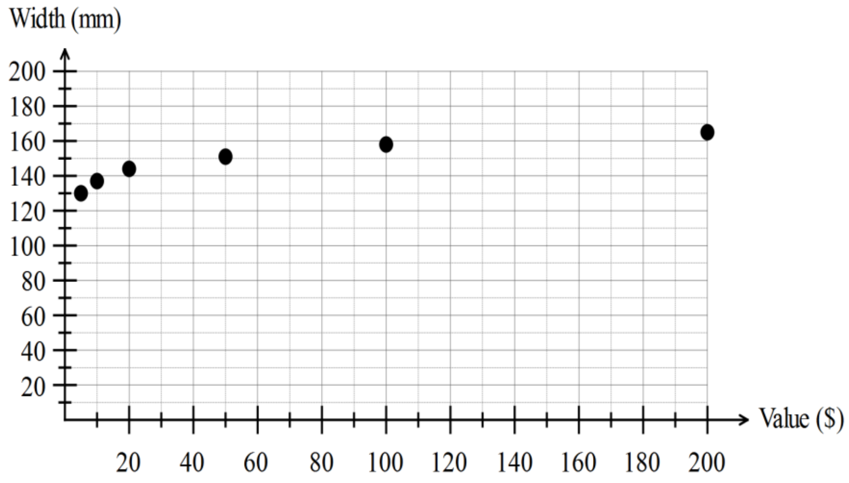


2. Describe the pattern seen in the locations of these points.

   The points are in a line parallel to the horizontal axis.

3. Plot the points showing the relationship between width and the value of the note.

   See graph below



4. Describe the pattern seen in the locations of these points.

   The points are rising slowly as the value increases.

Activity 4

1. Use the results of your investigation to suggest an answer to the original question.

If the Australian government was to introduce a $200 note, what might its dimensions be?

Give reasons for your decision.

It should be 65 mm long because all bank notes are so long.

It should be 165 mm wide because that is also a 7 mm increase in width.

2. Extend the graphs above and add the points represented by your answer to the previous question.

   See extra points on graphs in Activity 3.

3. If instead on introducing a $200 note, the government decided to introduce an $80 note, what might its dimensions be? Would the pattern be retained?

   65 mm long.

   The width is a problem because you cannot go 7 mm up from the $50 note or 7 mm down from the $100 note. Anything in between changes the pattern.

Use the activities provided to assist you to answer the following question:

If the Australian government was to introduce a $200 note, what might its dimensions be?

Activity 1

Using Australian banknotes or the internet, locate the information needed to complete the table.

Width and length of the Australian bank notes

Note	*$5*	*$10*	*$20*	*$50*	*$100*
Width (mm)					
Length (mm)					

Activity 2

1. Write a sentence to describe how the lengths of banknotes change as their value increases.

2. Write a sentence to describe how the widths of banknotes change as their value increases.

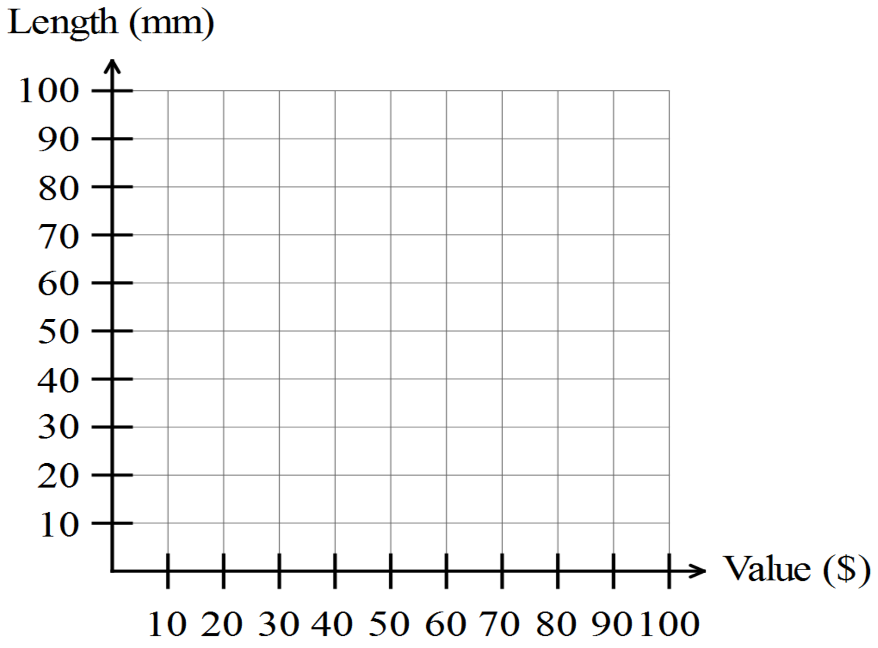
3. Complete the table below. On the first row, enter the amounts by which the notes increase in value from left to right. On the second row, record the number of times the second note increases in value from the one before it.

Note	*$5*	*$10*	*$20*	*$50*	*$100*
Amount of increase		*$5*			
Multiple of increase		*twice*			

4. Identify and describe any patterns in the increases in the table in Question 3.

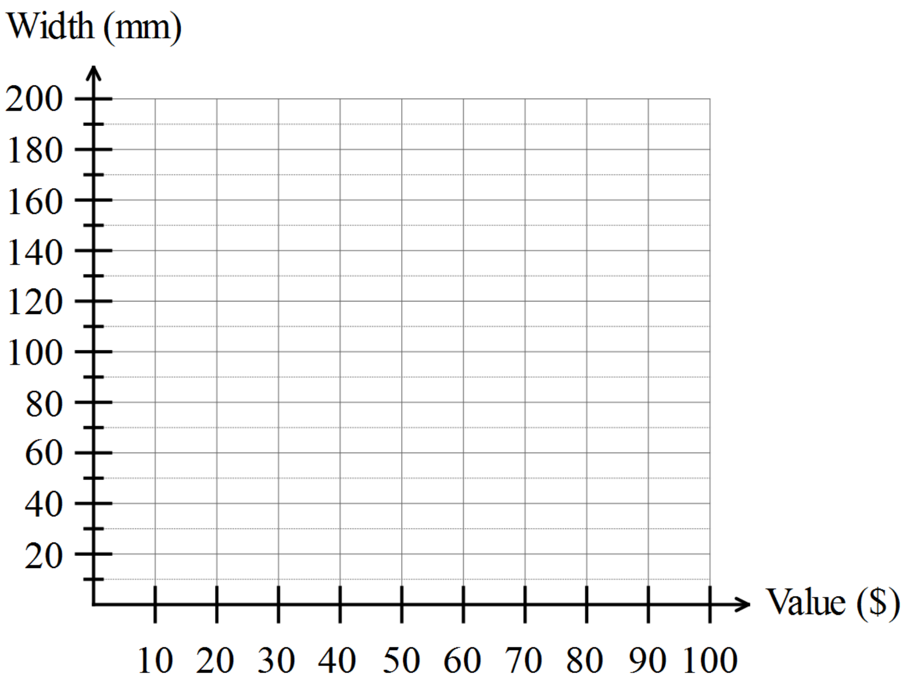
Activity 3

1. Using the data from Activity 1, plot the points showing the relationship between length and the value of the note.



2. Describe the pattern seen in the locations of these points.

3. Plot the points showing the relationship between width and the value of the note.



4. Describe the pattern seen in the locations of these points.

Activity 4

1. Use the results of your investigation to suggest an answer to the original question.

If the Australian government was to introduce a $200 note, what might its dimensions be?

Give reasons for your decision.

2. Extend the graphs above and add the points represented by your answer to the previous question.

3. If instead on introducing a $200 note, the government decided to introduce an $80 note, what might its dimensions be? Would the pattern be retained?

Department of Education

YEAR 7 MATHEMATICS

Multi-Strand Activity

Rectangles

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 6: RECTANGLES

Overview

In this task students are encouraged to firstly review their understanding of the basic features of a rectangle. The purpose of this task is to support students in their development of the formula for the area of a rectangle and to encourage them to use algebra to formulate perimeter and area.

Students will need

- calculators

Relevant content descriptions from the Western Australian Curriculum

- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Students can demonstrate

- *understanding* when they
    - determine algebraic expressions for perimeter and area as in Activities 3 and 4.
- *reasoning* when they
    - apply their knowledge of rectangles to justify which shapes are not rectangles as in Activity 1
- *problem solving* when they
    - interpret the diagram of the tennis court and answer the questions posed in Activity 5

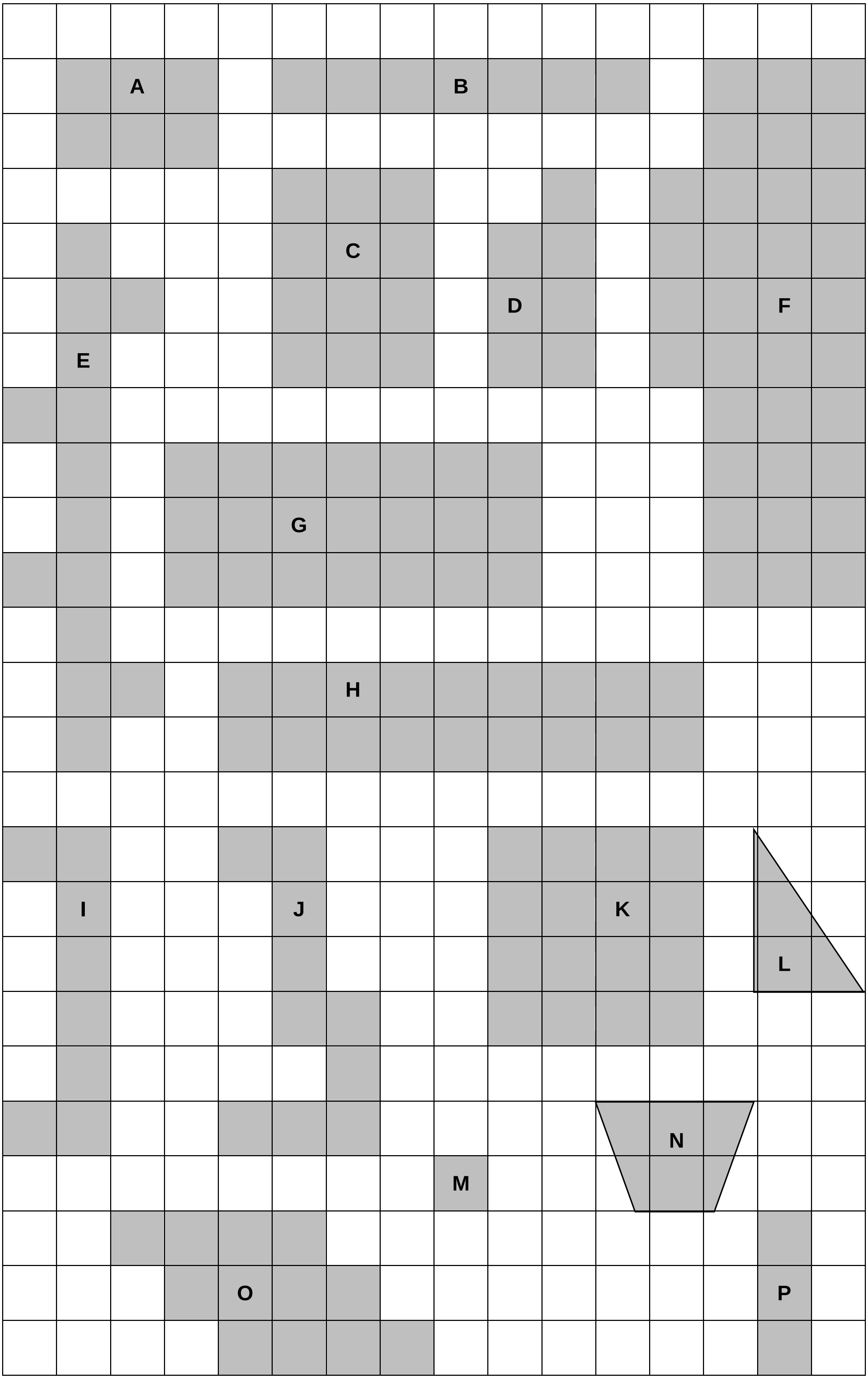
Activity 1: Review of rectangle properties

On the following page there are several 2-D figures. For each figure, indicate in the table if the figure is a rectangle. If the figure is not a rectangle give a reason in the final column.

	Rectangle?	If relevant, give reasons why the shape is not a rectangle.
A	YES	
B	YES	
C	YES	
D	NO	It has more than 4 sides.
E	NO	It has more than 4 sides.
F	NO	It has more than 4 sides.
G	YES	
H	YES	
I	NO	It has more than 4 sides.
J	NO	It has more than 4 sides.
K	YES	
L	NO	It has only 3 sides.
M	YES	
N	NO	The inside angles are not 90 degrees.
O	NO	It has more than 4 sides.
P	YES	

Name three features you could use to decide if a 2-D shape is a rectangle.

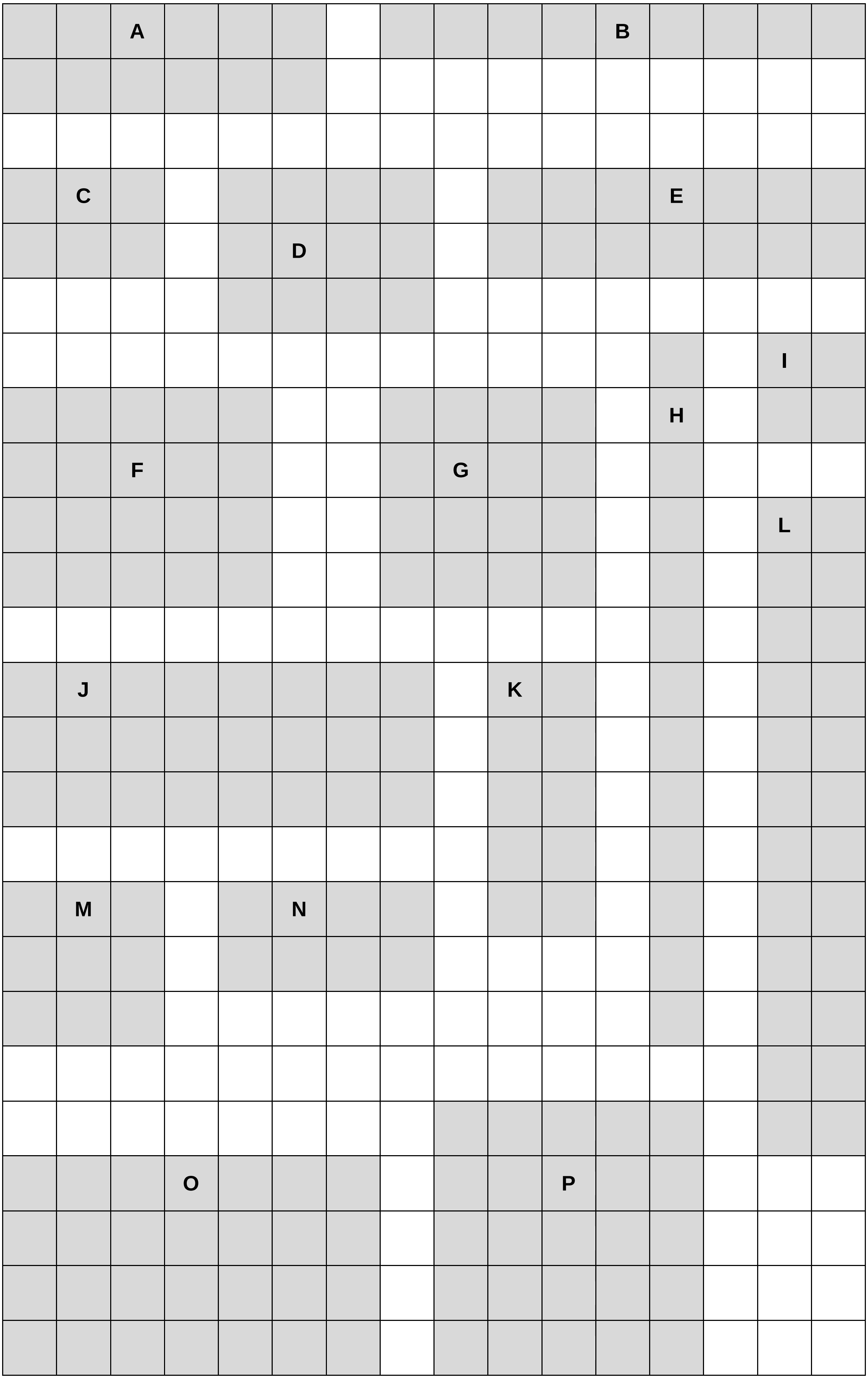
1. The shape has 4 sides.
2. The shape is closed.
3. All the inside angles are 90 degrees


A
B
C
D
F
E
G
H
I
J
K
L
N
M
O
P

Activity 2

Use the rectangles on the following page to complete the table summarising the dimensions of these rectangles. On the grid each small square is 1cm by 1 cm.

	length (cm)	width (cm)	perimeter (cm) (distance around the outside of the shape)	area (cm^2) (number of small squares inside the rectangle)
A	6	2	16	12
B	9	1	20	9
C	3	2	10	6
D	4	3	14	12
E	7	2	18	14
F	5	4	18	20
G	4	4	16	16
H	13	1	28	13
I	2	2	8	4
J	8	3	22	24
K	5	2	14	10
L	12	2	28	24
M	3	3	12	9
N	4	2	12	8
O	7	4	22	28
P	5	5	20	25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Activity 3

Consider the perimeter of rectangles.

1. Describe in words what you need to do to calculate the perimeter of any rectangle.

   Add the lengths of all the sides of the rectangle.

   Add the length to the width (breadth) and then double your answer.

   Double the length, double the width and add these two doubles.

2. Use your method for the previous question to determine the perimeter of -

   (a) a rectangular piece of paper which is 25 cm wide and 15 cm long

   25 + 15 + 25 + 15 = 80 cm

   (b) a rectangular path which is 100 cm wide and 550 cm long.

   100 X 2 + 550 X 2 = 200 + 1100 = 1300 cm

3. Using the symbol w to represent the width of any rectangle and l to represent the length of any rectangle, write a rule for P, the perimeter.

   $P = 2 \times w + 2 \times l$

4. Write the rule from Question 3 in another way.

   $P = 2 \times (w + l)$

   $P = 2w + 2l$

5. Show the use of one of your rules from Question 3 or 4 to determine the perimeter of the following rectangular shapes.

   (a) a rectangular piece of paper which is 29 cm wide and 19 cm long.

   $P = 2 \times w + 2 \times l$
   = 2 x 29 + 2 x 19
   = 58 + 38
   = 96 cm

   (b) a rectangular path which is 150 cm wide and 650 cm long.

   $P = 2 \times w + 2 \times l$
   = 2 x 150 + 2 x 650
   = 300 + 1300
   = 1600 cm

6. Use your rule for P from Question 3 to write an algebraic expression when the perimeter is 50 cm and the length and width are not known (they are represented by l and w).

$50 = 2 \times w + 2 \times l$

Activity 4

Consider the area of rectangles.

1. Write down what you understand by area.

The area measures coverage of the surface within the boundaries of the shape.

2. Describe in words what you need to do to calculate the area of any rectangle.

Multiply the length by the width (breadth).

3. Use your method for the previous question to determine the area of -

(a) a rectangular piece of paper which is 20 cm wide and 15 cm long

20 cm x 15 cm = 300 cm^2

(b) a rectangular path which is 1 m wide and 5.5 m long.

1 x 5.5 = 5.5 m^2

4. Using the symbol w to represent the width of any rectangle and l to represent the length of any rectangle, write a rule for A, the area.

$A = w \times l$

5. Write the rule from Question 4 in another way.

$A = wl$ *or* $A = l \times w$ *or* $A = lw$

6. Show the use of one of your rules from Question 4 or 5 to determine the area of the following rectangular shapes

(a) a rectangular piece of paper which is 25 cm wide and 20 cm long.

$A = w \times l$
$= 25 \times 20$
$= 500\ m^2$

(b) a rectangular path which is 1 m wide and 6.5 m long.

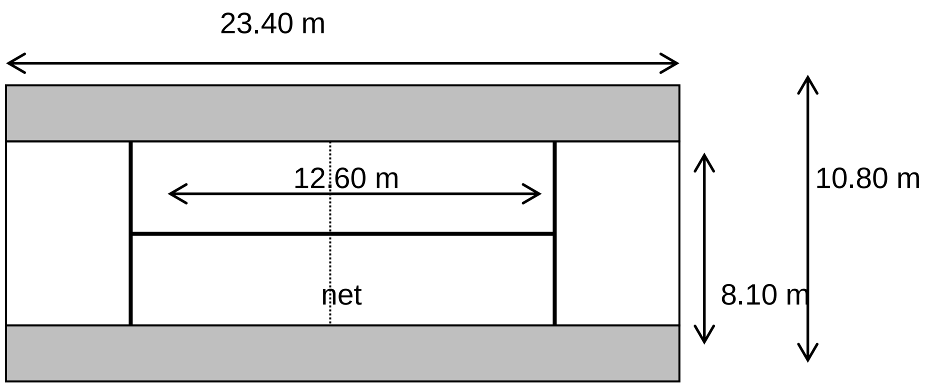
$A = w \times l$
$= 1 \times 6.5$
$= 6.5\ \text{m}^2$

7. Use your rule for A from Question 3 to write an algebraic expression when the area is 50 cm^2 and the length and width are not known (they are represented by l and w).

$50 = w \times l$

Activity 5

When people play singles in tennis they do not use the tramlines but in doubles the tramlines are used. The diagram of a tennis court is provided below and the area shaded is referred to as the tramlines. There are lines on the court as shown on the diagram below except at the net.



1. Determine the area of the singles court.

Area = width x length
= 8.1 x 23.4
= 189.54 m^2

2. How much larger is the doubles court than the singles court?

The simplest method is to find the area of the tramlines, since this represents the difference. Students may also find the total area of each and then find the difference.

Width = 10.8 – 8.1
= 2.7 m
Area = width x length
= 2.7 x 23.4
= 63.18 m^2

3. It costs $60 per m^2 to cover the doubles court. What would be the total cost of covering the doubles court?

Cost = number of m^2 x 60
= (23.4 x 10.8) x 60
= 252.72 x 60
= $15 163.20

4. It costs $5 per metre to paint the lines on the court. What is the total cost of painting all the lines?

Total of all lines = 23.4 x 4 + 10.8 x 2 + 12.6 + 2 x 8.1 = 144 m
Cost = 144 x $5 = $720

Activity 1: Review of rectangle properties

On the following page there are several 2-D figures. For each figure, indicate in the table if the figure is a rectangle. If the figure is not a rectangle give a reason in the final column.

	Rectangle?	If relevant, give reasons why the shape is not a rectangle.
A		
B		
C		
D		
E		
F		
G		
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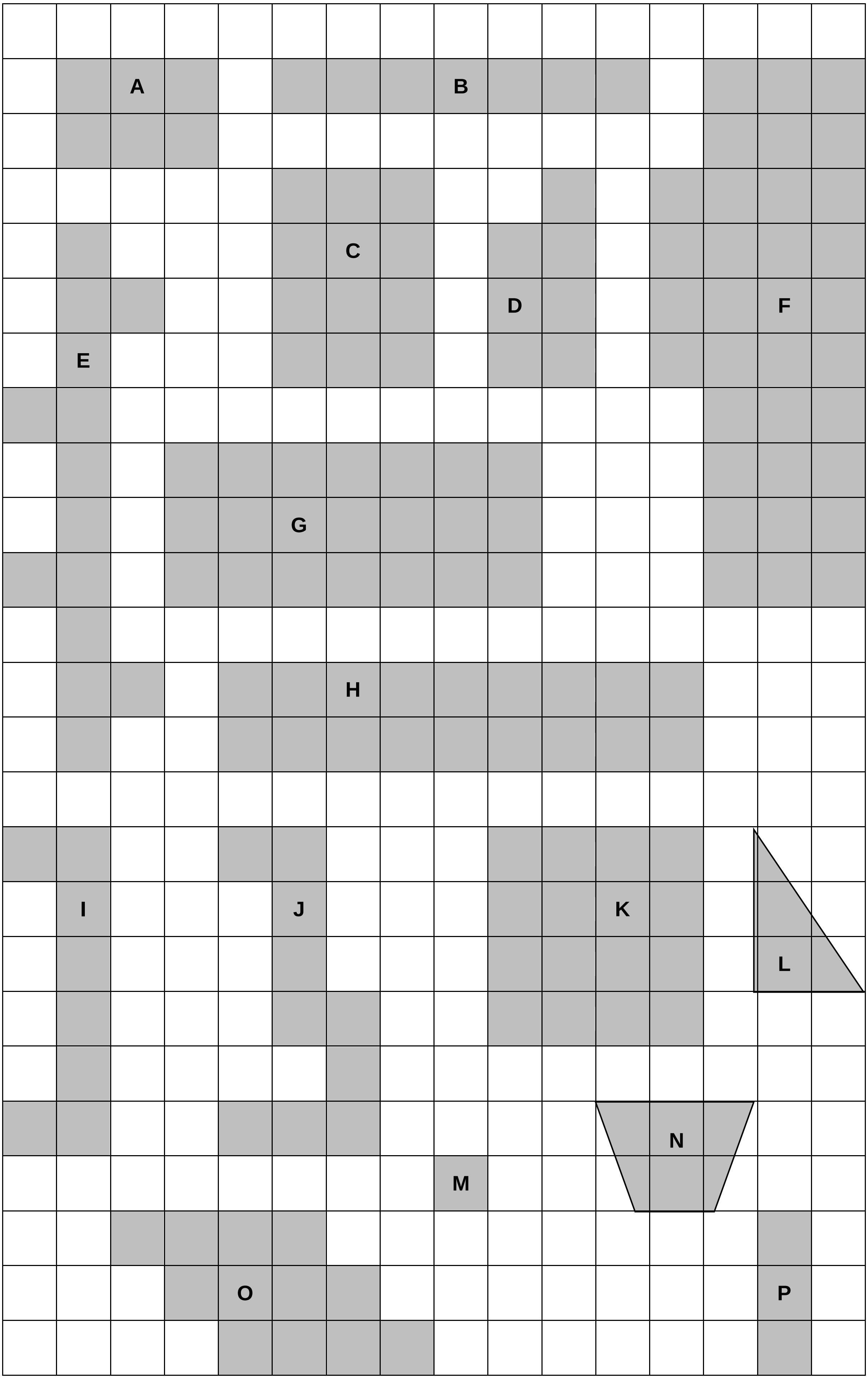
O		
P		

Name three features you could use to decide if a 2-D shape is a rectangle.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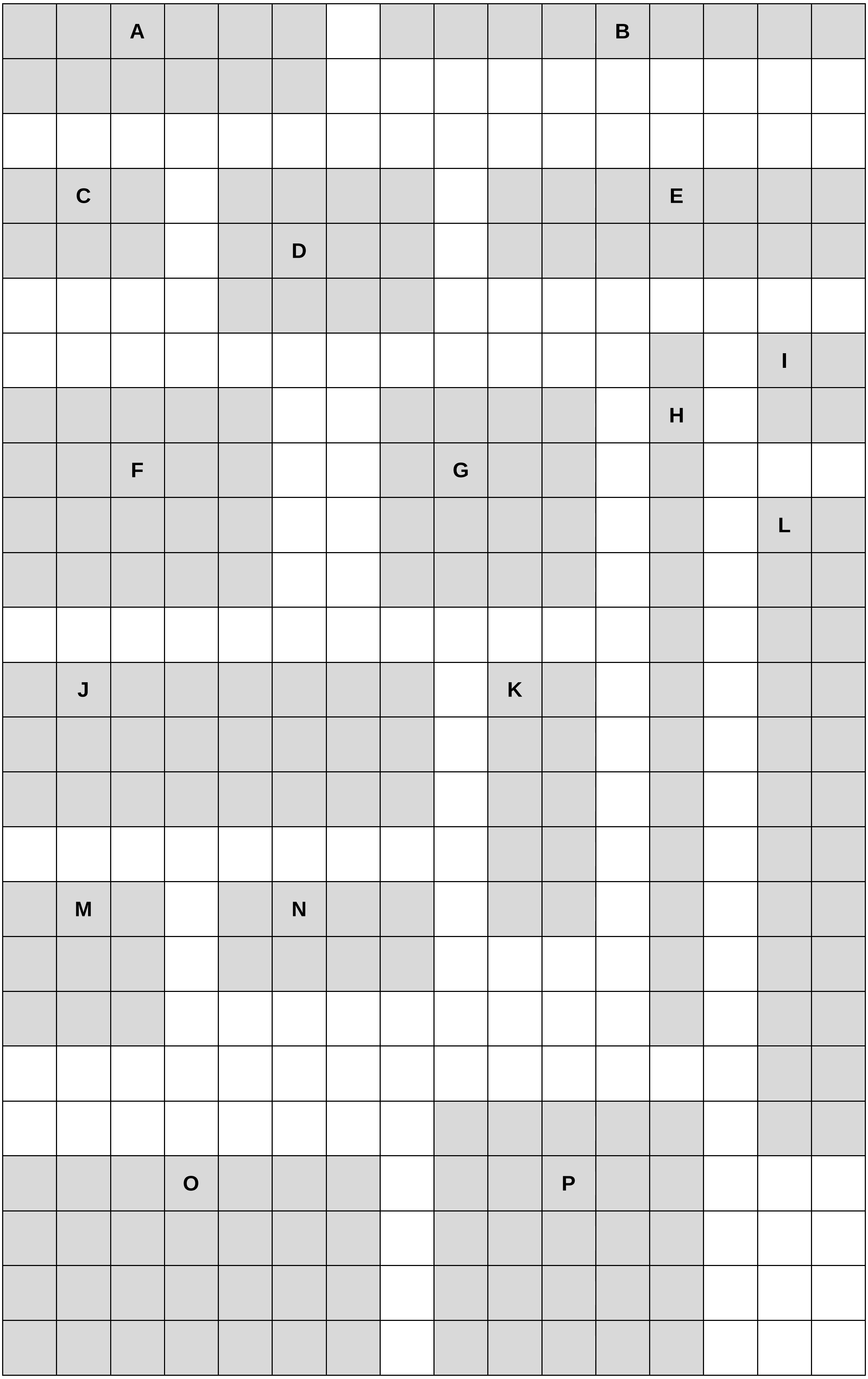
Activity 2

Use the rectangles on the following page to complete the table by summarising the dimensions of these rectangles. On the grid each small square is 1cm by 1 cm.

	length (cm)	width (cm)	perimeter (cm) (distance around the outside of the shape)	area (cm^2) (number of small squares inside the rectangle)
A	6	2	16	12
B				
C				
D				
E				
F				
G				
H				
I				
J				
K				
L				
M				
N				
O				
P				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Activity 3

Consider the perimeter of rectangles.

1. Describe in words what you need to do to calculate the perimeter of any rectangle.

2. Use your method for the previous question to determine the perimeter of -

   (a) a rectangular piece of paper which is 25 cm wide and 15 cm long

   (b) a rectangular path which is 100 cm wide and 550 cm long.

3. Using the symbol w to represent the width of any rectangle and l to represent the length of any rectangle, write a rule for P, the perimeter.

4. Write the rule from Question 3 in another way.

5. Show the use of one of your rules from Question 3 or 4 to determine the perimeter of the following rectangular shapes.

   (a) a rectangular piece of paper which is 29 cm wide and 19 cm long.

   (b) a rectangular path which is 150 cm wide and 650 cm long.

6. Use your rule for P from Question 3 to write an algebraic expression when the perimeter is 50 cm and the length and width are not known (they are represented by l and w).

<u>Activity 4</u>

Consider the area of rectangles.

1. Write down what you understand by area.

2. Describe in words what you need to do to calculate the area of any rectangle.

3. Use your method for the previous question to determine the area of

   (a) a rectangular piece of paper which is 20 cm wide and 15 cm long.

   (b) a rectangular path which is 1 m wide and 5.5 m long.

4. Using the symbol w to represent the width of any rectangle and l to represent the length of any rectangle, write a rule for A, the area.

5. Write the rule from Question 4 in another way.

6. Show the use of one of your rules from Question 4 or 5 to determine the area of the following rectangular shapes.

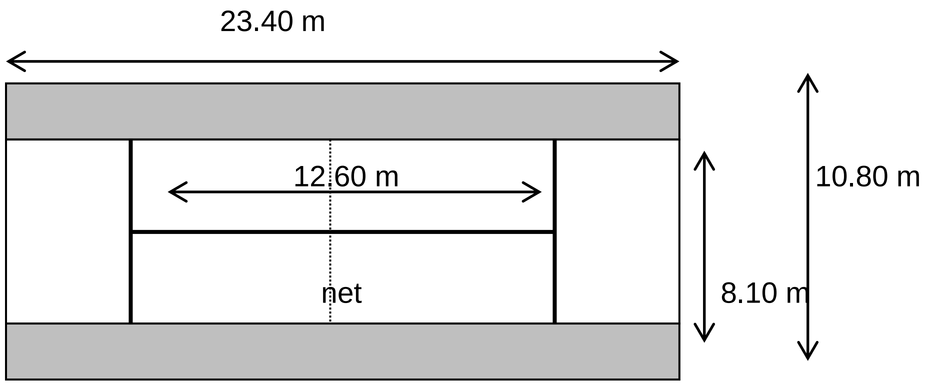
   (a) a rectangular piece of paper which is 25 cm wide and 20 cm long.

   (b) a rectangular path which is 1 m wide and 6.5 m long.

7. Use your rule for A from Question 3 to write an algebraic expression when the area is 50 cm^2 and the length and width are not known (they are represented by l and w)

Activity 5

When people play singles in tennis they do not use the tramlines but in doubles the tramlines are used. The diagram of a tennis court is provided below and the area shaded is referred to as the tramlines. There are lines on the court as shown on the diagram below except at the net.



1. Determine the area of the singles court.

2. How much larger is the doubles court than the singles court?

3. It costs $60 per m^2 to cover the doubles court. What would be the total cost of covering the doubles court?

4. It costs $5 per metre to paint the lines on the court. What is the total cost of painting all the lines?

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Maximum Area

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 7: MAXIMUM AREA

Overview

In this task students will examine the dimensions of rectangles with fixed perimeters to determine when the area is a maximum. It is assumed that students are able to calculate the area and perimeter of rectangles and this task should help their consolidation of these concepts.

Students may need

- calculators
- extra grid paper (1 cm x 1 cm)

Relevant content descriptions from the Western Australian Curriculum

- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Students can demonstrate

- *fluency* when they
  - calculate areas of rectangles
  - can symbolically represent the rules they have described
- *understanding* when they
  - describe patterns seen in the tables they create
  - can symbolically represent the rules they have described
  - connect the relationship between maximum area and rectangle dimensions

Introduction

As part of this investigation, you are given a fixed perimeter for a rectangle and will need to use this to create different rectangles and calculate their area. Unless indicated otherwise, all lengths and widths should be whole numbers.

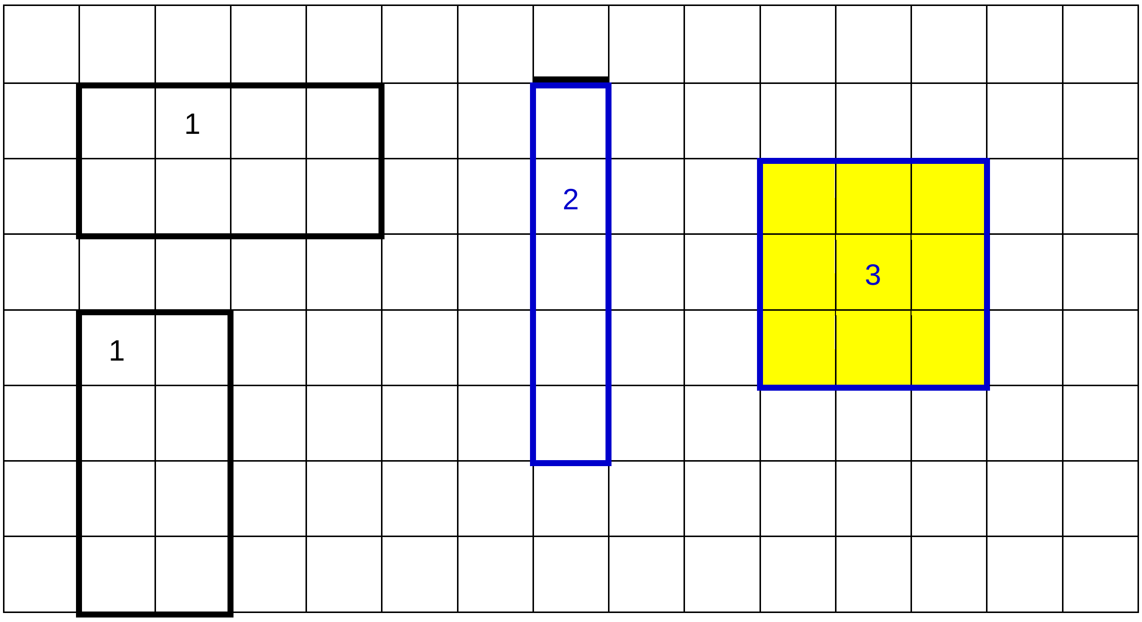
A rectangle which is 2 cm wide and 3 cm long can be rotated to face the other direction so that it looks to be 3 cm wide and 2 cm long. In this investigation, these are considered to "be the same".

Activity 1

On the grid provided each row is 1 cm high and each column is 1 cm wide. Two rectangles with a perimeter of 12 cm are drawn on the grid and they are "the same". Draw two other different rectangles with perimeters of 12 cm. Remember that the widths and lengths must be whole numbers.

Complete the table below.

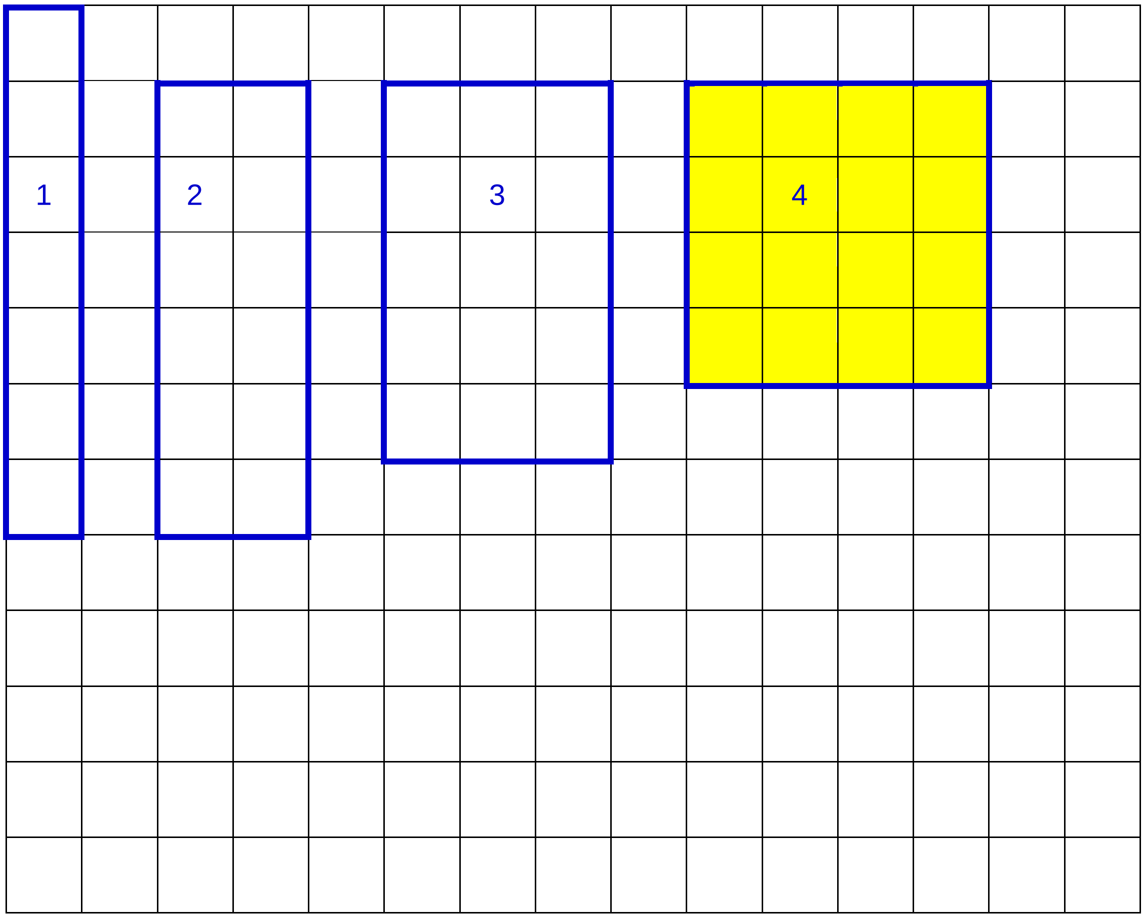
rectangle	width	length	perimeter	area
1	2 cm	4 cm	12 cm	8 cm^2
2	5 cm	1 cm	12 cm	5 cm^2
3	3 cm	3 cm	12 cm	9 cm^2



Highlight the rectangle with the greatest area.

Activity 2

On the grid provided each row is 1 cm high and each column is 1 cm wide. Draw as many different rectangles as you can, all with a perimeter of 16 cm. Remember that the widths and lengths must be whole numbers and if a new rectangle is made by moving another one around then they are considered the same.



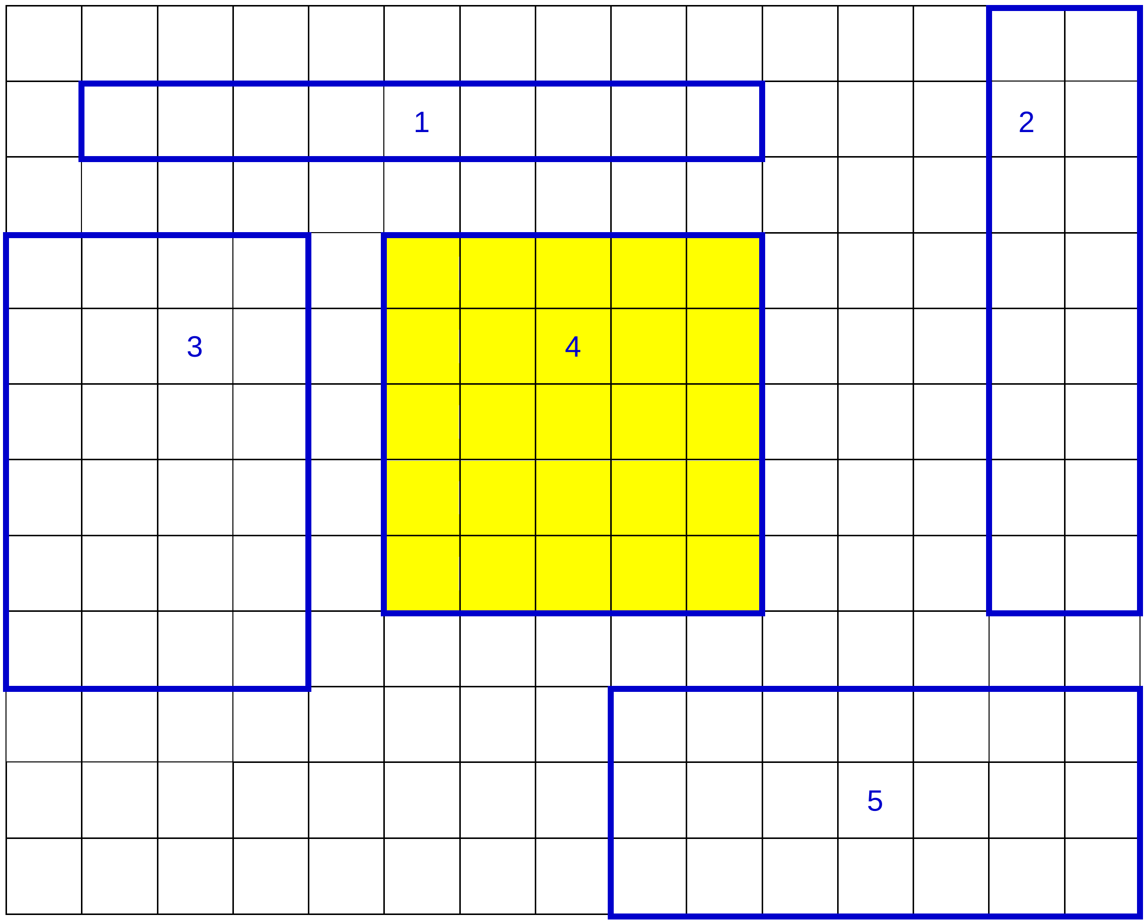
Complete the table below.

rectangle	width	length	perimeter	area
1	7 cm	1 cm	16 cm	7 cm^2
2	6 cm	2 cm	16 cm	12 cm^2
3	5 cm	3 cm	16 cm	15 cm^2
4	4 cm	4 cm	16 cm	16 cm^2

Highlight the rectangle with the greatest area.

Activity 3

On the grid provided each row is 1 cm high and each column is 1 cm wide. Draw as many different rectangles as you can, all with a perimeter of 20 cm. Remember that the widths and lengths must be whole numbers and, if a new rectangle is made by moving one around, then they are considered the same.



Complete the table below.

rectangle	width	length	perimeter	area
1	1 cm	9 cm	20 cm	9 cm^2
2	8 cm	2 cm	20 cm	16 cm^2
3	6 cm	4 cm	20 cm	24 cm^2
4	5 cm	5 cm	20 cm	25 cm^2
5	3 cm	7 cm	20 cm	21 cm^2

Highlight the rectangle with the greatest area.

Activity 4

Consider a rectangle with a perimeter of 24 cm. Examine the patterns of width and length in the tables for Activities 1-3 and use these patterns to complete the table below to list as many different rectangles as possible. If necessary, draw your rectangles on grid paper.

rectangle	width	length	perimeter	area
1	1 cm	11 cm	24 cm	11 cm^2
2	2 cm	10 cm	24 cm	20 cm^2
3	3 cm	9 cm	24 cm	27 cm^2
4	4 cm	8 cm	24 cm	32 cm^2
5	5 cm	7 cm	24 cm	35 cm^2
6	6 cm	6 cm	24 cm	36 cm^2

Highlight the rectangle with the greatest area.

What do you notice about the dimensions of the rectangle with the greatest area?

The width and the length are the same.

Activity 5

1. Does your observation apply to the rectangles drawn in Activities 1 to 3? Yes

2. Write a sentence to summarise this observation for all rectangles.

   The rectangle with the greatest area for a fixed perimeter has sides all equal; it is a square.

3. What would be the maximum possible area for a rectangle with a perimeter of -

   (a) 28 cm?
   49 cm^2

   (b) 32 cm?
   64 cm^2

4. Describe how you can determine the maximum possible area that a rectangle could have if given the perimeter.

   Firstly you divide the perimeter by 4; this gives the length of the side of the square. Then, you square this result for the area.

5. If A represents the area of a rectangle and P represents the perimeter, write an algebraic expression for -

   (a) the length of the rectangle with the greatest possible area

   Length = –

   (b) the greatest possible area for the rectangle with the perimeter of P.

   Area = $\left(\frac{P}{4}\right)^2$

6. Use the rules you have developed in item 5 above to determine the dimensions (length, width and area) of the rectangle with the greatest area when the perimeter is -

   (a) 80 m

   $$Area = \left(\frac{P}{4}\right)^2 = \left(\frac{80}{4}\right)^2 = 20^2 = 400\,\text{m}^2$$

   (b) 200 m.

   $$Area = \left(\frac{P}{4}\right)^2 = \left(\frac{200}{4}\right)^2 = 50^2 = 2500\,\text{m}^2$$

<u>Activity 6</u>

Consider the results from Activities 1 to 4.

(a) How many different rectangles could you draw when the perimeter was 16 cm?

   4

(b) How many different rectangles could you draw when the perimeter was 20 cm?

   5

(c) Place your results in the table below and continue the patterns in each row.

Perimeter (cm)	12	16	20	24	28	32	36
Number of different rectangles	3	4	5	6	7	8	9

(d) Describe how you can use the perimeter to determine the number of different rectangles -

(i) in words

Divide the perimeter by four to determine the number of different rectangles.

(ii) in algebraic symbols

$n = -$ is the perimeter and n represents the number of rectangles.

(e) How many different rectangles are possible when the perimeter is 100 cm?

25

(f) If there are 100 different rectangles possible, what is the perimeter?

400 cm

Introduction

As part of this investigation, you are given a fixed perimeter for a rectangle and will need to use this to create different rectangles and calculate their area. Unless indicated otherwise, all lengths and widths should be whole numbers.

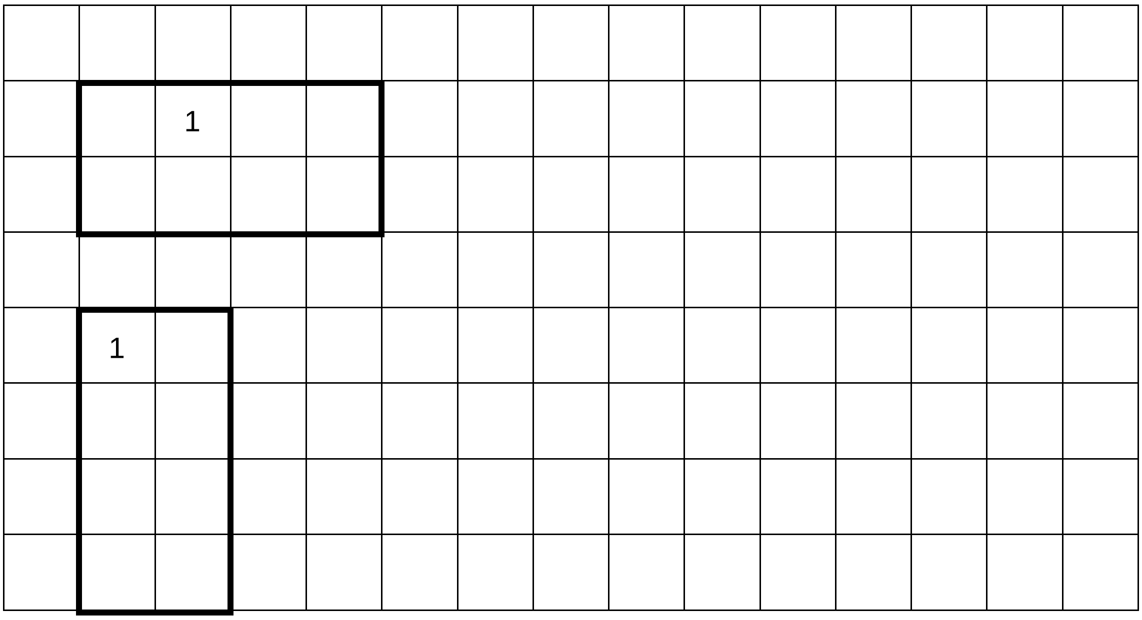
A rectangle, which is 2 cm wide and 3 cm long, can be rotated to face the other direction so that it looks to be 3 cm wide and 2 cm long. In this investigation, these are considered to "be the same".

<u>Activity 1</u>

On the grid provided each row is 1 cm high and each column is 1 cm wide. Two rectangles with a perimeter of 12 cm are drawn on the grid and they are "the same". Draw two other different rectangles with perimeters of 12 cm. Remember that the widths and lengths must be whole numbers.

Complete the table below.

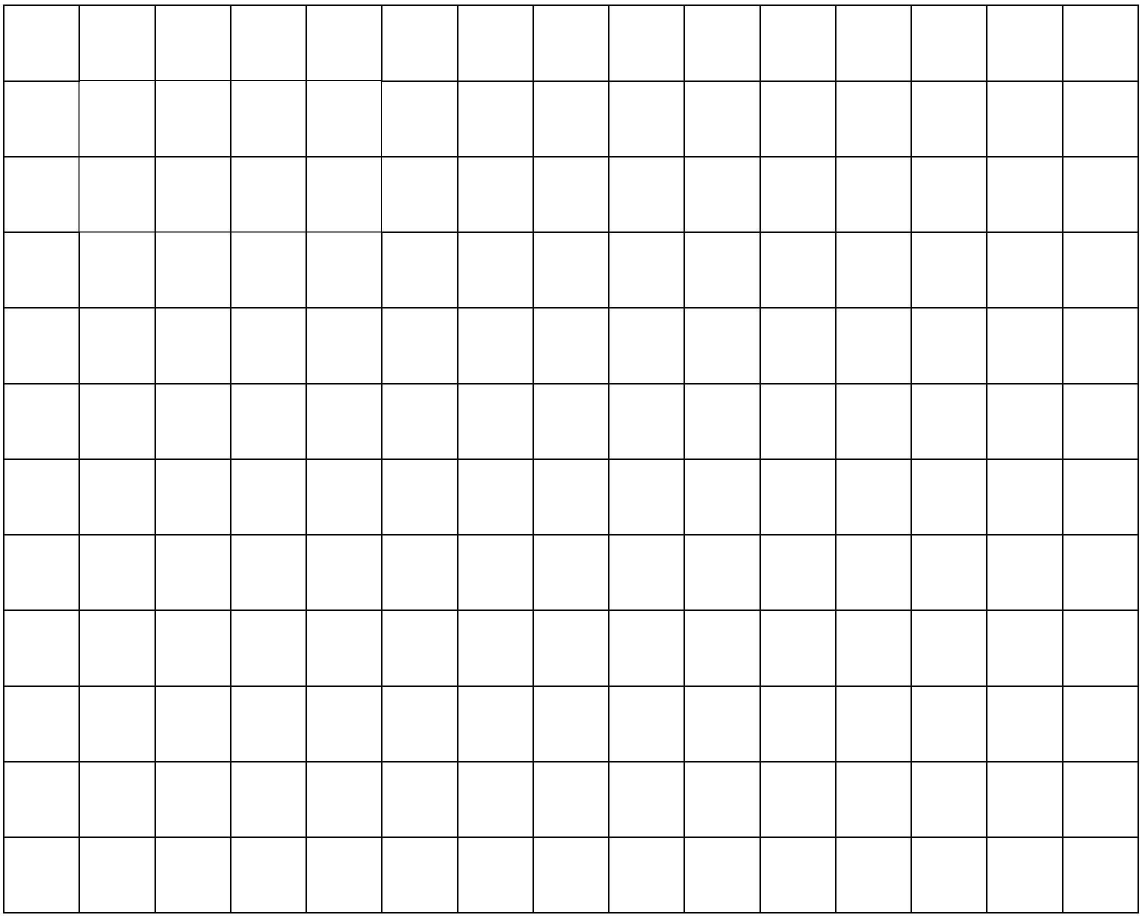
rectangle	width	length	perimeter	area
1			12 cm	8 cm^2
2			12 cm	
3			12 cm	



Highlight the rectangle with the greatest area.

<u>Activity 2</u>

On the grid provided each row is 1 cm high and each column is 1 cm wide. Draw as many different rectangles as you can, all with a perimeter of 16 cm. Remember that the widths and lengths must be whole numbers and if a new rectangle is made by moving another one around, then they are considered the same.



Complete the table below.

rectangle	width	length	perimeter	area
1			16 cm	

Highlight the rectangle with the greatest area.

Activity 3

On the grid provided each row is 1 cm high and each column is 1 cm wide. Draw as many different rectangles as you can, all with a perimeter of 20 cm. Remember that the widths and lengths must be whole numbers and if a new rectangle is made by moving one around then they are considered the same.

Complete the table below.

rectangle	width	length	perimeter	area
1			20 cm	

Highlight the rectangle with the greatest area.

Activity 4

Consider a rectangle with a perimeter of 24 cm. Examine the patterns of width and length in the tables for Activities 1-3 and use these patterns to complete the table below to list as many different rectangles as possible. If necessary, draw your rectangles on grid paper.

rectangle	width	length	perimeter	area
1			24 cm	

Highlight the rectangle with the greatest area.

What do you notice about the dimensions of the rectangle with the greatest area?

Activity 5

1. Does your observation apply to the rectangles drawn in Activities 1 to 3?

2. Write a sentence to summarise this observation for all rectangles.

3. What would be the maximum possible area for a rectangle with a perimeter of -

   (a) 28 cm?

   (b) 32 cm?

4. Describe how you can determine the maximum possible area that a rectangle could have if given the perimeter.

5. If A represents the area of a rectangle and P represents the perimeter, write an algebraic expression for -

   (a) the length of the rectangle with the greatest possible area

   (b) the greatest possible area for the rectangle with the perimeter of P.

6. Use the rules you have developed in item 5 above to determine the dimensions (length, width and area) of the rectangle with the greatest area when the perimeter is -

   (a) 80 m

   (b) 200 m

Activity 6

Consider the results from Activities 1 to 4.

(a) How many different rectangles could you draw when the perimeter was 16 cm?

(b) How many different rectangles could you draw when the perimeter was 20 cm?

(c) Place your results in the table below and continue the patterns in each row.

Perimeter (cm)	12	16	20	24			
Number of different rectangles	3						

(d) Describe how you can use the perimeter to determine the number of different rectangles

(i) in words

(ii) in algebraic symbols

(e) How many different rectangles are possible when the perimeter is 100 cm?

(f) If there are 100 different rectangles possible, what is the perimeter?

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

YEAR 7 MATHEMATICS

Multi-Strand Activity

Bigger Hand

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 8: BIGGER HAND

Overview

This activity requires students to consider different measurements and interpretations of the word "bigger". This activity could be done as a revision of area and perimeter as it only requires the skills students should have developed prior to Year 7, and students need to be re-focussed for measurement work in Year 7. It is anticipated that this task could take one to two 50-minute lessons.

Students will need

- calculators
- rulers
- string (should be available and brought out when students suggest it)

Relevant content descriptions from the Western Australian Curriculum

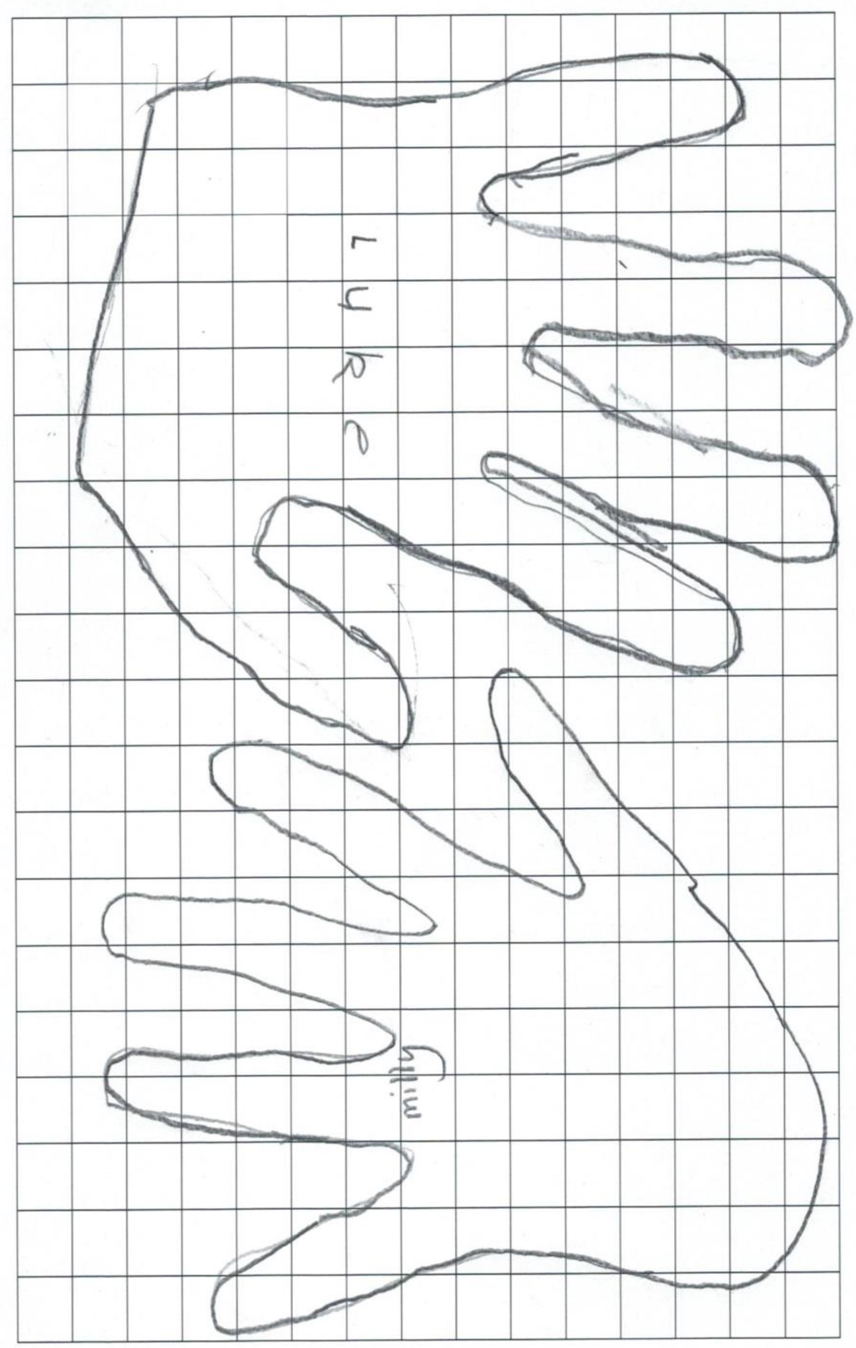
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
- Solve problems involving comparison of lengths and areas using appropriate units (ACMMG137) [Year 6]
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159) ** [as revision and preparation]
- Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)

Students can demonstrate

- understanding when they
  - can determine and use a process to calculate area in Problem 3, Question 6
- reasoning when they
  - provide answers and explanations in the reflection questions
- problem solving when they
  - determine methods to answer questions posed in the problems provided
  - interpret and use the information provided to determine glove size in Problem 2
  - decide which measurements to consider for Problem 1
  - link prior calculations to determine a process to solve Problems 5-7

Activity 1

On the grid below, Luke and Milly have drawn an outline of their hands. The grid was originally 1 cm x 1 cm. Who has the bigger hand? How do you know?



In deciding who has the bigger hand, what features and measurements could you use? Complete the table to show possibilities. Use each "square" as 1 cm long and 1 cm wide.

Measurements approximated in this table.

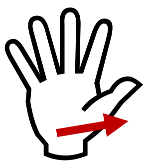
Number	Feature	Measurement		Whose is larger? Luke's or Millie's?
		Luke	Milly	
1	Width at the base	6 cm	4 cm	Luke
2	Length of thumb	4 cm	3.5 cm	Luke
3	Length from base to end longest finger	5 cm	4 cm	Luke
4	Length longest finger	6.5 cm	5.5 cm	Luke
5	Width middle finger	1.6 cm	1.3 cm	Luke
6	Length little finger	5 cm	4 cm	Luke
7	Width of thumb	1.5 cm	1.2 cm	Luke
8	Number of squares covered	80	67	Luke
9	Width at widest point	7.5 cm	6.5 cm	Luke
10	Width of handspan	12 cm	11.5 cm	Luke

<u>Activity 2</u>

Luke, Millie and their parents all play hockey and want to know what size gloves they would need to buy. Use the glove-sizing chart provided to determine the glove size for each person in the family. The hand measurement used is the distance across the palm from the knuckle of the little finger to the knuckle of the forefinger. Mum's measurement is 8.5 cm and Dad's measurement is 9.7 cm. In Australia, measurements are in cm [1 inch = 2.54 cm]. Complete the measurement column in centimetres.

Glove-sizing chart

Measurement (inches)	Measurement (cm)	Size (alpha)	Size (numeric)
2.125-2.625	5.4-6.7	XS	7
2.625-3.125	6.7-7.9	S	8
3.125-3.625	7.9-9.2	M	9
3.625-4.125	9.2-10.5	L	10
4.125-4.625	10.5-11.7	XL	11
4.625-5.125	11.7-13.0	XXL	12



Luke Size S

Millie Size XS

Mum Size 9

Dad Size 10

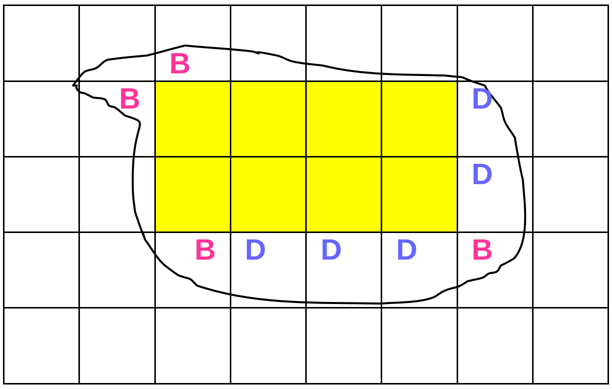
Problem 3

Whose hand has the greater area?

On the 1 cm x 1 cm grid paper, draw an outline of your non-writing hand as Luke and Millie have done.

Use the steps below to determine the area of your hand. Use the diagram as a guide.
An estimate is needed for the fractional parts of the square. Use the closest fraction.

- Colour all the full squares inside the outline in **colour A.**
- Colour all the "half" squares inside the outline in **colour B.**
- Colour all the "quarter" squares inside the outline in **colour C.**
- Colour all the "three-quarter" squares inside the outline in **colour D.**



- Count all the parts of squares and complete the table below.

Fractional part	Whole squares	Half-squares	Quarter-squares	Three-quarter squares
Number				

- Write a worded description using symbols for operations and the information in the table to show how you can determine the area of your hand.

Number of whole squares + Number of half squares ÷ 2 + Number of quarter squares ÷ 4 + Number of three-quarter squares ÷ –
Some students will give a correct response in a less concise manner.

- Use your previous description to calculate the area of your hand.

Answers as appropriate.

Activity 4

Is the hand with the greater area also the hand with the greater perimeter?

Consider:

- What do we mean by the perimeter of our hand?

The distance around the outside. How long is the trace of the outline?

- How do we measure the perimeter of our hand?

Get a piece of string, place it around the outline. Measure the string

- Estimate the perimeter of your hand and write your estimate.

- Measure the perimeter of your hand.

Collect data from 6 other classmates and complete the table provided.

Various answers as required in the table.

Person	Name	Estimated perimeter	Measured perimeter	Difference
1				
2				
3				
4				
5				
6				
7				

Most students find their estimates quite inaccurate; probably because it is not something they often do. Students should distinguish between overestimating and underestimating and also understand the significance of a negative answer when determining the difference.

Problems

1. What size of hockey gloves would you need? Show how you arrived at your decision.

   Measure the width of the palm (cm) as shown in the diagram earlier. Check in the second column of the table.

2. Does the person whose hand has the greater area also need the larger size of hockey gloves? Explain how you arrived at this conclusion.

   Locate the student with the greater area.
   Locate the student needing the larger size of gloves.
   Do they match?

3. Another glove size chart indicates that boys with a hand size of 5.5 inches to 6.5 inches should buy the XS size. How is this statement a different recommendation from the indication given in the previous glove-sizing chart? Explain.

   This hand size is 14 cm to 16.51 cm and according to the previous chart they would need a larger size than is available.
   Perhaps they are referring to the whole way round and this would be double the previous XS width plus the width of the hand.

4. Would all sports use the same glove size chart? Give evidence to support your answer.

   Probably not.
   Boxing gloves are more padded. The chart for boxing gloves gives a mass for the gloves according to the circumference of the hand. A professional boxer with a palm circumference of 18 cm would wear 10-ounce gloves.
   Cricket glove size is determined by the length of the hand from the wrist to the tip of the longest finger.

Reflection Questions

1. How accurate is your calculation for area? Explain.

   Not very accurate as each part of the square was an estimate based on opinion rather than any measurement.

2. How could you determine that the area of your hand is greater or less than the area of Luke's hand without calculating the area of Luke's hand?

   Place your had over Luke's hand and see if it covers the area of his hand.

3. How good were the estimates for the perimeters of the students' hands?
   Think of three different measures you could calculate to support your answer to this question.

Students could look at the mean of their estimates and compare it with the mean of the measurement. Students could also compare the differences; what is the minimum and maximum difference, what is the range of the differences. They could compare their differences with those of another student to see who was the "best" estimator.

4. Explain each of the following situations.

   The hand with the smallest area is not the hand with the smallest perimeter.

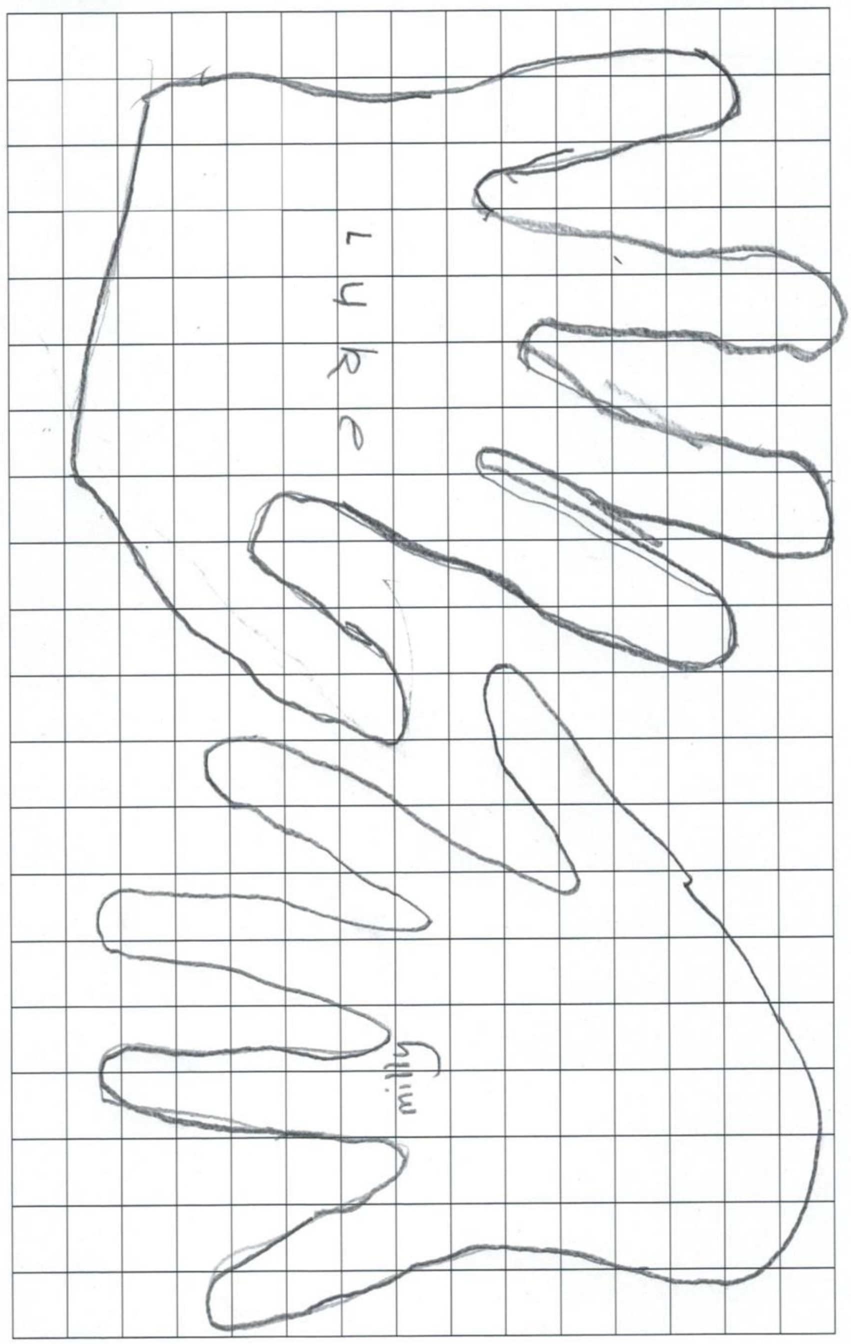
   Hand or fingers could be long and thin.

   The hand with the greatest area is not the hand with the greatest perimeter.

   Fingers or hand could be short and wide.

Activity 1

On the grid below, Luke and Milly have drawn an outline of their hands. The grid was originally 1 cm x 1 cm. Who has the bigger hand? How do you know?



In deciding who has the bigger hand, what features and measurements could you use? Complete the table to show possibilities. Use each "square" as 1 cm long and 1 cm wide.

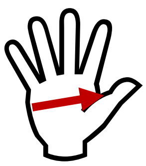
Number	Feature	Measurement		Whose is larger? Luke's or Milly's?
		Luke	Milly	
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Activity 2

Luke, Milly and their parents all play hockey and want to know what size gloves they would need to buy. Use the glove-sizing chart provided to determine the glove size for each person in the family. The hand measurement used is the distance across the palm from the knuckle of the little finger to the knuckle of the forefinger. Mum's measurement is 8.5 cm and Dad's measurement is 9.7 cm. In Australia, measurements are in cm [1 inch = 2.54 cm]. Complete the measurement column in centimetres.

Glove-sizing chart

Measurement (inches)	Measurement (cm)	Size (alpha)	Size (numeric)
2.125-2.625		XS	7
2.625-3.125		S	8
3.125-3.625		M	9
3.625-4.125		L	10
4.125-4.625		XL	11
4.625-5.125		XXL	12



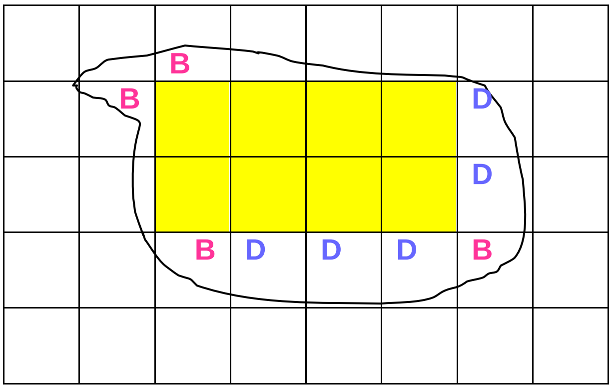
Activity 3

Whose hand has the greater area?

On the 1 cm x 1 cm grid paper, draw an outline of your non-writing hand as Luke and Milly have done.

Use the steps below to determine the area of your hand. Use the diagram as a guide. An estimate is needed for the fractional parts of the square. Use the closest fraction.

- Colour all the full squares inside the outline in **colour A**.
- Colour all the "half" squares inside the outline in **colour B.**
- Colour all the "quarter" squares inside the outline in **colour C.**
- Colour all the "three-quarter" squares inside the outline in **colour D.**



- Count all the parts of squares and complete the table below.

Fractional part	Whole squares	Half-squares	Quarter-squares	Three-quarter squares
Number				

- Write a worded description using symbols for operations and the information in the table to show how you can determine the area of your hand.

- Use your previous description to calculate the area of your hand.

Activity 4

Is the hand with the greater area also the hand with the greater perimeter?

Consider:

- What do we mean by the perimeter of our hand?

- How do we measure the perimeter of our hand?

- Estimate the perimeter of your hand and write your estimate.

- Measure the perimeter of your hand.

Collect data from 6 other classmates and complete the table provided.

Person	Name	Estimated perimeter	Measured perimeter	Difference
1				
2				
3				
4				
5				
6				
7				

Problems

1. What size of hockey gloves would you need? Show how you arrived at your decision.

2. Does the person whose hand has the greater area also need the larger size of hockey gloves? Explain how you arrived at this conclusion.

3. Another glove size chart indicates that boys with a hand size of 5.5 inches to 6.5 inches should buy the XS size. How is this statement a different recommendation from the indication given in the previous glove-sizing chart? Explain.

4. Would all sports use the same glove size chart? Give evidence to support your answer.

Reflection Questions

1. How accurate is your calculation for area? Explain.

2. How could you determine that the area of your hand is greater or less than the area of Luke's hand without calculating the area of Luke's hand?

3. How good were the estimates for the perimeters of the students' hands? Think of three different measures you could calculate to support your answer to this question.

4. Explain each of the following situations.

   The hand with the smallest area is not the hand with the smallest perimeter.

   The hand with the greatest area is not the hand with the greatest perimeter.

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Parallelograms

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 9: PARALLELOGRAMS

Overview

This task provides an opportunity for students to review the features of parallelograms as well as establish and apply the formula for their area. Students should be familiar with the features of rectangles and be able to calculate their area, and this is reviewed at the beginning of the task.

Students will need

- rulers
- scissors
- glue
- calculators

Relevant content descriptions from the Western Australian Curriculum

- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)
- Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

Students can demonstrate

- fluency when they
  - determine and use measurements to complete the tables for rectangles and parallelograms
- understanding when they
  - identify the features of a parallelogram
  - recognise that rectangles are also parallelograms
  - apply what they have learnt to determine which statements are true/false in the reflection
- reasoning when they
  - explain why rectangles are also parallelograms (e.g., Activity 2)
- problem solving when they
  - determine the formulae for area and perimeter (e.g., Activity 3: 5, 8)

In naming the sides for quadrilaterals used in this exercise, the length is considered as the bottom, base or horizontal length and the width refers to the vertical measurement as shown in the diagram. In some activities it might be necessary to rearrange the parallelogram to identify a width and length to make this clear.

Activity 1: Revising the features of rectangles

1. Using a ruler, accurately draw a rectangle with a length of 11 cm and a width of 3 cm.

2. Showing your calculations, determine the perimeter and area of the rectangle that you have drawn.

   Area = 3 cm x 11 cm = 33 cm^2

   Perimeter = 3 cm + 3 cm + 11 cm + 11 cm = 28 cm

3. List the features common to all rectangles.

   Closed figure with 4 sides
   All internal angles are 90 degrees
   Opposite pairs of sides are equal
   Opposite pairs of sides are parallel

4. Locate and name four items of rectangular shape in the room.

   This page

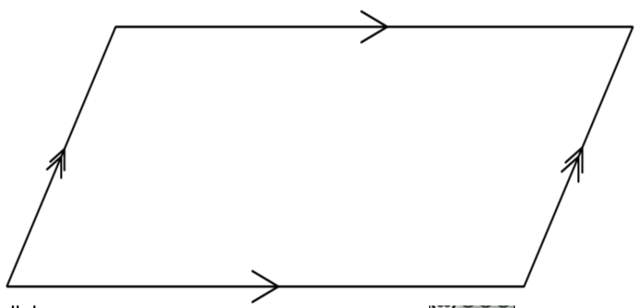
   Look for doors, windows, lids of desks, book covers, etc.

Activity 2

1. Why is the following shape called a parallelogram?

   It has 4 sides

   Opposite sides are parallel



2. Give three features of the sides of a parallelogram.

Parallelograms have 4 sides.

Opposite sides of a parallelogram are equal.

Opposite sides of parallelogram are parallel.

3. Why is a rectangle also classified as a parallelogram?

A rectangle has all the features of a parallelogram.

4. What feature of a parallelogram would make it also a rectangle?

All internal angles are of 90 degrees

5. Locate and measure the height of the parallelogram. Is the height equal to one of the sides? What angle does the height make with the length (base)?

The height is 3.5 cm

It is not equal to one of the sides.

The height makes an angle of 90 degrees to the base.

Activity 3

You are given two pages of parallelograms. On the second page the parallelograms are the same size and shape as on the first page.

1. From the first page, cut out each parallelogram and paste it into your workbook. Place the parallelograms under each other.

2. For each parallelogram that has been pasted into your workbook -
   - locate the matching parallelogram on the second page
   - cut out the matching parallelogram
   - cut a triangle off one end of the parallelogram and add it to the other end to make a rectangle [Hint: Work out how to do it before cutting.]
   - paste the rectangle you have made into your workbook.

3. You should now have 6 parallelograms and next to each one a rectangle that you have made from the parallelogram of the same size.

4. Determine the measurements of these **rectangles** and complete the table.

Rectangle	A	B	C	D	E	F
Width	4.1 cm	4 cm	6.5 cm	4.7 cm	2.5 cm	3.8 cm
Length	7.2 cm	5.8 cm	4.7 cm	7.3 cm	5 cm	12.5 cm
Perimeter	22.6 cm	19.6	22.4 cm	24 cm	15 cm	32.6 cm
Area	29.52 cm^2	23.2 cm^2	30.55 cm^2	34.31 cm^2	12.5 cm^2	47.5 cm^2
Height	4.1 cm	4 cm	6.5 cm	4.7 cm	2.5 cm	3.8 cm

Note: The width, height and lengths depend on the orientation used and may vary. Students' answers for area and perimeter should not vary

5. Using the following letters to represent these variables:
w : width h : height l: length A: area P : perimeter

(a) Write a rule using symbols to calculate perimeter.

$P = w + w + l + l$ OR $P = 2w + 2l$ OR $P = 2l + 2h$

(b) Write a rule using symbols to calculate area.

$A = l \times w$ OR $A = l \times h$ OR $A = lw$ OR $A = lh$

6. When the parallelogram was cut and the pieces re-arranged to form a rectangle -

(a) were any pieces discarded? No

(b) was the area changed? No
Students should use this fact to enter the areas for the parallelograms

7. Determine the measurements of these parallelograms and complete the table. ***

Parallelogram	A	B	C	D	E	F
Slant height	5.1 cm	4.4 cm	6.7 cm	5.2 cm	3.9 cm	4.9 cm
Length	7.2 cm	5.8 cm	4.7 cm	7.3 cm	5 cm	12.5 cm
Perimeter	24.6 cm	20.4 cm	22.8 cm	25 cm	17.8 cm	34.8
Area	29.52 cm^2	23.2 cm^2	30.55 cm^2	34.31 cm^2	12.5 cm^2	47.5 cm^2

Height	4.1 cm	4 cm	6.5 cm	4.7 cm	2.5 cm	3.8 cm

8. Using the following letters to represent these variables -
s : slant height h : height l: length A: area P : perimeter

(a) write a rule using symbols to calculate perimeter

$P = s + s + l + l$ OR $P = 2s + 2l$ OR $P = 2 \times s + 2 \times l$ OR $P = 2(s + l)$

Note: Height is not used

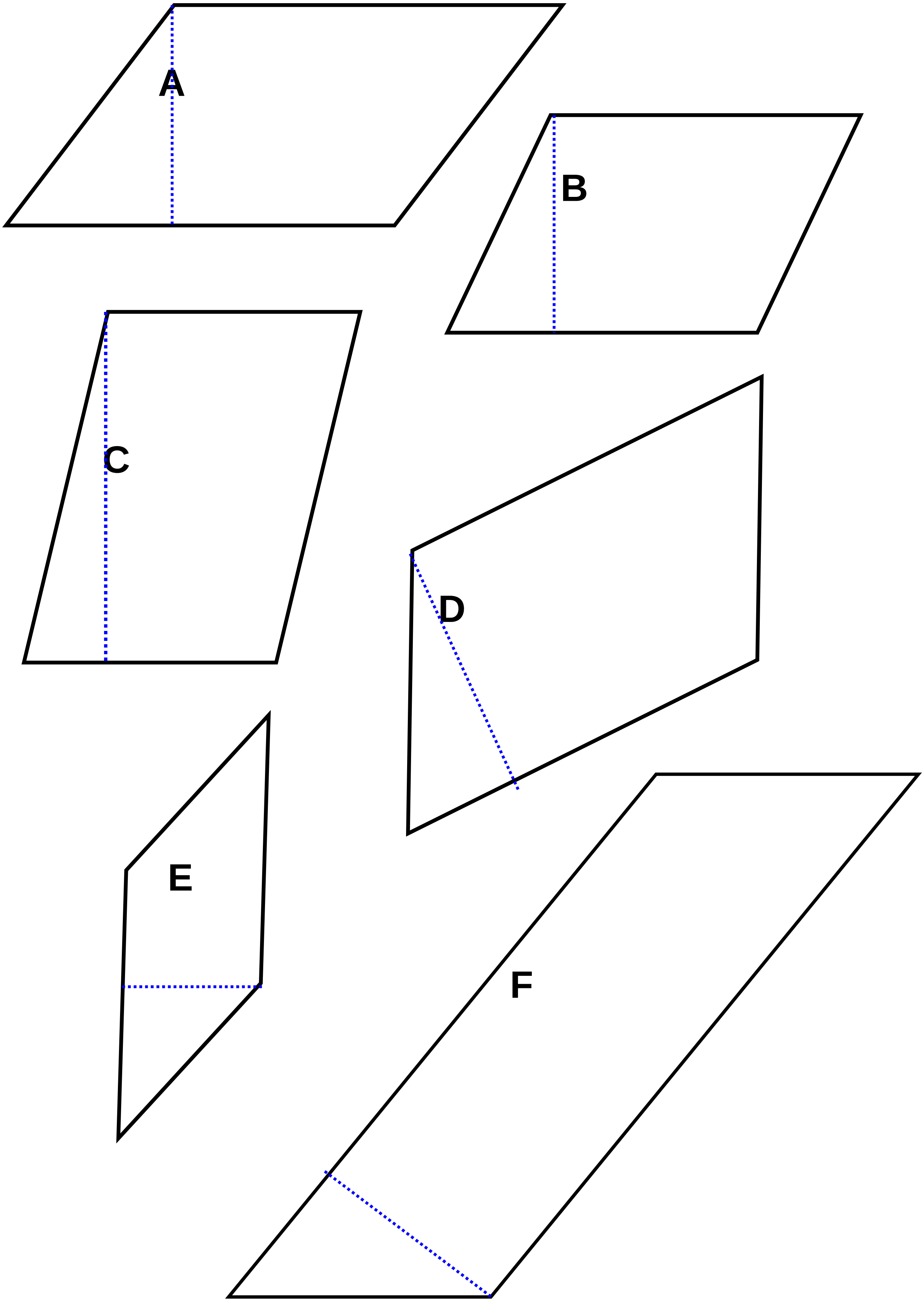
(b) write a rule using symbols to calculate area.

$A = w \times h$ OR $A = wh$ OR $A = hw$ OR $A = h \times l$ OR $A = hl$

9. What is the general rule to calculate the area of a parallelogram?
Use words to describe this rule.

To calculate the area of a parallelogram, multiply the width by the height (perpendicular height)

*** On the next page, the heights are indicated for the table above.


A
B
C
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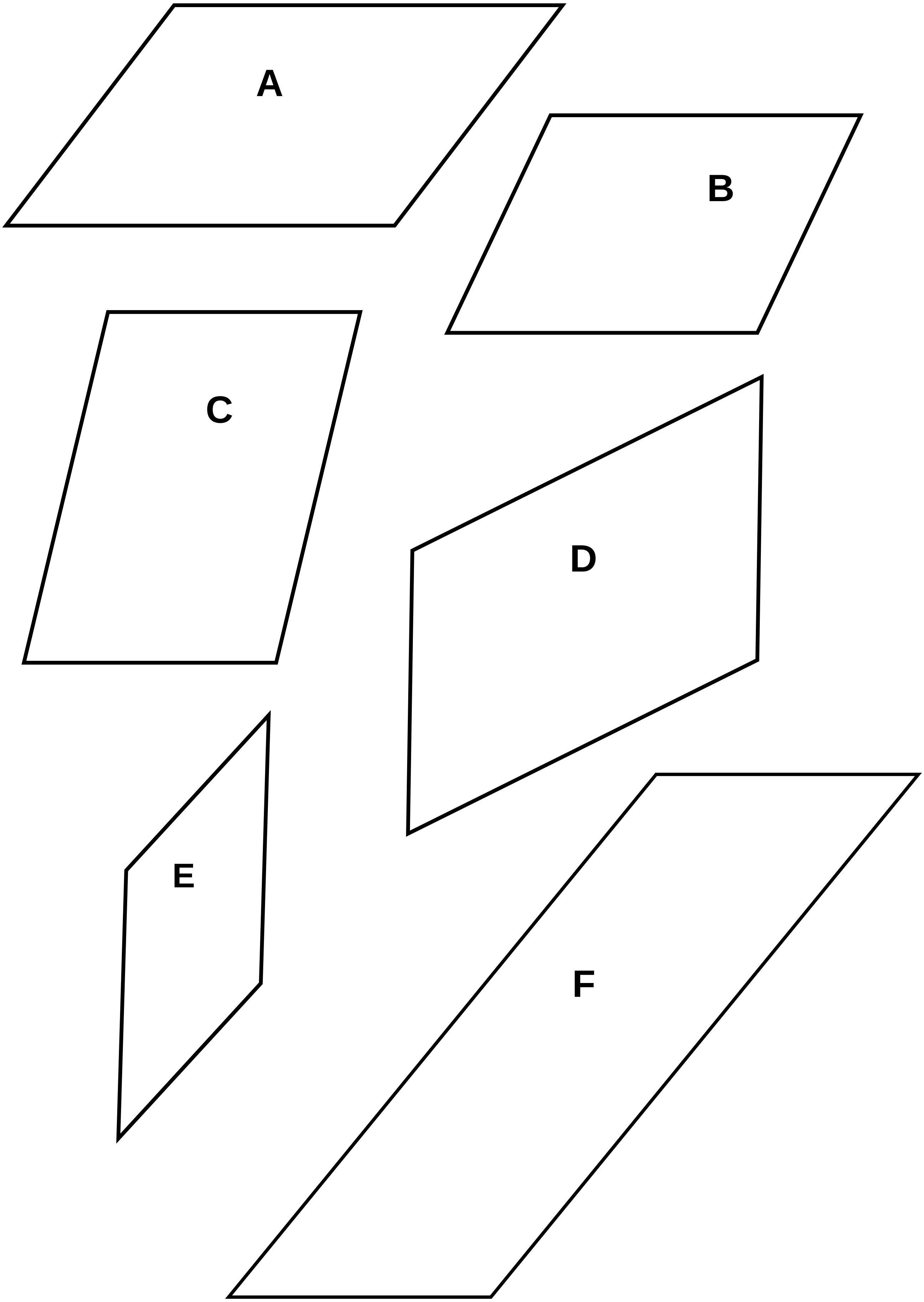
E
F


A
B
C
D
E
F

Reflection

In this activity parallelograms were cut into two and the parts rearranged to form rectangles. No parts were removed so the areas were the same for both figures.

Was the perimeter of the rectangle the same as the perimeter of the parallelogram?

No

List the variables which were the same for the parallelogram and its matching rectangle.

Height Length Area

List the variable which was different for the parallelogram and its matching rectangle.

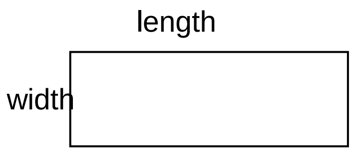
Perimeter

Compare your results with those of another student.

Consider the following statements about rectangles and parallelograms. Determine if they are TRUE or FALSE.

1. Rectangles and parallelograms are quadrilaterals. TRUE
2. All rectangles are parallelograms. TRUE
3. All parallelograms are rectangles. FALSE
4. The height of a rectangle is equal to the length of one of its sides. TRUE
5. The height of a parallelogram is equal to the length of one of its sides. FALSE
6. The area of a rectangle = width x height. TRUE
7. The area of a parallelogram = width x height. TRUE
8. The area of a rectangle = width x length. TRUE
9. The area of a parallelogram = width x length. TRUE
10. The names width and length can be interchanged. TRUE
11. The height of a parallelogram is perpendicular to one of the sides. TRUE
12. In a non-rectangular parallelogram, the height is not used in the calculation of the perimeter. TRUE

In naming the sides for quadrilaterals used in this exercise, the length is considered as the bottom, base or horizontal length and the width refers to the vertical measurement as shown in the diagram. In some activities it might be necessary to rearrange the parallelogram to identify a width and length to make this clear.



Activity 1: Revising the features of rectangles

1. Using a ruler, accurately draw a rectangle with a length of 5 cm and a width of 3 cm.

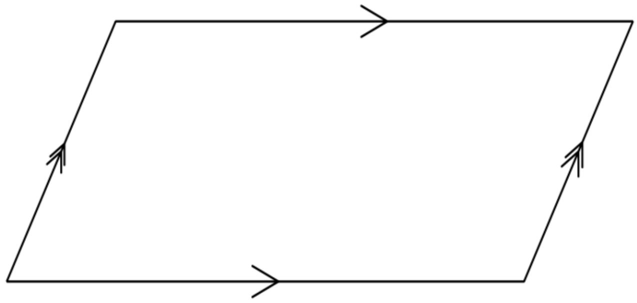
2. Showing your calculations, determine the perimeter and area of the rectangle that you have drawn.

3. List the features common to all rectangles.

4. Locate and name four items of rectangular shape in the room.

Activity 2

1. Why is the following shape called a parallelogram?



2. Give three features of the sides of a parallelogram.

3. Why is a rectangle also classified as a parallelogram?

4. What feature of a parallelogram would make it also a rectangle?

5. Locate and measure the height of the parallelogram. Is the height equal to one of the sides? What angle does the height make with the length (base)?

Activity 3

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1. From the first page, cut out each parallelogram and paste it into your workbook. Place the parallelograms under each other.

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   - paste the rectangle you have made into your workbook.

3. You should now have 6 parallelograms and next to each one a rectangle that you have made from the parallelogram of the same size.

4. Determine the measurements of these **rectangles** and complete the table.

Rectangle	A	B	C	D	E	F
Width						
Length						
Perimeter						
Area						
Height						

5. Using the following letters to represent these variables -
   w : width h : height l: length A: area P : perimeter

   (a) write a rule using symbols to calculate perimeter

   (b) write a rule using symbols to calculate area.

6. When the parallelogram was cut and the pieces re-arranged to form a rectangle -

(a) were any pieces discarded?

(b) was the area changed?

7. Determine the measurements of these parallelograms and complete the table.

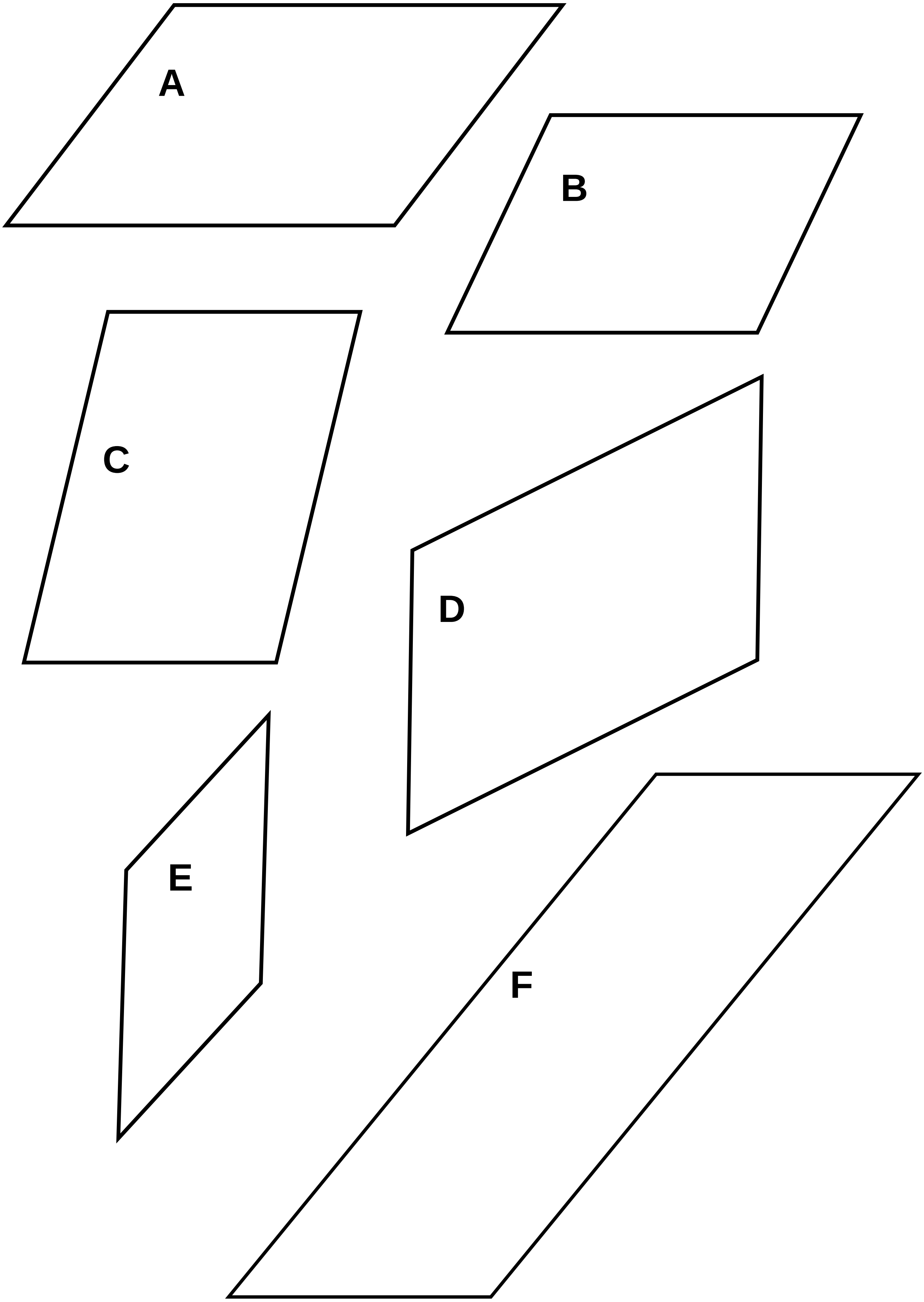
Parallelogram	A	B	C	D	E	F
slant height						
Length						
Perimeter						
Area						
Height						

8. Using the following letters to represent these variables -
s : slant height h : height l: length A: area P : perimeter

(a) write a rule using symbols to calculate perimeter

(b) write a rule using symbols to calculate area.

9. What is the general rule to calculate the area of a parallelogram?
Use words to describe this rule.


A
B
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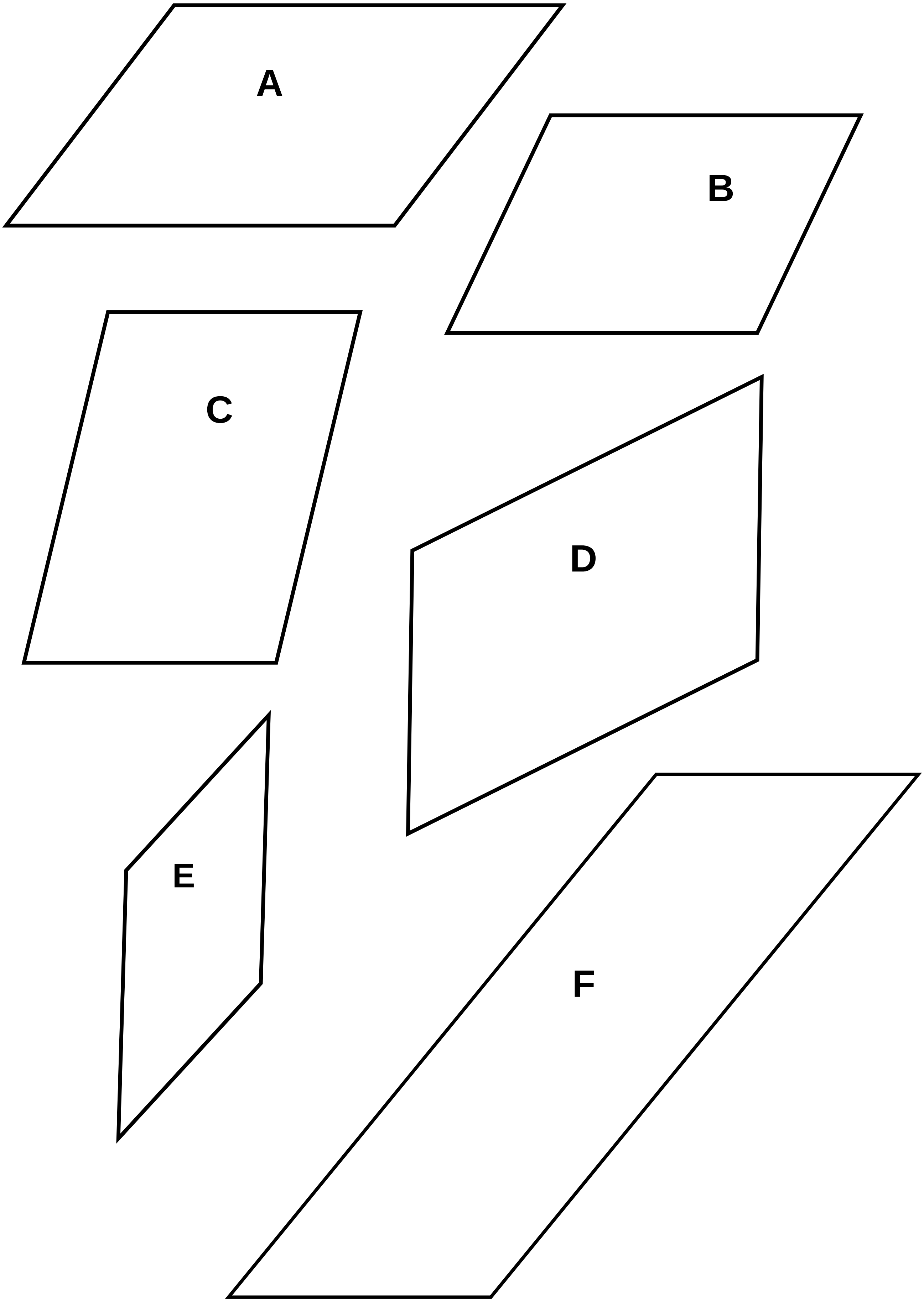

A
B
C
D
E
F

Reflection

In this activity parallelograms were cut into two and the parts rearranged to form rectangles. No parts were removed so the areas were the same for both figures.

Was the perimeter of the rectangle the same as the perimeter of the parallelogram?

List the variables which were the same for the parallelogram and its matching rectangle.

List the variables which were different for the parallelogram and its matching rectangle.

Compare your results with those of another student.

Consider the following statements about rectangles and parallelograms. Determine if they are TRUE or FALSE.

1. Rectangles and parallelograms are quadrilaterals.
2. All rectangles are parallelograms.
3. All parallelograms are rectangles.
4. The height of a rectangle is equal to the length of one of its sides.
5. The height of a parallelogram is equal to the length of one of its sides.
6. The area of a rectangle = width x height
7. The area of a parallelogram = width x height
8. The area of a rectangle = width x length
9. The area of a parallelogram = width x length
10. The names width and length can be interchanged.
11. The height of a parallelogram is perpendicular to one of the sides.
12. In a non-rectangular parallelogram, the height is not used in the calculation of the perimeter.

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Cash Out

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 10: CASH OUT

Overview

This task is designed to support the students' development of mental computation using statements of equivalence, as well as to reinforce aspects of financial literacy. These computations provide some basis for using the laws of arithmetic. There is also the opportunity to create a sample space by listing the possible outcomes of an event. Students should attempt this task without using their calculators.

Students will not need any extra equipment

Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)
- Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)

Students can demonstrate

- *understanding* when they
  - can determine the numbers of arrangements in Activity 1 and Activity 2
- *reasoning* when they
  - explain their solution process in Activity 4
  - explain the relationship between the amount of money and the number of arrangements as in Activity 2
- *problem solving* when they
  - can determine a process to solve the problem provided in Activity 4
  - determine the arrangements in Activity 3

If you have a bank account, you can deposit or withdraw money using an ATM. This activity considers the total amounts and the notes that can be transacted.

Preparation activity

A transaction is an exchange of money between the bank and the account holder. Name the two types of transactions. Deposits and withdrawals

What is the name used to describe the event when you take money out of your account?

Withdrawal, Get cash out

What do the initials ATM stand for?

Automated teller machine

When the ATM is used to transfer money, only notes are transferred; not coins. Suggest a reason why coins are not transferred.

Require more room to store. Expensive to disperse.

If you wanted to get $100 out in $20 notes, how many notes would you receive?

5

If a customer received 20 notes, each one a $50 note, how much did they withdraw?

$1000

Most machines do not dispense $5 or $10 notes. Of the amounts listed below, identify the ones that you would not be able to request from such machines.

$15 $25 $50 $70 $80 $110 $125

Activity 1

An ATM only dispenses the $20, $50 and $100 notes. A customer wants to use the ATM to get $200. There are several different ways to get out $200; e.g., four $50 notes. Use the table below to show as many other ways as you can to withdraw $200 using only these notes.

Way	Number of $20 notes	Number of $50 notes	Number of $100 notes	Check total
1	0	4	0	4 x $50 = $200
2	0	0	2	2 x $100 = $200
3	10	0	0	10 x $20 = $200
4	5	2	0	5 x $20 + 2 x $50 = $200
5	0	2	1	2 x $50 + 1 x $100 = $200
6	5	0	1	5 x $20 + 1 x $100 = $200

Note: There are 6 different ways of giving out $200.

1. For each amount listed, determine the number of different ways that the amount can be given out. Show the notes used.

   (a) $20 One way: one $20 note

   (b) $50 One way: one $50 note

   (c) $75 Can't be done

2. There are 3 ways by which $100 can be give out at this ATM. What are they?

   one $100 note

   two $50 notes

   five $20 notes

Activity 2

A similar ATM only dispenses the $20, $50 and $100 notes. To withdraw $500, one arrangement of notes is shown below. Use this format to identify 9 other ways to have $500 in only these three notes.

$500 = 3 x $100 + 4 x $50
$500 = 3 x $100 + 2 x $50 + 5 x $20
$500 = 2 x $100 + 4 x $50 + 5 x $20
$500 = 2 x $100 + 2 x $50 + 10 x $20
$500 = 2 x $100 + 15 x $20
$500 = 5 x $100
$500 = 1 x $100 + 6 x $50 + 5 x $20
$500 = 1 x $100 + 4 x $50 + 10 x $20
$500 = 1 x $100 + 2 x $50 + 15 x $20
$500 = 10 x $50

Do you agree with the statement below? Justify your conclusion.

As the amount to be withdrawn increases, the number of different arrangements of notes will also increase.

This is true on some occasions.

If the amount went from $500 to $510, there would be no increase because $510 is not possible.

If the amount went from $500 to $600 then there would be at least all the $500 arrangements plus either a $100, two $50 or five $20.

Going from $20 to $50 did not increase the number of arrangements.

Activity 3

Using only the $20, $50 and $100 notes, how many different arrangements can be used to make $300. Show all of these arrangements.

Way	Number of $20 notes	Number of $50 notes	Number of $100 notes	Check total
1	15	0	0	15 x $20 = $300
2	10	2	0	10 x $20 + 2 x $50 = $300
3	10	0	1	10 x $20 + 1 x $100 = $300
4	5	4	0	5 x $20 + 4 x $50 = $300
5	5	2	1	5 x $20 + 2 x $50 + 1 x $100 = $300
6	5	0	2	5 x $20 + 2 x $100 = $300
7	0	6	0	6 x $50 = $300
8	0	4	1	4 x $50 + 1 x $100 = $300
9	0	2	2	2 x $50 + 2 x $100 = $300
10	0	0	3	3 x $100 = $300

On review, students could discuss the benefits of listing these outcomes systematically.

Activity 4

If there are two Australian banknotes hidden in an envelope (denomination unknown), what might be the value of the money in the envelope? Prepare an argument to convince another student that you have considered all possibilities. The argument could take the form of a mathematical presentation.

The amounts in blue are the possibilities.

+	$5	$10	$20	$50	$100
$5	$10	$15	$25	$55	$105
$10	$15	$20	$30	$60	$110
$20	$25	$30	$40	$70	$120
$50	$55	$60	$70	$100	$150
$100	$105	$110	$120	$150	$200

If you have a bank account, you can deposit or withdraw money using an ATM. This activity considers the total amounts and the notes that can be transacted.

Preparation activity

A transaction is an exchange of money between the bank and the account holder. Name the two types of transactions.

What is the name used to describe the event when you take money out of your account?

What do the initials ATM stand for?

When the ATM is used to transfer money, only notes are transferred; not coins. Suggest a reason why coins are not transferred.

If you wanted to get $100 out in $20 notes, how many notes would you receive?

If a customer received 20 notes, each one a $50 note, how much did they withdraw?

Most machines do not dispense $5 or $10 notes. Of the amounts listed below, identify the ones that you would not be able to request from such machines.

$15	$25	$50	$70	$80	$110	$125

Activity 1

An ATM only dispenses the $20, $50 and $100 notes. A customer wants to use the ATM to get $200. There are several different ways to get out $200; e.g., four $50 notes. Use the table below to show as many other ways as you can to withdraw $200 using only these notes.

Way	Number of $20 notes	Number of $50 notes	Number of $100 notes	Check total
1	0	4	0	4 x $50 = $200
2				
3				
4				
5				
6				

Note: There are 6 different ways of giving out $200.

1. For each amount listed, determine the number of different ways that the amount can be given out. Show the notes used.

   (a) $20

   (b) $50

   (c) $75

2. There are 3 ways by which $100 can be give out at this ATM. What are they?

Activity 2

A similar ATM only dispenses the \$20, \$50 and \$100 notes. To withdraw \$500, one arrangement of notes is shown below. Use this format to identify 9 other ways to have \$500 in only these three notes.

\$500 = 3 x \$100 + 4 x \$50

Do you agree with the statement below? Justify your conclusion.

As the amount to be withdrawn increases, the number of different arrangements of notes will also increase.

Activity 3

Using only the $20, $50 and $100 notes, how many different arrangements can be used to make $300. Show all of these arrangements.

Activity 4

If there are two Australian banknotes hidden in an envelope (denomination unknown), what might be the value of the money in the envelope? Prepare an argument to convince another student that you have considered all possibilities. The argument could take the form of a mathematical presentation.

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Coins

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 17: COINS

Overview

This task uses the Australian coin currently in circulation to provide a context for calculating with decimals and listing elements of sample spaces. It also provides an opportunity for students not yet familiar with Australian coins to review their knowledge and understanding of the coins in circulation. All operations with decimals have been designed for a standard level of Year 7 and should be performed without a calculator.

Students will need

- calculators for checking answers
- access to the internet (Activity 1) – data supplied in solutions

Relevant content descriptions from the Western Australian Curriculum

- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
- Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)
- Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers (Year 6: ACMNA128)

Students can demonstrate

- *fluency* when they
    - determine and perform the operations necessary in Activity 1
- *understanding* when they
    - make connections between the variables – value, mass and width
- *reasoning* when they
    - justify their decisions in Activities 3 and 5
- *problem solving*
    - in Activities 3 – 6 when they can identify appropriate procedures to solve the problems posed.

Activity 1

Investigate the mass and width of the Australian coins currently in circulation and use the data to complete the table provided. One source of information is the Royal Australian Mint. [http://www.ram int.gov.au/]

Coin	5c	10c	20c	50c	$1	$2
Mass (g)	2.83	5.65	11.30	15.55	9.00	6.60
Width (mm)	19.41	23.60	28.65	31.65	25.00	20.50

Ensure students realise the difference between say, 6.6 and 6.60 (nearest place values).

Check your data with that of another student. These values will be needed in further activities.

Activity 2

Use non-calculator methods to determine the following:

(a) If you had just two coins of the same value, what is -

(i) the greatest amount of money you might have?

$2 + $2 = $4

(ii) the least amount of money you might have?

5c + 5c = 10c

(b) If you had just two coins of the different value, what is -

(i) the greatest amount of money you might have?

$1 + $2 = $3

(ii) the least amount of money you might have?

5c + 10c = 15c

(c) If you had a handful of 5c coins which weighed no more than 100 g, what is the greatest amount of money you could have?

Each coin weighs 2.83 g. Try 30 coins. 2.83 x 30 = 84.9 g.
100 - 84.9 = 15.1 so another 5 coins and that makes 2.83 x 5 g = 14.15 g
84.9 g + 14.15 g = 99.05 g and no more coins needed to make 100 g.
Total number of coins is 35 which is $1.75

Activity 3

For each of the following statements decide if the statement is TRUE or FALSE and give mathematical reasons to explain your decision.

(a) The greater the value of the coin, the greater is the mass of the coin.

FALSE: The statement is true from 5c to 50c but the $1 coin has less mass than the 50c coin.

(b) The smaller the width of the coin, the less it is worth.

If you line up the coins from the widest to the narrowest the order will be:
50c 20c $1 10c $2 5c
These are not decreasing in value so the statement is FALSE.

(c) The greater the mass of the coin, the wider it is.

Lining up the coins in order of mass gives	2.83	5.65	6.60	9.00	11.10	15.55
Putting the coin width below the mass:	19.41	23.60	20.50	25.00	28.65	31.65

These are nearly in order but the $2 coin which has a mass of 6.60 g is not as wide as the 10c coin which is heavier. So the statement is FALSE.

(d) The mass of thirty 10c coins is greater than the mass of ten 50c coins.

Thirty 10c coins have a mass of 30 x 5.65, which is 169.5 g.
Ten 50c coins have a mass of 10 x 15.55, which is 155.5 g.
The statement is TRUE.

(e) 9 kg of $1 coins is worth the same as 6.6 kg of $2 coins.

9 kg of $1 coins is 9000 g and each coin has a mass of 9 g so there are 1000 coins and this will be worth $1000.
6.6 kg of $2 coins is 6600 g and each coin has a mass of 6.6 g so there will be 1000 coins and this is worth $2000.

The statement is FALSE

Activity 4

Use non-calculator methods for these calculations. Show your calculation methods.

(a) If you had one of each coin what is the -

(i) total value? 5c + 10c + 20c + 50c + $1 + $2 = $3.85

(ii) total mass? 2.83 + 5.65 + 11.3 + 15.55 + 9.0 + 6.6 = 50.93 g

(b) How much heavier is the 50c coin than the 10c coin?

15.55 – 5.65 = 9.9 g

(c) If you have forty 20c coins organised so the coins are touching and are in a straight line -

(i) what is the total value?

40 x 20c = $8

(ii) how long is the line?

40 x 28.65 = 286.5 x 4 = 800 + 320 + 24 + 2 = 1146 mm

(iii) what is the total mass?

40 x 11.3 = 113 x 4 = 400 + 40 + 12 = 452 g

(d) Which single coin is closest in mass to the total mass of the 5c coin and the 10c coin?

The total mass of the 5c con and the 10c coin = 2.83 + 5.65 = 8.48 g.
The closest mass to this is 9.0 g so it is the $1 coin.

(e) What is the best way to divide the coins into two groups so the mass of each group is about the same?

Group 1: 50c, 5c, $2 has a mass of 24.98 g.
Group 2: 10c, 20c, $1 has a mass of 25.95 g.

Activity 5

Assume for this activity that all the coins are flat and lined up next to each so that they are touching. Rank these rows of coins in the order in which you would like to have them. Show justification for your decision. However, before you commence this activity, predict which row will be worth the most.

Row 1: 2 metres of $1 coins

Row 2: 1 metre of $2 coins

Row 3: 5 metres of 50c coins

Row 4: 20 metres of 5c coins

Row 1: 2 m is 2000 mm and each coin is 25 mm wide.
Four coins will be 100 mm so there will be 20 x 4 coins in 2000 mm
The value of 80 coins is $80

Row 2: 1 m is 1000 mm and each coin is 20.5 mm wide.
10 coins is 205 mm in width and 50 coins is 1025 mm in width
Need to take 2 coins off, so 48 coins in total
These 48 coins have a value of $96

Row 3: 5 m is 5000 mm and each coin is 31.65 mm wide.
100 coins is 3165 mm in width so 1835 mm left
For 50 coins width is 50 x 31.65 = 316.5 x 5 = 300 x 5 + 16 x 5 + 0.5 x 5 = 1582.5
Still 1835 –1582.5 width remaining, which is 252.5 mm
Try 8 coins: 8 x 31.65 = 253.2. Too much, so 7 coins
157 coins and this is $78.50

Row 4: 20 m is 20 000 mm and each coin is 19.41 mm wide.
1000 coins is 19 410 mm in width so 590 mm left
Each coin is approximately 20 mm so try 30 coins
For 30 coins width is 30 x 19.41 = 194.1 x 3 = 100 x 3 + 90 x 3 + 4.1 x 3 = 582.3
No room for another coin so 1030 coins
The value of 1030 coins is 1030 x 5 ÷ 100 = $51.50

If you select the rows according to value the order of worth is Row 2, Row 1, Row 3, Row 4.

Activity 6: Investigation

I have two coins in my pocket.

What amount of money might I have?

To guide you in this investigation, here are some steps to take.

1. Check that you understand the problem

Think of some questions that need to be asked, and some things that need to be checked before you can proceed.

Are the coins Australian?
Are the coins currently in circulation?

2. Think!

Do you have an answer to this question?

Is there more than one answer? YES

How will you know that you have all the answers? Various responses.

Is there a systematic way to check all the answers have been found?

3. Showing your answers

What is the best way to show your answers?
Show them below.

Add	**5c**	**10c**	**20c**	**50c**	**$1**	**$2**
5c	10c	15c	25c	55c	$1.05	$2.05
10c	15c	20c	30c	60c	$1.10	$2.10
20c	25c	30c	40c	70c	$1.20	$2.20
50c	55c	60c	70c	$1	$1.50	$2.50
$1	$1.05	$1.10	$1.20	$1.50	$2	$3
$2	$2.05	$2.10	$2.20	$2.50	$3	$4

Are your answers clear for others to read and understand?

There are 21 different possibilities:
10c 15c 20c 25c 30c 40c 55c 60c 70c $1
$1.05 $1.10 $1.20 $1.50 $2 $2.05 $2.10 $2.20 $2.50 $3 $4

4. Extension

Assume a $5 coin is added to the Australian coins in circulation. Show a systematic way to identify all answers to the original question asked in Activity 6.

Add	5c	10c	20c	50c	$1	$2	$5
5c	10c	15c	25c	55c	$1.05	$2.05	$5.05
10c	15c	20c	30c	60c	$1.10	$2.10	$5.10
20c	25c	30c	40c	70c	$1.20	$2.20	$5.20
50c	55c	60c	70c	$1	$1.50	$2.50	$5.50
$1	$1.05	$1.10	$1.20	$1.50	$2	$3	$6
$2	$2.05	$2.10	$2.20	$2.50	$3	$4	$7
$5	$5.05	$5.10	$5.20	$5.50	$6	$7	$10

Activity 1

Investigate the mass and width of the Australian coins currently in circulation and use the data to complete the table provided. One source of information is the Royal Australian Mint. [http://www.ramint.gov.au/]

Coin						
Mass (g)						
Width (mm)						

Check your data with that of another student. These values will be needed in further activities.

Activity 2

Use non-calculator methods to determine the following:

(a) If you had just two coins of the same value, what is -

(i) the greatest amount of money you might have?

(ii) the least amount of money you might have?

(b) If you had just two coins of the different value, what is -

(i) the greatest amount of money you might have?

(ii) the least amount of money you might have?

(c) If you had a handful of 5c coins which weighed no more than 100 g, what is the greatest amount of money you could have?

Activity 3

For each of the following statements decide if the statement is TRUE or FALSE and give mathematical reasons to explain your decision.

(a) The greater the value of the coin, the greater is the mass of the coin.

(b) The smaller the width of the coin, the less it is worth.

(c) The greater the mass of the coin, the wider it is.

(d) The mass of thirty 10c coins is greater than the mass of ten 50c coins.

(e) 9 kg of $1 coins is worth the same as 6.6 kg of $2 coins.

Activity 4

Use non-calculator methods for these calculations. Show your calculation methods.

(a) If you had one of each coin what is the -

(i) total value?

(ii) total mass?

(b) How much heavier is the 50c coin than the 10c coin?

(c) If you have forty 20c coins organised so the coins are touching and are in a straight line -

(i) what is the total value?

(ii) how long is the line?

(iii) what is the total mass?

(d) Which single coin is closest in mass to the total mass of the 5c coin and the 10c coin?

(e) What is the best way to divide the coins into two groups so the mass of each group is about the same?

Activity 5

Assume for this activity that all the coins are flat and lined up next to each so that they are touching. Rank these rows of coins in the order in which you would like to have them. Show justification for your decision. However, before you commence this activity, predict which row will be worth the most.

Row 1: 2 metres of $1 coins

Row 2: 1 metre of $2 coins

Row 3: 5 metres of 50c coins

Row 4: 20 m of 5c coins

Activity 6: Investigation

I have two coins in my pocket.
What amount of money might I have?

To guide you in this investigation, here are some steps to take.

1. Check that you understand the problem

Think of some questions that need to be asked, and some things that need to be checked before you can proceed.

2. Think!

Do you have an answer to this question?

Is there more than one answer?

How will you know that you have all the answers?

Is there a systematic way to check all the answers have been found?

3. Showing your answers

What is the best way to show your answers?
Show them below.

Are your answers clear for others to read and understand?

4. Extension

Assume a $5 coin is added to the Australian coins in circulation. Show a systematic way to identify all answers to the original question asked in Activity 6.

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Squares

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 20: SQUARES

Overview

In this task students will examine the effect of linear scaling on the area of a square. The activities bring together Number and Algebra with Measurement and Geometry and provide an opportunity for students to use square numbers and square roots.

Students will need

- calculators

Relevant content descriptions from the Western Australian Curriculum

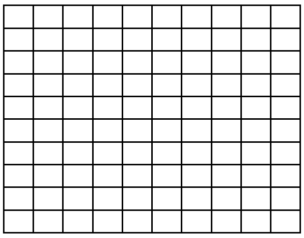
- Investigate and use square roots of perfect square numbers (ACMNA150)
- solve problems involving the comparison of lengths and areas using appropriate units (Year 6: ACMMG137)
- create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

Students can demonstrate

- *fluency* when they
    - calculate the length of the sides of a square given the area of the square
- *understanding* when they
    - identify changes in scale for the sides and areas of the squares
- *reasoning* when they
    - determine strategies and procedures to complete Activities 5 and 6
- *problem solving* when they
    - design an investigation for Activity 5

Review activity

1. Let the diagram below be a scaled diagram of a square which in actual size is 1 cm x 1cm. Each side is divided into 10 equal parts.

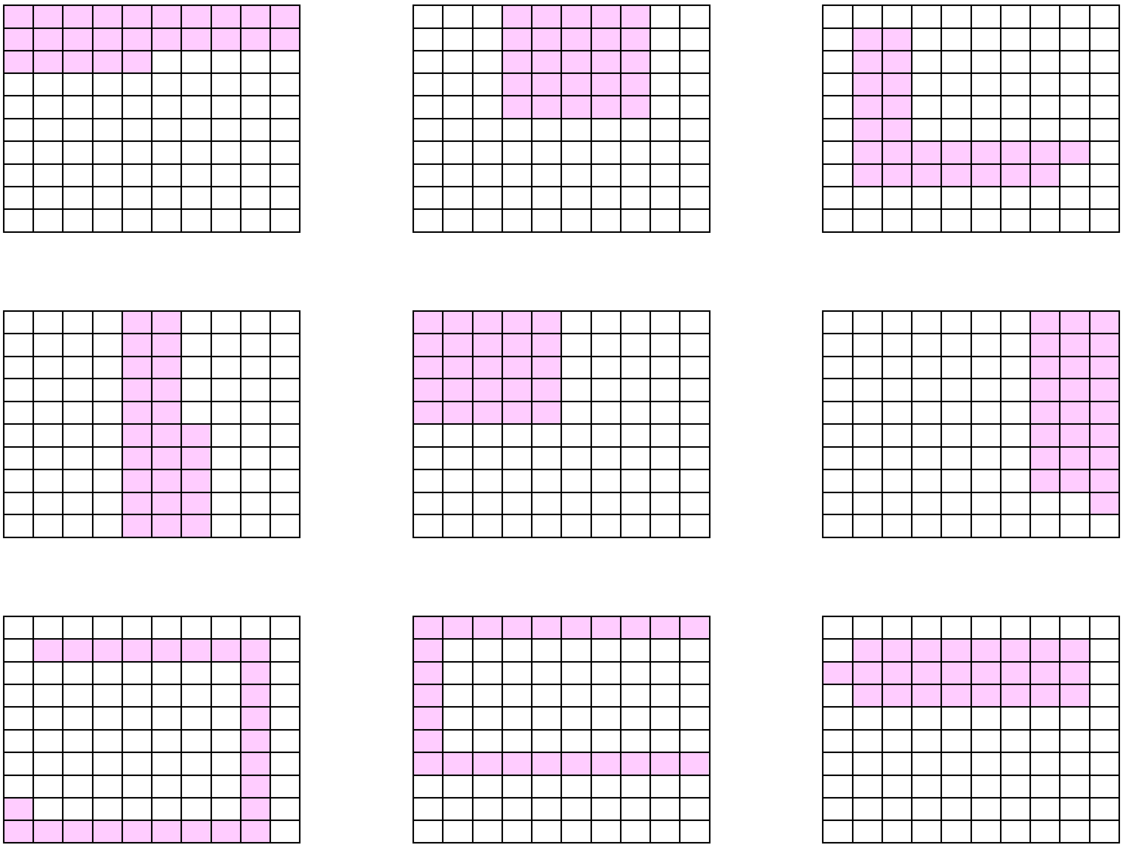


Explain why each of the smaller squares represents a square millimetre (mm^2).

There are 10 mm in 1 cm and each small square is one-tenth of a cm long, so 1 mm. It is a square with dimensions 1 mm x 1 mm so each square is 1 mm^2.

What does each row represent? 10 mm^2.

2. For each diagram below, shade in a different region of a continuous area of 25 mm^2.
Possible examples:



Does an object which has an area of 25 mm^2 have to be in the shape of a square? No

Activity 1: Investigating the area of squares

The table is used to record the dimensions of squares and their areas.

Length of side (m)	1	2	4	8	16
Area of square (m^2)	1	4	16	64	256

(a) Determine the values to complete the first row of the table.

(b) Calculate the area of the last square and record the data in the second row.

(c) From left to right on the first row of the table how is the length of the side changing?

For each new column the length of the side is double that in the previous column.

(d) From left to right on the second row of the table how is the area of the square changing?

The area is four times the area in the previous column.

(e) Use the tables below to check if the conclusion from part (d) also holds when the sides are different.

Length of side (m)	3	6	12	24	48
Area of square (m^2)	9	36	144	576	2304

Length of side (m)	5	10	20	40	80
Area of square (m^2)	25	100	400	1600	6400

What can you conclude?

If you double the length of the side of a square, the area of the new square is four times the area of the original square.

Activity 2

Length of side (m)	1	3	9	27	81
Area of square (m^2)	1	9	81	729	6561

(a) Calculate the lengths of the sides of the four squares and record the data in the first row above. Complete both rows by continuing the pattern.

(b) From left to right on the first row of the table how is the length of the side changing?

For each new column the length of the side is triple that in the previous column

(c) From left to right on the second row of the table how is the area of the square changing?

The area is nine times the area in the previous column

(d) Write a conclusion to summarise the patterns evident in this information.

If you triple (multiply by 3) the length of the side of a square, the area of the new square is nine times the area of the original square.

(e) Use the tables below to check if the conclusion from part (d) also occurs when the sides are different.

Length of side (m)	2	6	18	54	162
Area of square (m^2)	4	36	324	2916	26 244

Length of side (m)	5	15	45	135	405
Area of square (m^2)	25	225	2025	18 225	164 025

Did these calculations confirm the conclusion you made in part (d)?

Yes.

Activity 3

Length of side (m)	1	4	16	64	256
Area of square (m^2)	1	16	256	4096	65 536

(a) Calculate the lengths of the sides of the four squares and record the data in the first row. Use the pattern to complete the table.

(b) From left to right on the first row of the table, how is the length of the side changing?

Each length is quadruple (4 times) the length of the side in the previous column.

(c) From left to right on the second row of the table how is the area of the square changing?

The area is sixteen times the area in the previous column.

(d) Write a conclusion to summarise the patterns evident in this information.

If you enlarge the sides of a square by a factor of 4 (quadruple) then the area of the new square is 16 times the area of the original square.

(e) Use the tables below to check if the conclusion from part (d) also occurs when the sides are different.

Length of side (m)	2	8	32	128
Area of square (m^2)	4	64	1024	16 384

Length of side (m)	5	20	80	320
Area of square (m^2)	25	400	6400	102 400

Did these calculations confirm the conclusion you made in part (d)?

Yes.

Activity 4

Summarise your conclusions from the above investigation by completing the statements below.

When the sides of a square are doubled in length then the area is __4____ times the area of the original square.

When the sides of a square are trebled (3 times the size) in length then the area is ___9___ times the area of the original square.

When the sides of a square are quadrupled (4 times the size) in length then the area is ___16___ times the area of the original square.

This information is now summarised in a table to make the pattern clearer.

Increase to sides of a square	X 2	X 3	X 4
Matching increase in the area of the square	X 4	X 9	X 16

What type of numbers occur in the second row?

Square numbers

How do the numbers in the second row relate to the numbers in the first row?

The numbers in the second row are the squares of the numbers in the first row.

Continue the pattern to determine the change in the area of a square if -

(a) the sides are multiplied by 5

The new area is 25 times the original area.

(b) the sides are multiplied by 10

The new area is 100 times the original area.

If one square has an area which is 81 times the size of the area of another square, how many times longer are the sides of the larger square?

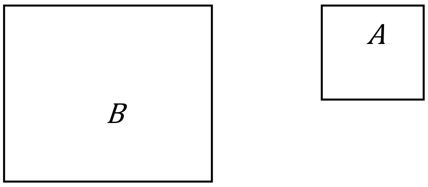
Need the square root of 81; i.e., 9, which can be shown as $\sqrt{81} = 9$

Activity 5

Investigate the following statement:

To reduce the area of any square to a quarter of its size, the sides need to be halved.

Show evidence to support your investigation.



Area of square B	100	64	256	16	400	10 000
Length of square B	10	8	16	4	20	100
Square A has a quarter of area of Square B	25	16	64	4	100	2500
Length of Square A	5	4	8	2	10	50

The six examples support this statement.
The first and third rows show the original area and then a quarter of the area.
The second and fourth rows show the original length and then the length of the “shrunken” square.
You can see that the new length is half of the original length.

Activity 6

Consider the results from Activity 4.
If the lengths of sides of a square are each multiplied by a number, how many times greater is the area of the larger square compared to the original square?

The area is “the square of the number multiplying the side length” times the original area.

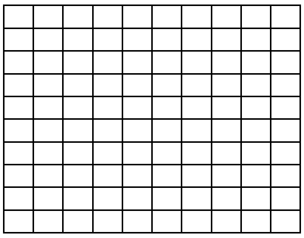
Explain, using algebra, why this works every time.

Say the side length is s.
Say the side length is multiplied by k

Area of first square = $s \times s$
New side = $s \times k$
New area = $s \times k \times s \times k = s \times s \times k \times k$
So the new area is $k \times k$ times the original area. That is k^2 times the original area.

Review activity

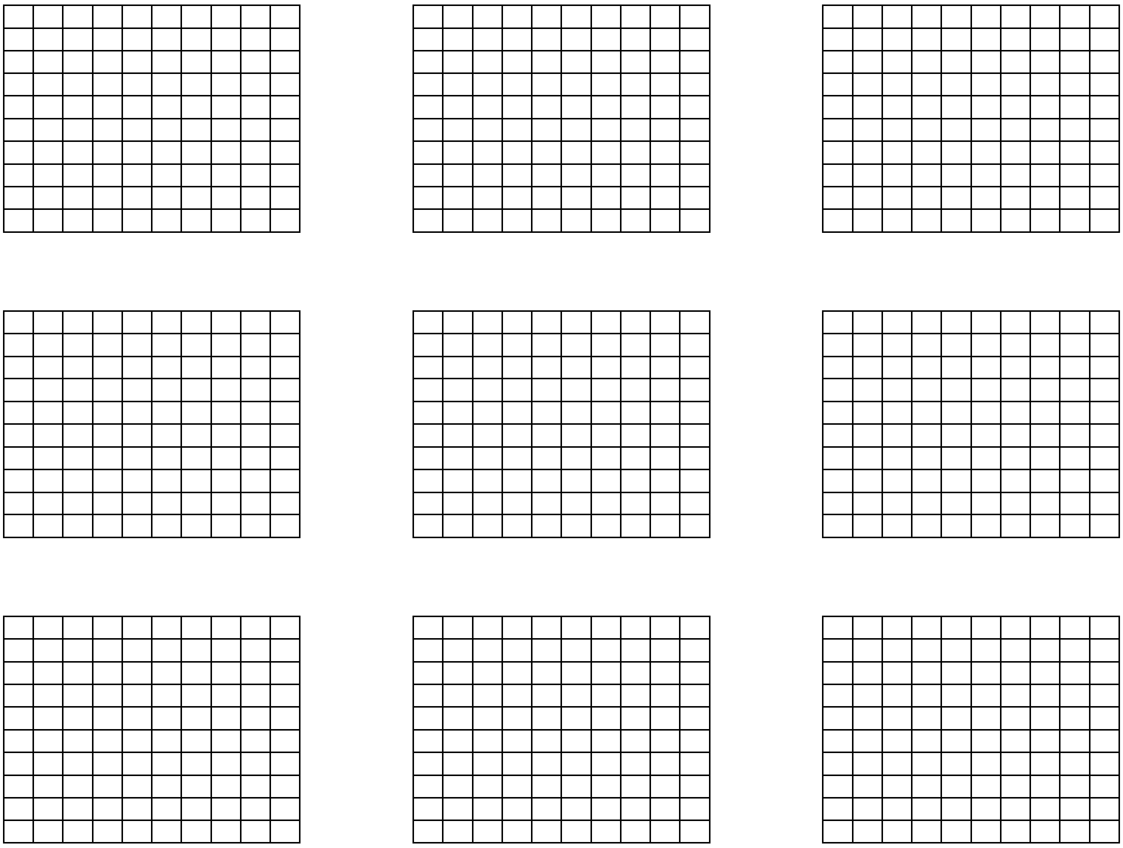
1. Let the diagram below be a scaled diagram of a square which in actual size is 1 cm x 1cm. Each side is divided into 10 equal parts.



Explain why each of the smaller squares represents a square millimetre (mm^2).

What does each row represent?

2. For each diagram below, shade in a different region of a continuous area of 25 mm^2.



Does an object which has an area of 25 mm^2 have to be in the shape of a square?

Activity 1: Investigating the area of squares

The table is used to record the dimensions of squares and their areas

Length of side (m)					
Area of square (m^2)	1	4	16	64	

(a) Determine the values to complete the first row of the table.

(b) Calculate the area of the last square and record the data in the second row.

(c) From left to right on the first row of the table how is the length of the side changing?

(d) From left to right on the second row of the table how is the area of the square changing?

(e) Use the tables below to check if the conclusion from part (d) also holds when the sides are different.

Length of side (m)					
Area of square (m^2)	9	36	144	576	2304

Length of side (m)					
Area of square (m^2)	25	100	400	1600	6400

What can you conclude?

Activity 2

Length of side (m)					
Area of square (m^2)	1	9	81	729	

(a) Calculate the lengths of the sides of the four squares and record the data in the first row above. Complete both rows by continuing the pattern.

(b) From left to right on the first row of the table how is the length of the side changing?

(c) From left to right on the second row of the table how is the area of the square changing?

(d) Write a conclusion to summarise the patterns evident in this information.

(e) Use the tables below to check of the conclusion from part (d) also occurs when the sides are different.

Length of side (m)					
Area of square (m^2)	4	36	324	2916	26 244

Length of side (m)					
Area of square (m^2)	25	225	2025	18 225	164 025

Did these calculations confirm the conclusion you made in part (d)?

Activity 3

Length of side (m)					
Area of square (m^2)	1	16	256	4096	

(a) Calculate the lengths of the sides of the four squares and record the data in the first row. Use the pattern to complete the table.

(b) From left to right on the first row of the table, how is the length of the side changing?

(c) From left to right on the second row of the table, how is the area of the square changing?

(d) Write a conclusion to summarise the patterns evident in this information.

(e) Use the tables below to check of the conclusion from part (d) also occurs when the sides are different.

Length of side (m)				
Area of square (m^2)	4	64	1024	16 384

Length of side (m)				
Area of square (m^2)	25	400	6400	102 400

Did these calculations confirm the conclusion you made in part (d)?

Activity 4

Summarise your conclusions from the above investigation by completing the statements below.

When the sides of a square are doubled in length then the area is ______ times the area of the original square.

When the sides of a square are trebled (3 times the size) in length then the area is ______ times the area of the original square.

When the sides of a square are quadrupled (4 times the size) in length then the area is ______ times the area of the original square.

This information is now summarised in a table to make the pattern clearer.

Increase to sides of a square	X 2	X 3	X 4
Matching increase in the area of the square			

What type of numbers occur in the second row?

How do the numbers in the second row relate to the numbers in the first row?

Continue the pattern to determine the change in the area of a square if -

(a) the sides are multiplied by 5

(b) the sides are multiplied by 10

If one square has an area which is 81 times the size of the area of another square, how many times larger are the sides of the larger square?

Activity 5

Investigate the following statement.

To reduce the area of any square to a quarter of its size, the sides need to be halved.

Show evidence to support your investigation.

Activity 6

Consider the results from Activity 4.
If the lengths of sides of a square are each multiplied by a number, how many times greater is the area of the larger square compared to the original square?

Explain, using algebra, why this works every time.

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Angles

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 22: ANGLES

Overview

This activity starts with a review of the concept of angles and then provides activities so that students can discover that vertically opposite angles are equal. In Activity 2, the students measure the angles formed when a line transverses a pair of parallel lines and examine the relationships formed. It is left to the teacher to develop the terms co-interior, corresponding and alternate; but this task provides background material for understanding these types of relationships between angles.

Students will need

- rulers
- protractors

Relevant content descriptions from the Western Australian Curriculum

- Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal (ACMMG163)
- Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)

Students can demonstrate

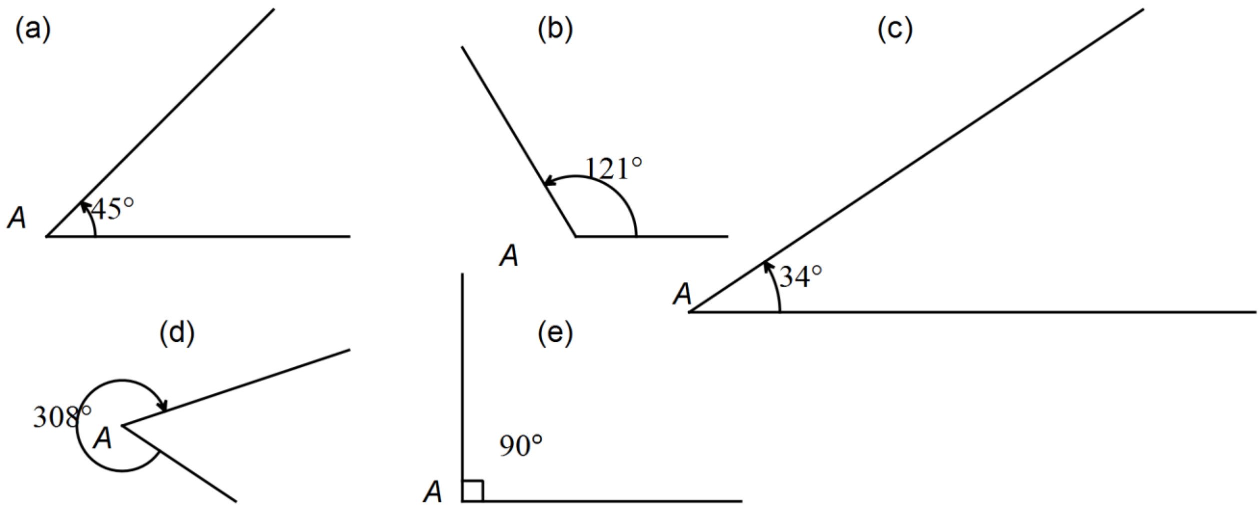
- *fluency* when they
  - measure angles accurately
- *understanding* when they
  - connect measurements of angles to known facts about angles on a straight line and angles in one full revolution
  - recognise opposite and adjacent angles
- *reasoning* when they
  - describe the relative positions of equal and supplementary angles
- *problem solving* when they
  - can independently execute the Extension activity

<u>Review activity</u>

1. Everyone stand up and face the front

- Turn 90^o clockwise
- Turn 180^o anticlockwise
- Turn 360^o anticlockwise
- At what angle do you need to turn to be back in the starting position?
  90^0 clockwise

2. Without measuring, identify the diagram showing the greatest angle?
   (d)



2. Write on each diagram the size of the angle.
   Label each vertex with the letter A.

3. Which of the following descriptions best defines what an angle measures?

The space between two lines

The area between two lines

The distance one line travels to meet another

The distance between two lines

The amount of turn as one line swings from another

Justify your choice of the best description.

Answers will vary.

Activity 1

There are four sets of intersecting lines shown below. The angles formed at the intersections are labelled with the letters m, n, w and h.

1. Measure the angles and enter your results in the table provided.

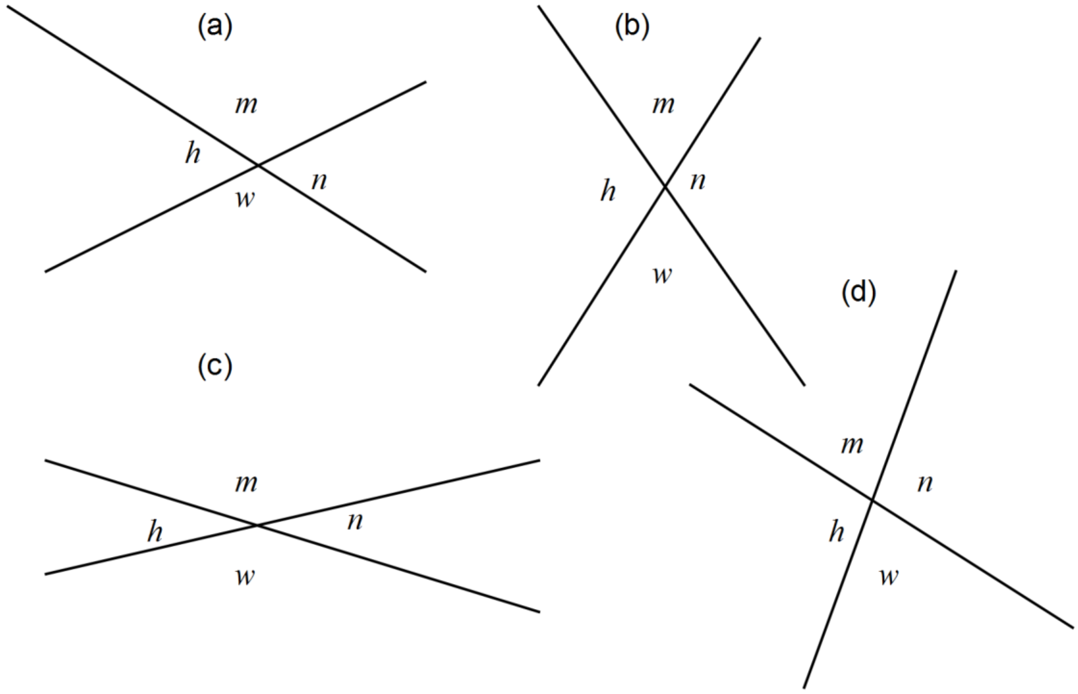


Diagram	m	n	w	h
(a)	121	59	121	59
(b)	68	112	68	112
(c)	150	30	150	30
(d)	77	103	77	103

2. Describe what you notice about the sizes of the four angles m, n, w and h.

In each diagram

- They all add to 360°
- Opposite angles are equal; e.g., $m = w$ and $n = h$
- Adjacent angles add to 180°.
  $m + n = 180°$, $n + w = 180°$, $w + h = 180°$, $h + m = 180°$

3. What do the following words mean?

(a) adjacent next to

(b) intersecting crossing

(c) opposite across the way from

4. In each diagram the letters were placed in similar positions so these answers will be the same for each diagram.

What is the letter given to -

(a) the angle vertically opposite to m? w

(b) the angle vertically opposite to h? n

5. Using the letters on the diagrams -

(a) name all the angles adjacent to m n, h

(b) name all the angles adjacent to h m, w

(c) name all the pairs of angles that add to 180° because they are on a straight line

$m + n = 180^{\circ}$ and $n + w = 180^{\circ}$ and $w + h = 180^{\circ}$ and $h + m = 180^{\circ}$

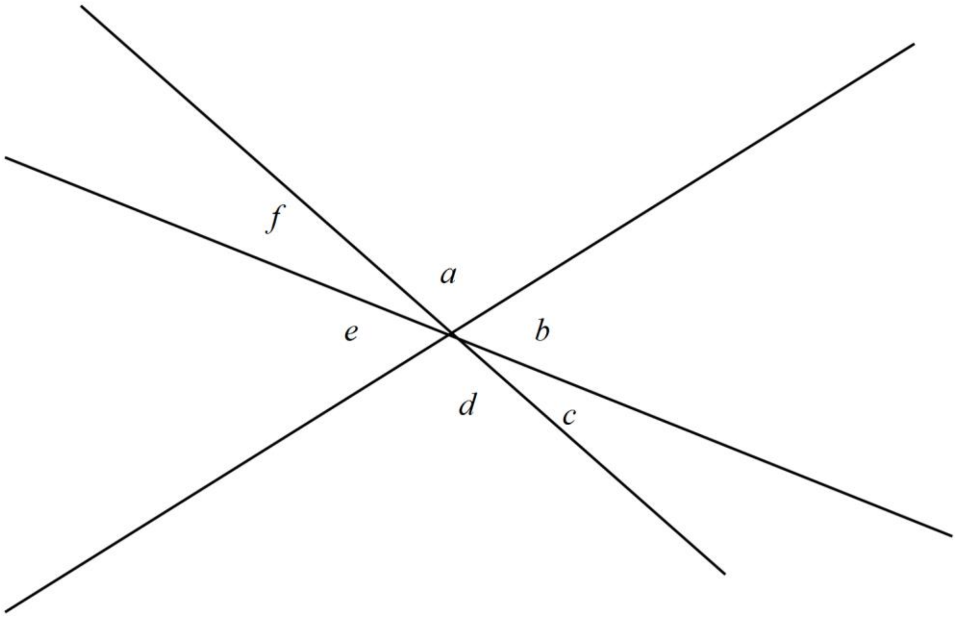
(d) name all the pairs of angles that are vertically opposite each other.

m and w; n and h.

6. Write a conclusion about this activity by completing this statement:

When two straight lines intersect, the opposite angles formed are equal.

Extension: This diagram shows three lines intersecting at the same point. Use the letters provided to identify all pairs of opposite angles, all sets of angles which are supplementary; i.e., add to 180°, and all angles adjacent to the marked angles.



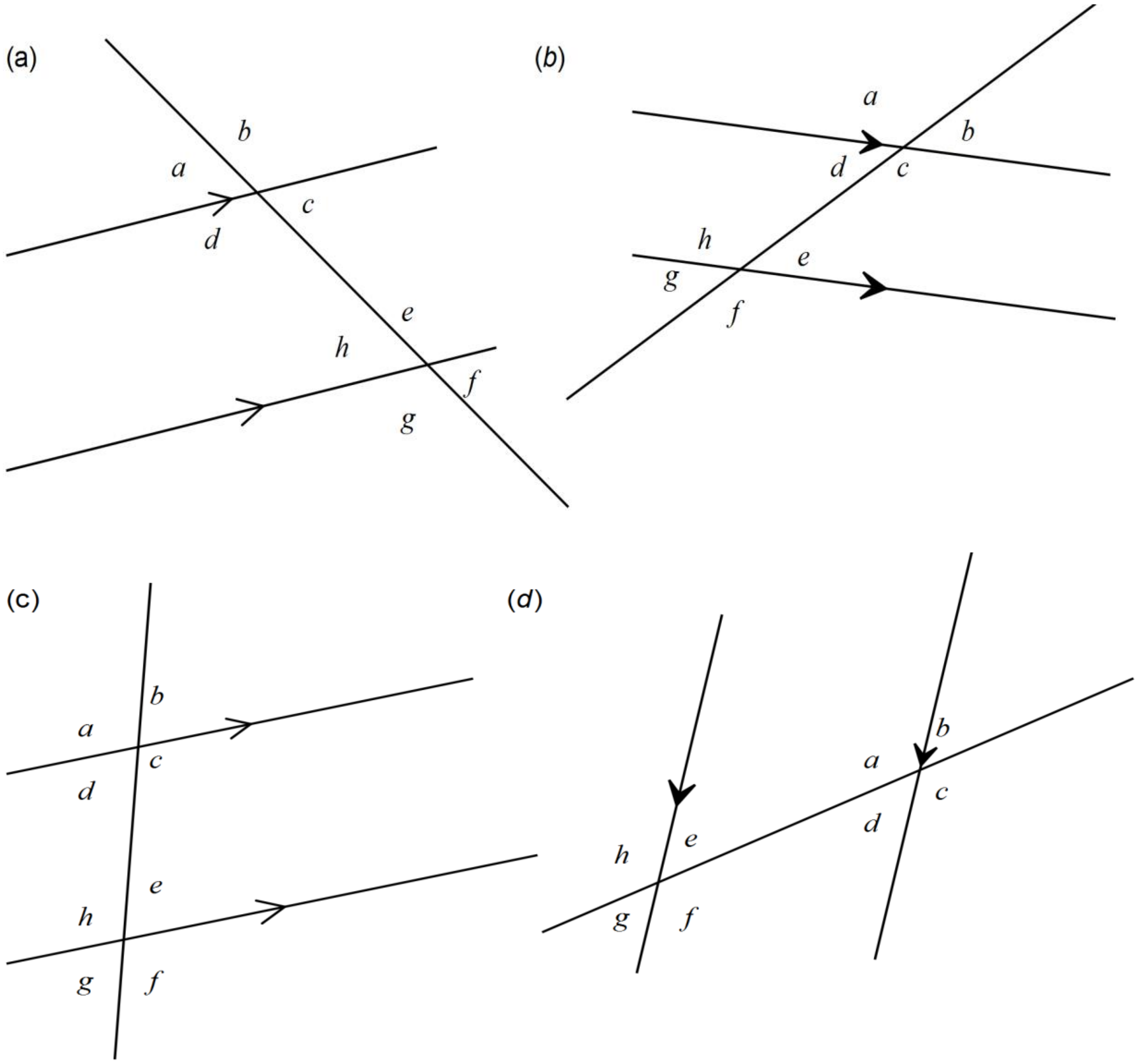
All pairs of vertically opposite angles are: *a* and *d*, *b* and *e*, *f* and *c*

All sets of supplementary angles are -
a , *b* and *c*,
b, *c* and *d*,
c, *d* and *e*,
d, *e* and *f*,
e, *f* and *a*,
f, *a* and *b*.

All sets of adjacent angles are -
a is adjacent to *b* and *f*,
b is adjacent to *c* and *a*,
c is adjacent to *d* and *b*,
d is adjacent to *c* and *e*,
e is adjacent to *d* and *f*,
f is adjacent to *e* and *a*.

Activity 2

1. In each of the following diagrams there is a pair of parallel lines marked with special symbols to indicate they are parallel, and there is a transversal.
For each diagram -
(i) identify the parallel lines and the transversal. [Hint: transverse means to go across]
(ii) measure all the angles marked with letters and write your results in the table provided.



No.	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*
(a)	60	120	60	120	120	60	120	60
(b)	135	45	135	45	45	135	45	135
(c)	106	74	106	74	74	106	74	106
(d)	128	52	128	52	52	128	52	128

2. The following directions apply equally to all the above diagrams in Activity 2.

(i) Write down all pairs of opposite angles.

a and c, b and d, f and h, e and g

(ii) What is the sum of each pair of angles?

$a + b = 180^o$	$b + c = 180^o$	$c + d = 180^o$	$d + a = 180^o$
$e + f = 180^o$	$f + g = 180^o$	$g + h = 180^o$	$h + e = 180^o$

What is the name given to angles which add up to this number?

Angles which add to 180^o are said to be supplementary.

(iii) What is the sum of angles $a + b + c + d$? 360^o $e + f + g + h$? 360^o

What does this number represent?

One full revolution or two straight angles,

(iv) Name two angles that are adjacent to angle f.

g *and* e

(v) Name two angles that are adjacent to angle b.

a *and* c

(vi) Name all the angles that are equal in size to angle a.

$a = c = f = h$

(vii) Name all the angles that are equal in size to angle b.

$b = d = g = e$

(viii) In the table, highlight in one colour, all the angles that are equal in size to angle a.

(ix) In the table, highlight in a different colour, all the angles that are equal in size to angle b.

(x) Describe the colour pattern of your highlighting.

The colour pattern is the same from diagram to diagram because $a = c = f = h$ and $b = d = g = e$ in every diagram.

(xi) Consider the following pairs of angles: a and h, b and e, c and f, d and g.

The angles in each pair are equal to each other.
Look at their position relative to each other and give a reason why they might be equal.

Two possible reasons that students may give are as follows.

For each pair they are on the same side of the transversal. They are also both above or both below the two parallel lines.

Because the lines are parallel they are sloping at the same angle to the horizontal. This means that the amount of turn from the parallel line to the transversal is the same in each case.

(xii) Consider the following pairs of angles: c and e, d and h

What is always the sum of each of these pairs of angles?

360^0

(xiii) Draw two more diagrams of your own and test the findings from this Activity.
Do you get the same results for your diagrams as you have for the supplied diagrams?

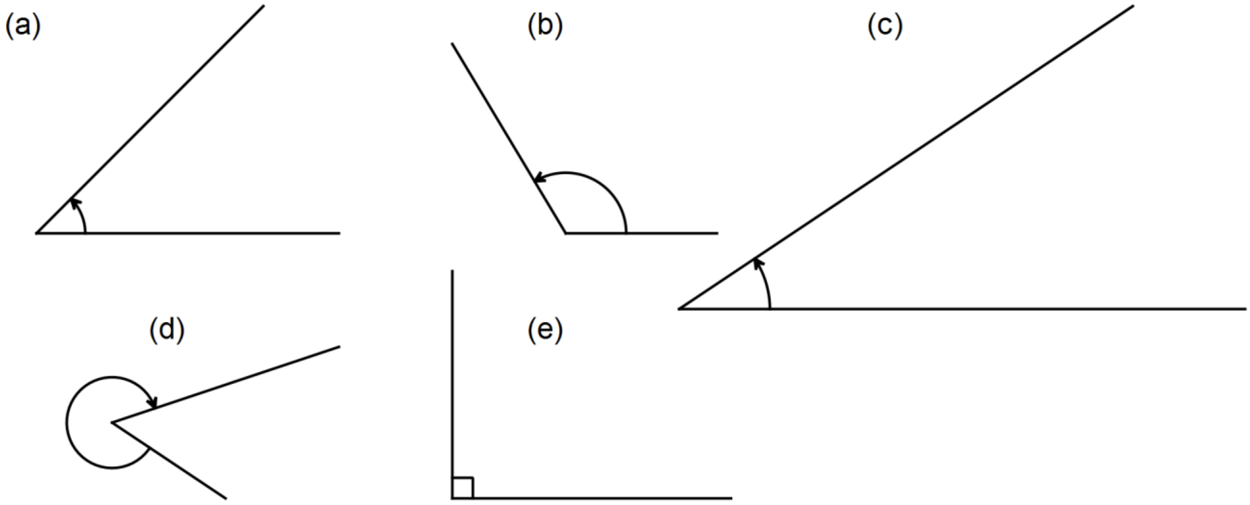
Answers will vary. Students should achieve the same results as for the diagrams provided. Not exactly the same sizes of angles but the same sets of equal, adjacent or supplementary angles.

Review activity

1. Everyone stand up and face the front.

- Turn 90^o clockwise
- Turn 180^o anticlockwise
- Turn 360^o anticlockwise
- At what angle do you need to turn to be back in the starting position?

Without measuring, identify the diagram showing the greatest angle?



2. Write on each diagram the size of the angle.
   Label each vertex with the letter A.

3. Which of the following descriptions best defines what an angle measures?

The space between two lines

The area between two lines

The distance one line travels to meet another

The distance between two lines

The amount of turn as one line travels to another

Justify your choice of the best description.

Activity 1

There are four sets of intersecting lines shown below. The angles formed at the intersections are labelled with the letters m, n, w and h.

1. Measure the angles and enter your results in the table provided.

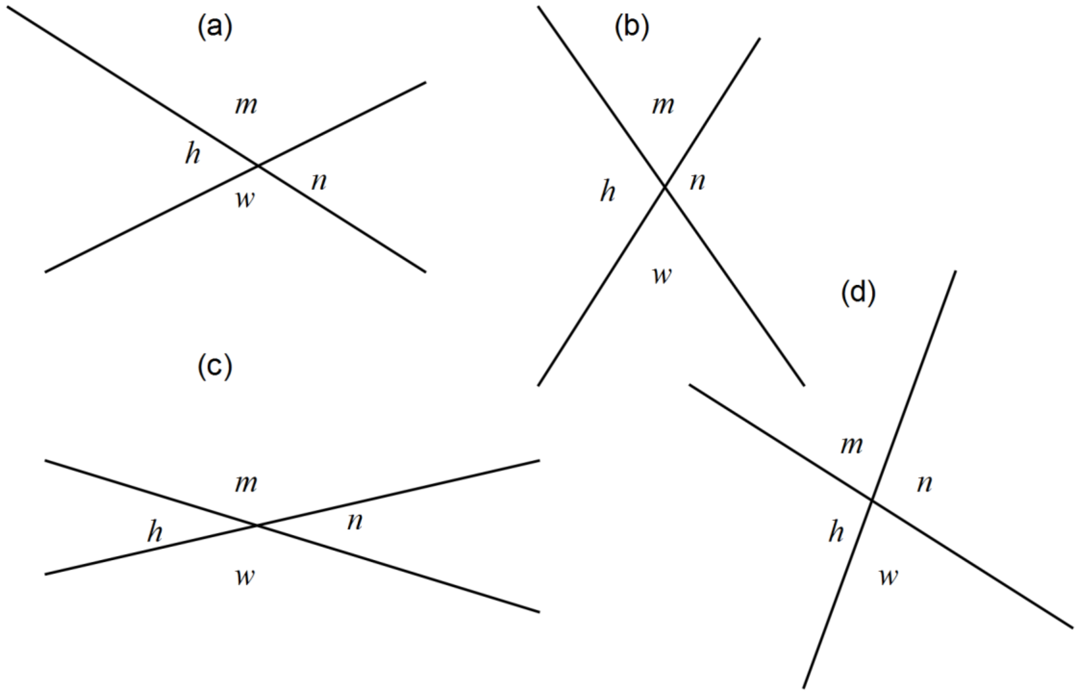


Diagram	***m***	***n***	***w***	***h***
(a)				
(b)				
(c)				
(d)				

2. Describe what you notice about the sizes of the four angles m, n, w and h.

3. What do the following words mean?

(a) adjacent

(b) intersecting

(c) opposite

4. In each diagram the letters were placed in similar positions so these answers will be the same for each diagram.

What is the letter given to -

(a) the angle vertically opposite to m?

(b) the angle vertically opposite to h?

5. Using the letters on the diagrams -

(a) name all the angles adjacent to m

(b) name all the angles adjacent to h

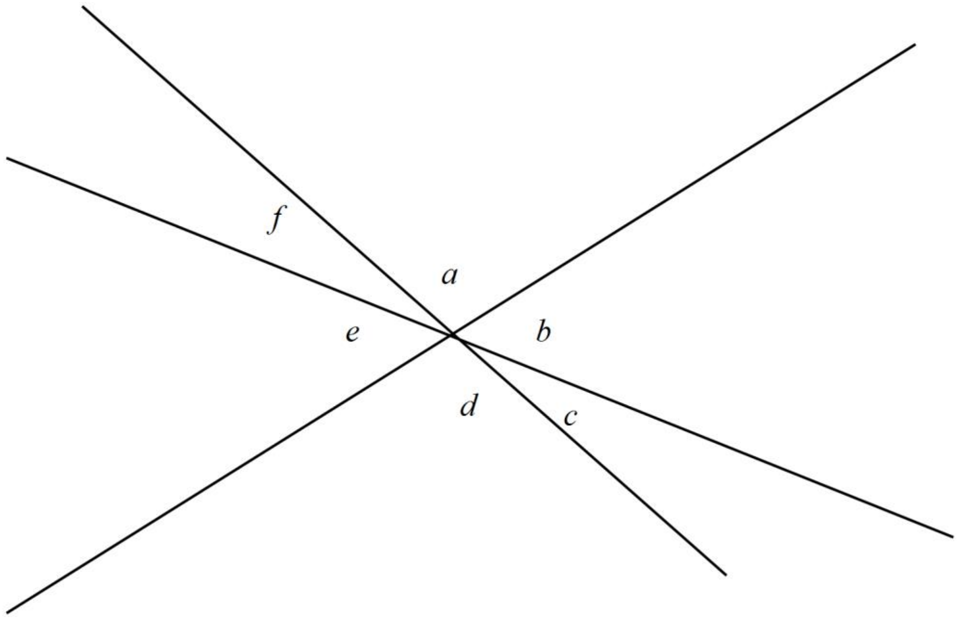
(c) name all the pairs of angles that add to 180^o because they are on a straight line

(d) name all the pairs of angles that are vertically opposite each other.

6. Write a conclusion about this activity by completing this statement:

When two straight lines intersect, the vertically opposite angles formed are . . .

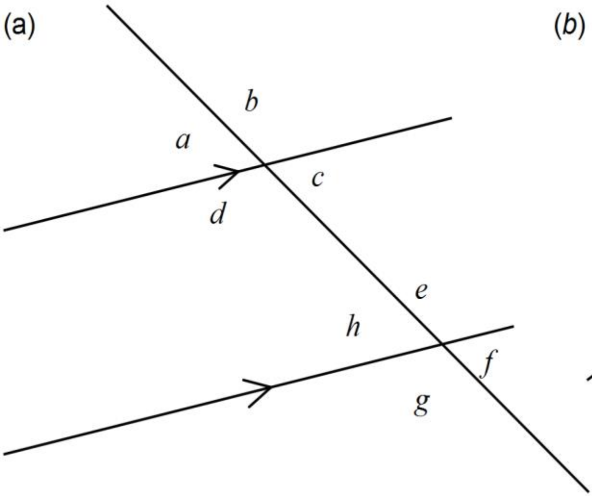
Extension: This diagram shows three lines intersecting at the same point. Use the letters provided to identify all pairs of vertically opposite angles, all sets of angles which are supplementary; i.e., add to 180°, and all angles adjacent to the marked angles.



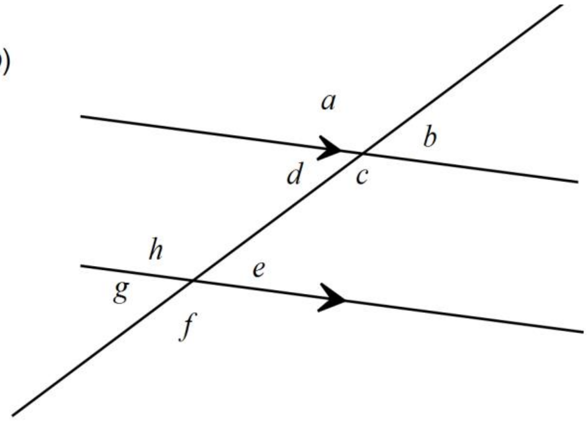
Activity 2

1. In each of the following diagrams there is a pair of parallel lines marked with special symbols to indicate they are parallel, and there is a transversal.
   For each diagram -
   (i) identify the parallel lines and the transversal. [Hint: transverse means to go across]
   (ii) measure all the angles marked with letters and write your results in the table provided.

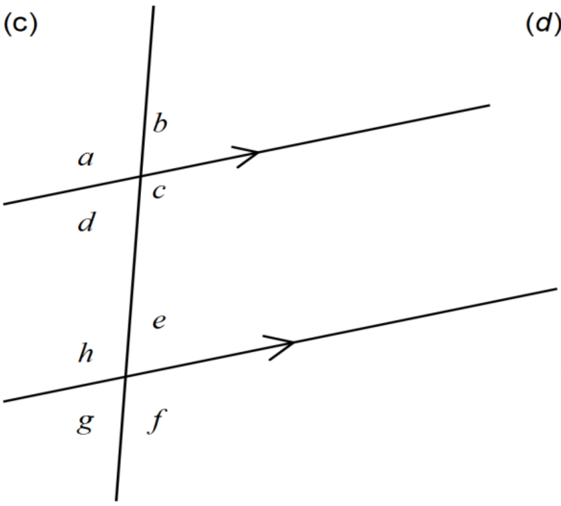
(a)



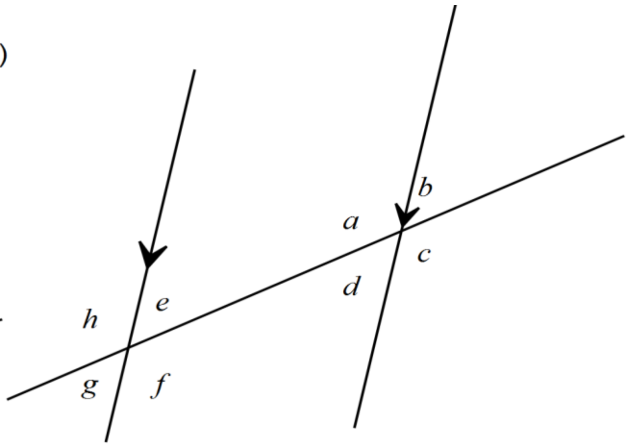
(*b*)



(c)



(*d*)



No.	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*
(a)								
(b)								
(c)								
(d)								

2. The following directions apply equally to all the above diagrams in Activity 2.

(i) Write down all pairs of opposite angles.

(ii) What is the sum of each pair of angles?

$a + b$ ___________ $b + c$ ________ $c + d$ _________ $d + a$ _________

$e + f$ ___________ $f + g$ ________ $g + h$ _________ $h + e$ _________

What is the name given to angles which add up to this number?

(iii) What is the sum of angles $a + b + c + d$? $e + f + g + h$?

What does this number represent?

(iv) Name two angles that are adjacent to angle f.

(v) Name two angles that are adjacent to angle b.

(vi) Name all the angles that are equal in size to angle a.

(vii) Name all the angles that are equal in size to angle b.

(viii) In the table, highlight in one colour, all the angles that are equal in size to angle a.

(ix) In the table, highlight in a different colour, all the angles that are equal in size to angle b.

(x) Describe the colour pattern of your highlighting.

(xi) Consider the following pairs of angles. a and h, b and e, c and f, d and g.

The angles in each pair are equal to each other.
Look at their position relative to each other and give a reason why they might be equal.

(xii) Consider the following pairs of angles. c and e, d and h

What do the angles in these pairs always add up to?

(xiii) Draw two more diagrams of your own and test the findings from this Activity.
Do you get the same results for your diagrams as you have for the supplied diagrams?

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Drawings

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 23: DRAWINGS

Overview

In this task students will need to combine their knowledge of geometric facts, features of shapes and understanding of measurement to draw polygons to the required specifications. These activities provide opportunities for robust discussions between students and for teachers to ask probing questions when students are unable to determine how to respond to the tasks provided. Ideally, the activities should be done by small groups of students.

Students will need

- rulers
- possibly extra grid paper

Relevant content descriptions from the Western Australian Curriculum

- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)
- Classify triangles according to their side and angle properties and describe quadrilaterals (ACMMG165)
- Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

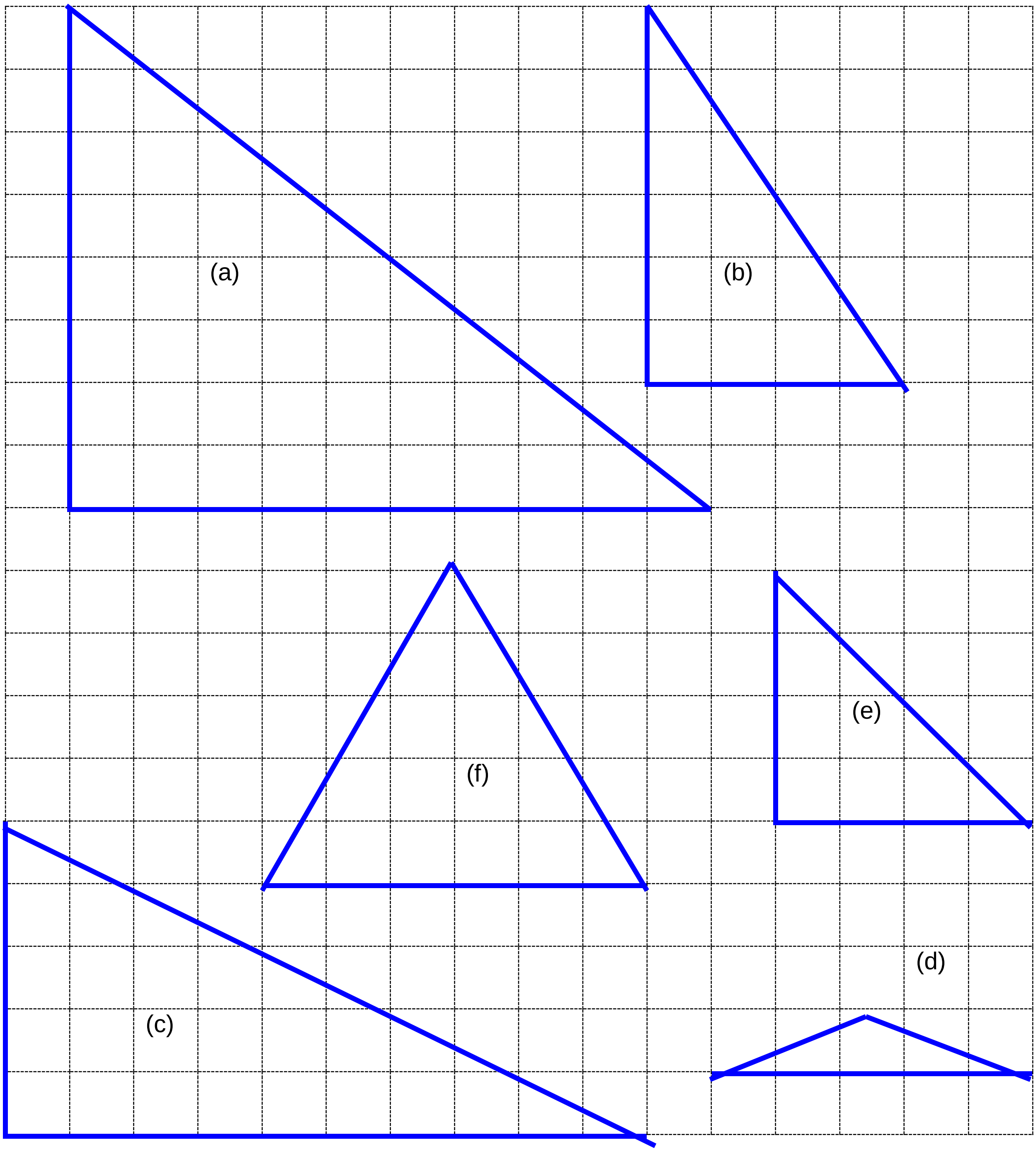
Students can demonstrate

- *understanding* when they
  - connect geometric features of shapes and their areas
  - represent the geometric relationships using variables
- *reasoning* when they
  - apply their discoveries about polygons in Activity 4 to draw conclusions
  - transfer their learning about the calculation of area and perimeter into new situations to determine the original dimensions of the shape

Activity 1: Triangles

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams. Label each diagram clearly.

(a) a right-angled triangle with a base equal to 10 cm and an area of 40 cm^2
(b) a right-angled triangle with an area of 12 cm^2 and a height of 6 cm
(c) a triangle with an area of 25 cm^2
(d) an obtuse-angled isosceles triangle (Student answers may vary in size and shape.)
(e) a right-angled isosceles triangle (Student answers may vary in size and shape.)
(f) an equilateral triangle with a perimeter of 18 cm.

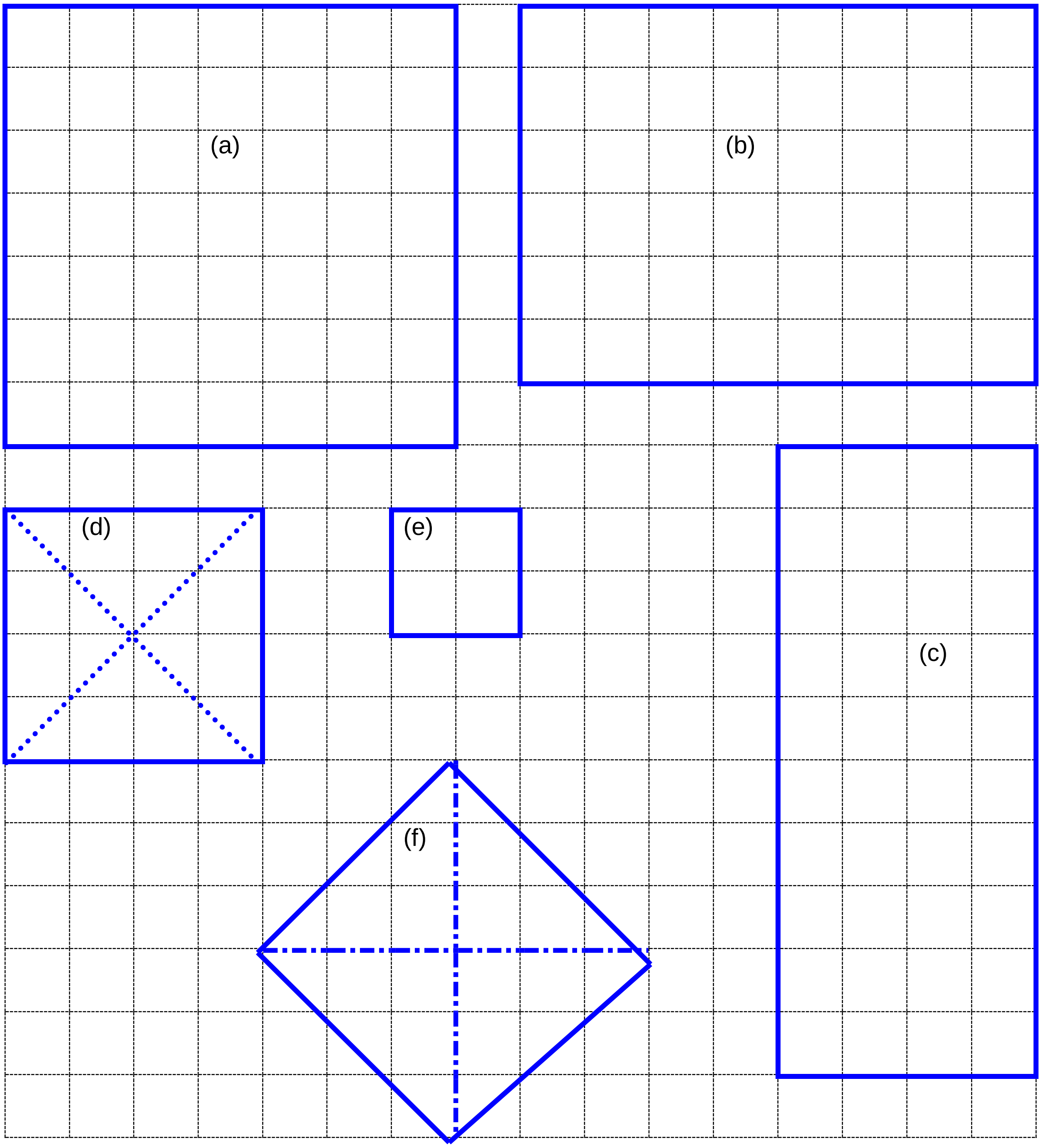


Activity 2: Drawing squares and rectangles

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams. Label each diagram clearly.

(a) a square with a perimeter of 28 cm
(b) a rectangle, which has a perimeter of 28 cm, but it is not a square
(c) another rectangle with a perimeter of 28 cm, but it has a different area to the rectangles in parts (a) and (b)
(d) a rectangle with diagonals which intersect at right angles
(e) a rectangle with four equal sides and four equal angles
(f) a rectangle with two diagonals which are both equal to 6 cm.

Student diagrams may vary in size and shape and still be appropriate for some of these.



Activity 3: Drawing other quadrilaterals

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams, and label each diagram clearly:

(a) a parallelogram of area 20 cm^2

(b) another parallelogram of area 20 cm^2 but with a different perimeter to the parallelogram in (a)

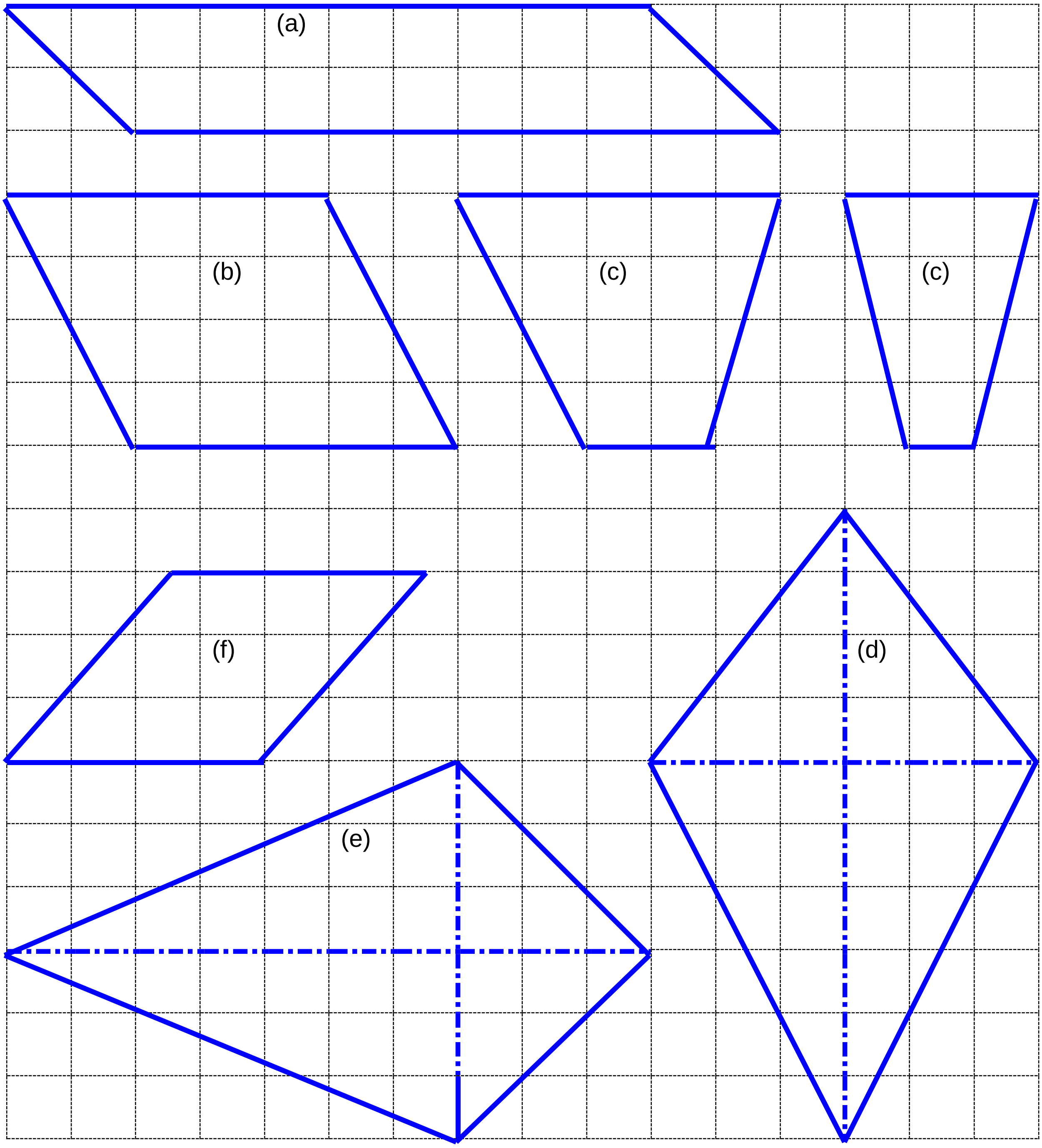
(c) two trapeziums with different areas but both having a height of 4 cm

(d) a kite with diagonals of 6 cm and 10 cm

(e) another kite with diagonals of 6 cm and 10 cm but with a different perimeter to the kite in (d)

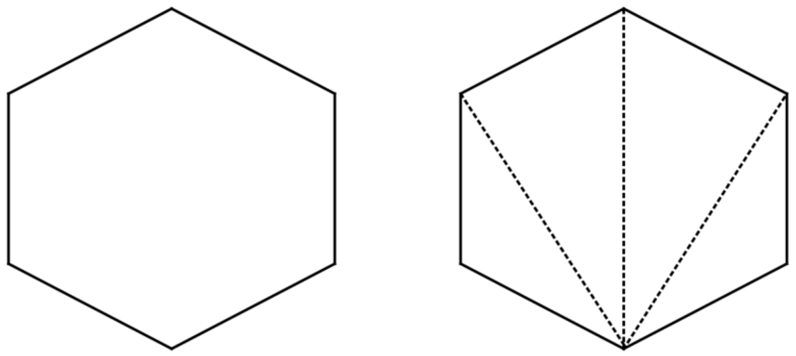
(f) a quadrilateral with four equal sides, each 4 cm long, but without four equal angles.

Student diagrams may vary in size and shape and still be appropriate for some of these.

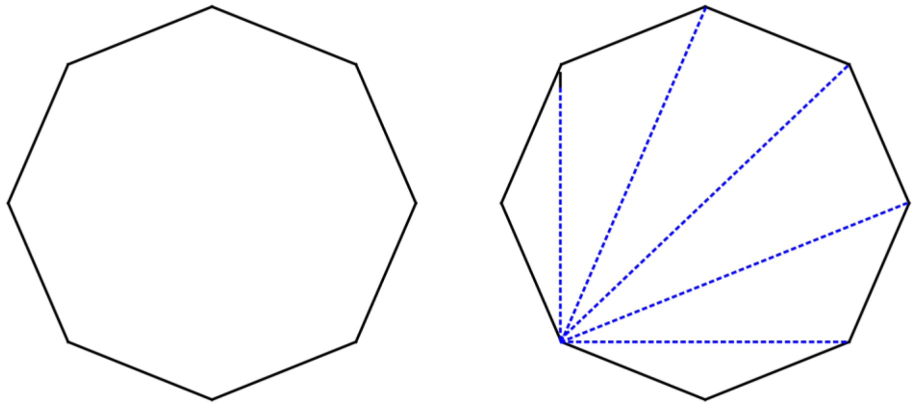


Activity 4: Drawing other polygons

The diagram below shows two hexagons of identical size.
In the regular hexagon on the right, diagonal lines are drawn from one vertex to other vertices to divide the hexagon into four triangles. The internal lines DO NOT cross.



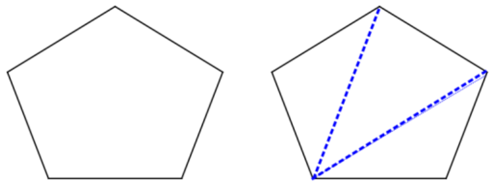
1. Repeat this process for the regular octagon drawn below.



How many lines did you use? 5

How many triangles did you make? 6

2. Repeat this process for a regular pentagon.



How many lines did you use? 2

How many triangles did you make? 3

3. Enter the results of this investigation into the table below. Draw diagrams to determine the data needed when there are 7 or 8 sides.

Polygon name	Number of sides	Number of vertices	Number of lines drawn	Number of triangles made
	p	v	k	t
triangle	3	3	0	1
quadrilateral	4	4	1	2
pentagon	5	5	2	3
hexagon	6	6	3	4
heptagon	7	7	4	5
octagon	8	8	5	6
nonagon	9	9	6	7
decagon	10	10	7	8

Use the pattern to complete the last two rows.

4. Describe in words the relationships between each of these features in each polygon.

(a) the number of sides and the number of vertices

The number of sides equals the number of vertices.

(b) the number of sides and the number of diagonal lines drawn

The number of sides is 3 more than the number of lines drawn.

(c) the number of diagonal lines drawn and the number of triangles made

The number of lines drawn is 1 less than the number of triangles drawn.

(d) the number of sides and the number of triangles made.

The number of sides is 2 more than the number of triangles drawn.

5. Use the letters provided in the table (p, v, k, t) to write the rule for the relationships between each of the features in each polygon.

   (a) the number of sides and the number of vertices $p = v$

   (b) the number of lines drawn and the number of sides $k = p - 3$

   (c) the number of triangles made and the number of lines drawn $t = k + 1$

   (d) the number of triangles made and the number of sides. $t = p - 2$

6. If you had a polygon with 20 sides,

   (a) how many vertices would it have? 20

   (b) how many diagonal lines could you draw as was done above? 17

   (c) how many triangles would you make with these lines? 18

7. If you had a MEGA-POLYGON and it was divided using the above process, into 100 triangles, how many sides would the polygon have? 102

Activity 1: Triangles

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams. Label each diagram clearly.

(a) a right-angled triangle with a base equal to 10 cm and an area of 40 cm^2
(b) a right-angled triangle with an area of 12 cm^2 and a height of 6 cm
(c) a triangle with an area of 25 cm^2
(d) an obtuse-angled isosceles triangle
(e) a right-angled isosceles triangle
(f) an equilateral triangle with a perimeter of 18 cm.

Activity 2: Drawing squares and rectangles

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams. Label each diagram clearly.

(a) a square with a perimeter of 28 cm
(b) a rectangle, which has a perimeter of 28 cm, but it is not a square
(c) another rectangle with a perimeter of 28 cm, but it has a different area to the rectangles in parts (a) and (b)
(d) a rectangle with diagonals which intersect at right angles
(e) a rectangle with four equal sides and four equal angles
(f) a rectangle with two diagonals which are both equal to 6 cm.

Activity 3: Drawing other quadrilaterals

On the grid below, use the fact that each row is 1 cm high and each column is 1 cm wide, to draw the following diagrams, and label each diagram clearly:

(a) a parallelogram of area 20 cm^2

(b) another parallelogram of area 20 cm^2 but with a different perimeter to the parallelogram in (a)

(c) two trapeziums with different areas but both having a height of 4 cm

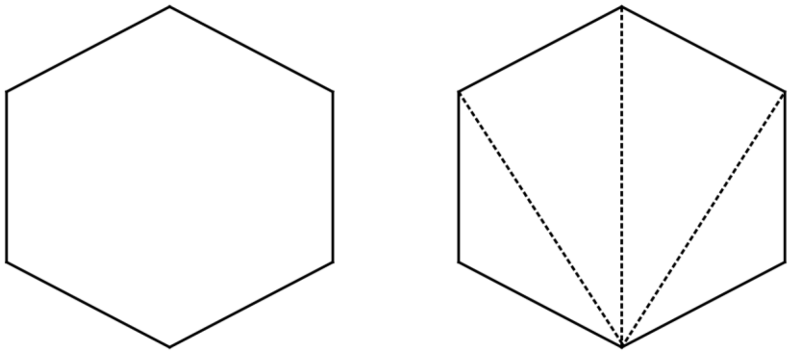
(d) a kite with diagonals of 6 cm and 10 cm

(e) another kite with diagonals of 6 cm and 10 cm but with a different perimeter to the kite in (d)

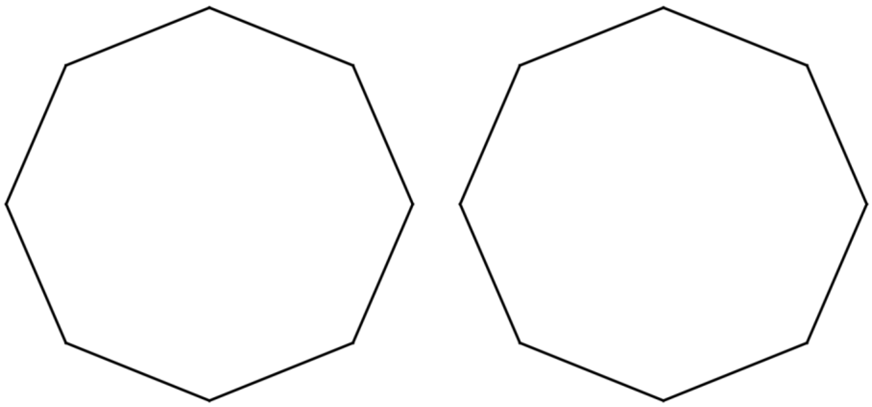
(f) a quadrilateral with four equal sides, each 4 cm long, but without four equal angles

Activity 4: Drawing other polygons

The diagram below shows two hexagons of identical size.
In the regular hexagon on the right, diagonal lines are drawn from one vertex to other vertices to divide the hexagon into four triangles. The internal lines DO NOT cross.



1. Repeat this process for the regular octagon drawn below.



How many lines did you use?

How many triangles did you make?

2. Repeat this process for a regular pentagon

How many lines did you use?

How many triangles did you make?

3. Enter the results of this investigation into the table below.
   Draw diagrams to determine the data needed when there are 7 or 8 sides.

Polygon name	Number of sides	Number of vertices	Number of lines drawn	Number of triangles made
	p	v	k	t
triangle	3		0	1
	4			
	5			
hexagon	6		3	4
	7			
octagon	8			

Use the pattern to complete the last two rows.

4. Describe in words the relationships between each of these features in each polygon.

   (a) the number of sides and the number of vertices

   (b) the number of sides and the number of diagonal lines drawn

   (c) the number of diagonal lines drawn and the number of triangles made

   (d) the number of sides and the number of triangles made

5. Use the letters provided in the table (*p, v, k, t*) to write the rule for the relationships between each of the features in each polygon.

   (a) the number of sides and the number of vertices

   (b) the number of lines drawn and the number of sides

   (c) the number of triangles made and the number of lines drawn

   (d) the number of triangles made and the number of sides

6. If you had a polygon with 20 sides,

   (a) how many vertices would it have?

   (b) how many diagonal lines could you draw as was done above?

   (c) how many triangles would you make with these lines?

7. If you had a MEGA-POLYGON and it was divided using the above process above, into 100 triangles, how many sides would the polygon have?

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Spinners

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 29: SPINNERS

Overview

In this task there is an opportunity for students to review angle measurement and probability and to reflect on chance events.

Students will need

- cardboard
- scissors
- compass
- protractor
- calculators

Relevant content descriptions from the Western Australian Curriculum

- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)
- Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)
- Construct and compare a range of data displays including stem-and-leaf plots and dot plots (ACMSP170)
- Estimate, measure and compare angles using degrees. Construct angles using a protractor. (ACMMG112) (Year 5)

Students can demonstrate

- *fluency* when they
  - measure and create angles for their spinners
  - determine the proportions of each outcome
  - plan and execute a graph of their results
- *understanding* when they
  - represent probability and proportions as fractions, decimals and percentages
- *reasoning* when they
  - explain the differences in the expected probability and the resulting proportions
- *problem solving* when they
  - calculate the angles for the spinner
  - design the spinner and determine the probability of each outcome
  - interpret the results of the two trials for the spinners

This task involves the creation and use of spinners. Each group will create their own spinner, test it and create a poster showing the results and conclusions from their experiments.

Solutions will vary according to the design of the spinner. A set of solutions is provided for the spinner drawn below and this will give an indication of possible student responses.

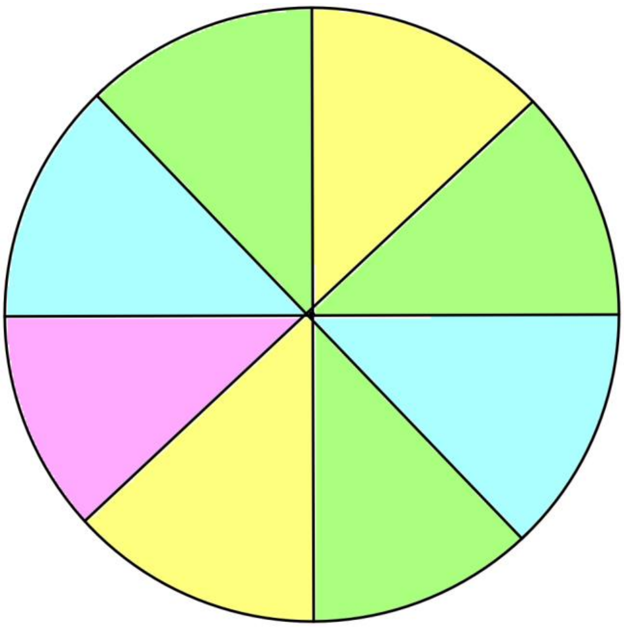
<u>Activity 1</u>

Create a spinner with the following features:

- the spinner is circular
- there are at least three different outcomes and no more than 8
- each spin is independent of the previous spin
- the outcome of each spin relies on chance.

[Suggestion: Divide your circular piece of cardboard into 8 equal sectors.]

Draw an accurate diagram of your spinner.



<u>Activity 2</u>

For each of your outcomes, state the theoretical probability that it will result from any trial of your spinner.

Below is the table of probabilities for the spinner above.

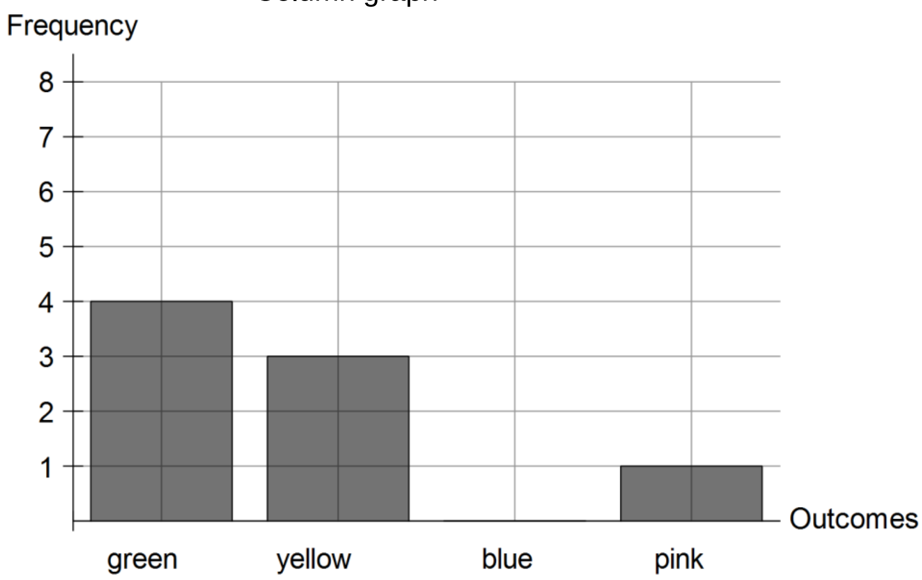
Outcome	green	yellow	blue	pink	
Theoretical probability	–	–	–	–	

Activity 3

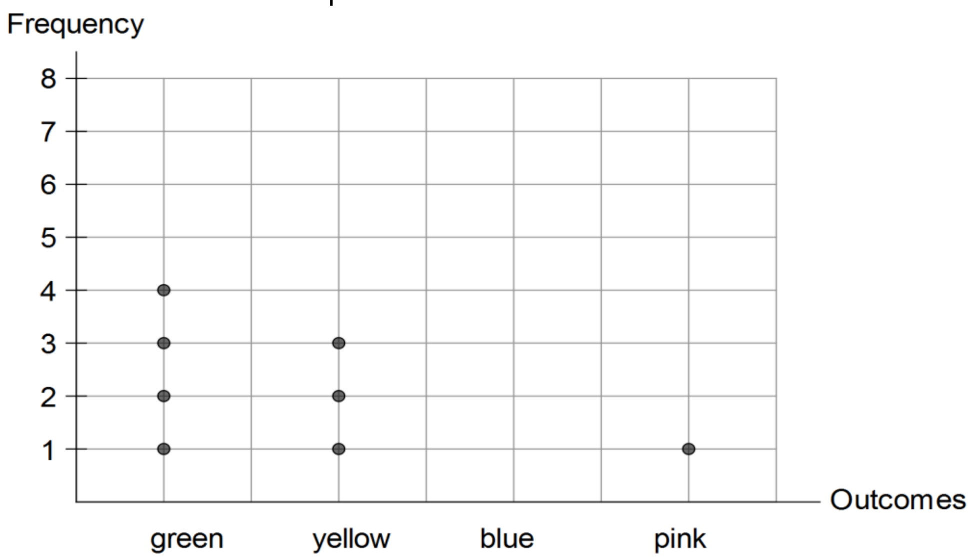
Spin your spinner 8 times and record your results in the table and as two different graphs.
Below is an example of possible results.

Outcome	Tally	Frequency
green	||||	4
yellow	|||	3
blue		
pink	|	1

Column graph



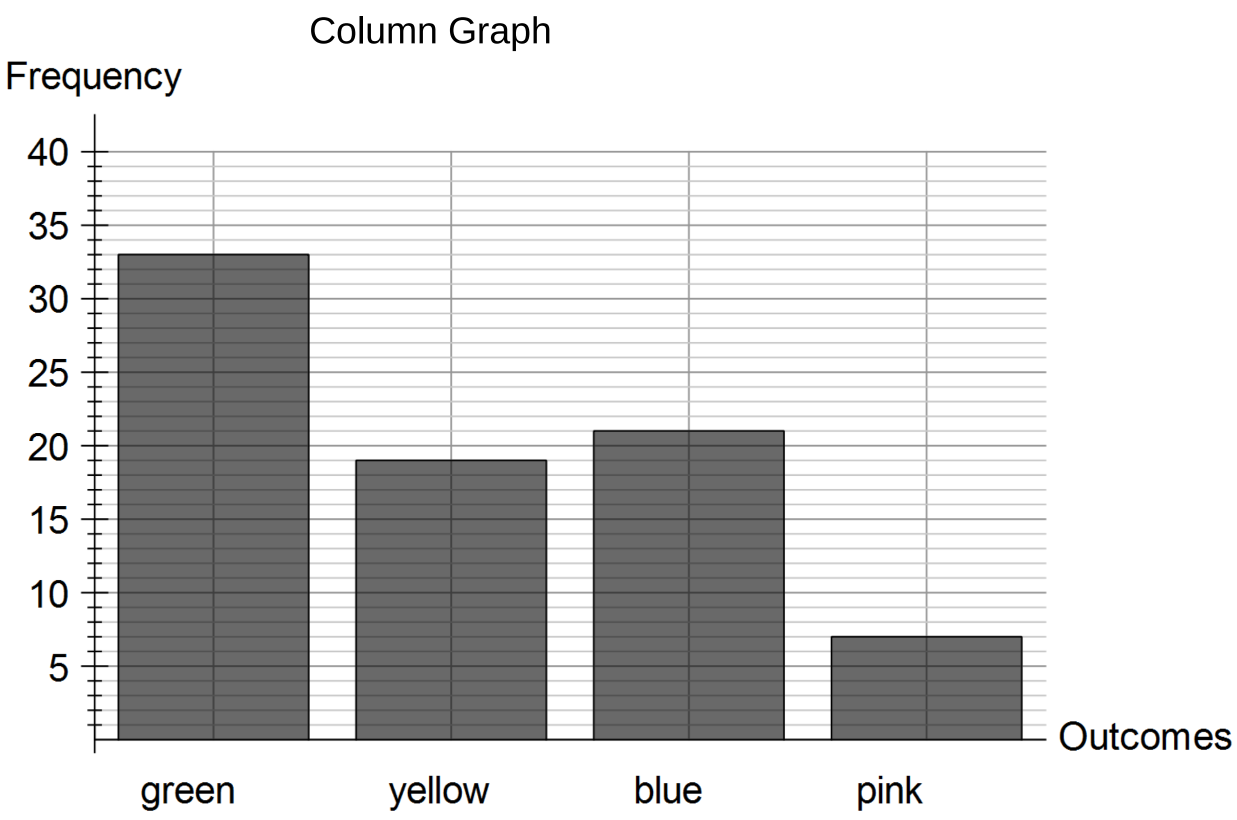
Dot plot



Activity 4

Spin your spinner 80 times, record your results in the table, and graph your results.

Outcome	Tally	Frequency
green	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	33
yellow	𝍸 𝍸 𝍸 ||||	19
blue	𝍸 𝍸 𝍸 𝍸 |	21
pink	𝍸 ||	7
	Total	80



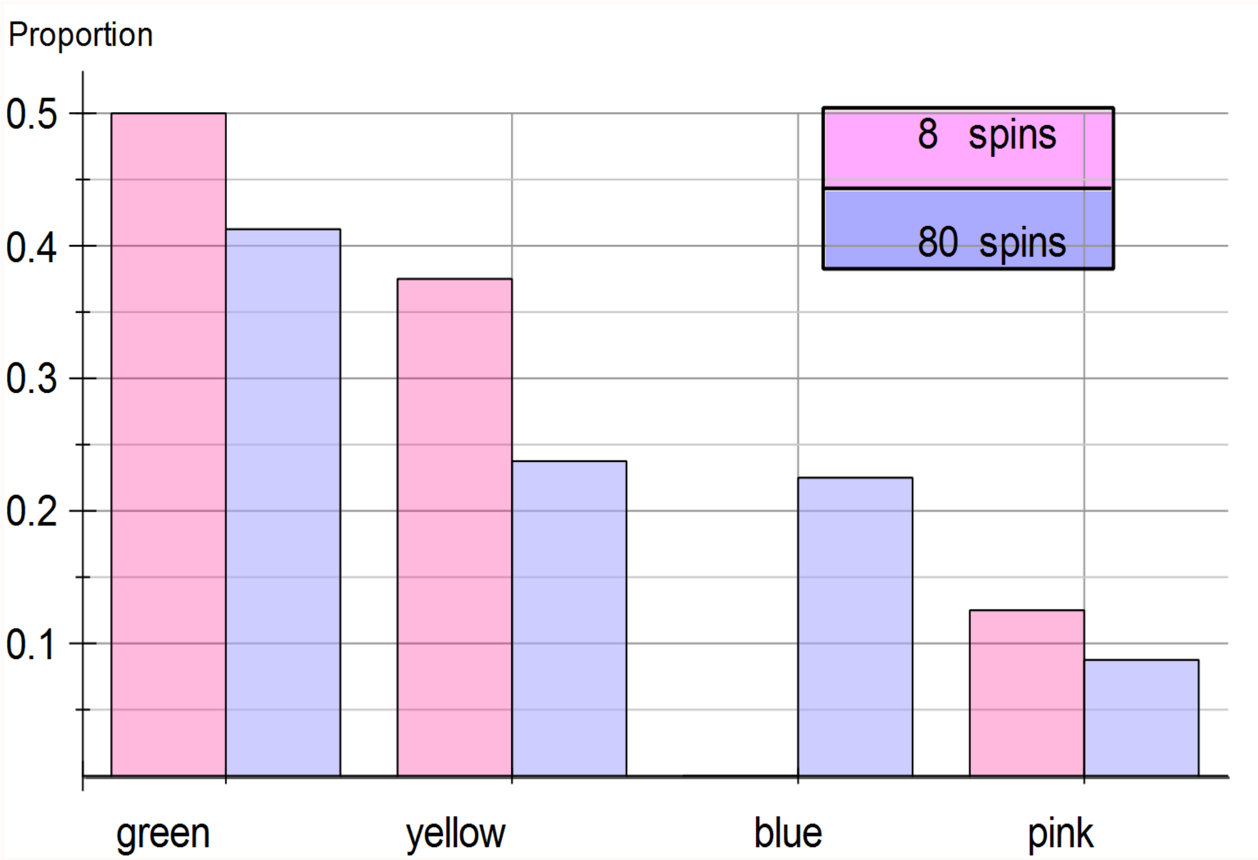
Activity 5

Summarise the results of your investigation in the table provided. Give the theoretical probability of each outcome as a fraction, decimal and percentage; and also the proportion for each outcome from your experiments as a percentage.

Outcome	Theoretical probability			Proportion for 8 spins	Proportion for 80 spins
	Fraction	Decimal	Percentage		
Green	–	0.375	37.5%	50%	41.25%
Yellow	–	0.25	25%	37.5%	23.75%
Blue	–	0.25	25%	0	22.5%
pink	–	0.125	12.5%	12.5%	8.75%

On one grid, draw a graph showing the proportions for each outcome for the 8 spins and the 80 spins. Make sure your graph has a legend and is clearly labelled.

Again the graph below is simply an example.



Activity 6

Prepare a poster showing the results of your experiments.

Examine the posters prepared by the other groups in your class.

Consider the following statements. State whether these statements are true according to the class results. If the class results do not support the statements provide the evidence and give a possible explanation.

1. *For the first 8 spins of the spinner the outcomes occurred in the same proportion as the theoretical probability that they should occur.*

This statement was not true for the spinner above. Theoretically there should have been two spins with “blue” as an outcome but there was none.

2. *Spinning the spinners 80 times gave the same proportions as spinning the spinner 8 times.*

This statement is not true for the spinner above. There was a higher proportion of green, yellow and pink when the spinner was spun 8 times than there was when the spinner was spun 80 times.

3. Which trial gave results closer to the theoretical probabilities. Why?

The 80-spin trial did result in the proportions being closer to the theoretical probabilities than the eight spins. The greater the number of trials, the more the results approximate the theoretical probabilities. Discuss this important principle of probability with students.

This task involves the creation and use of spinners. Each group will create their own spinner, test it and create a poster showing the results and conclusions from their experiments.

Activity 1

Create a spinner with the following features:

- the spinner is circular
- there are at least three different outcomes and no more than 8
- each spin is independent of the previous spin
- the outcome of each spin relies on chance

[Suggestion: Divide your circular piece of cardboard into 8 equal sectors.]

Draw an accurate diagram of your spinner.

Activity 2

For each of your outcomes, state the theoretical probability that it will result from any trial of your spinner.

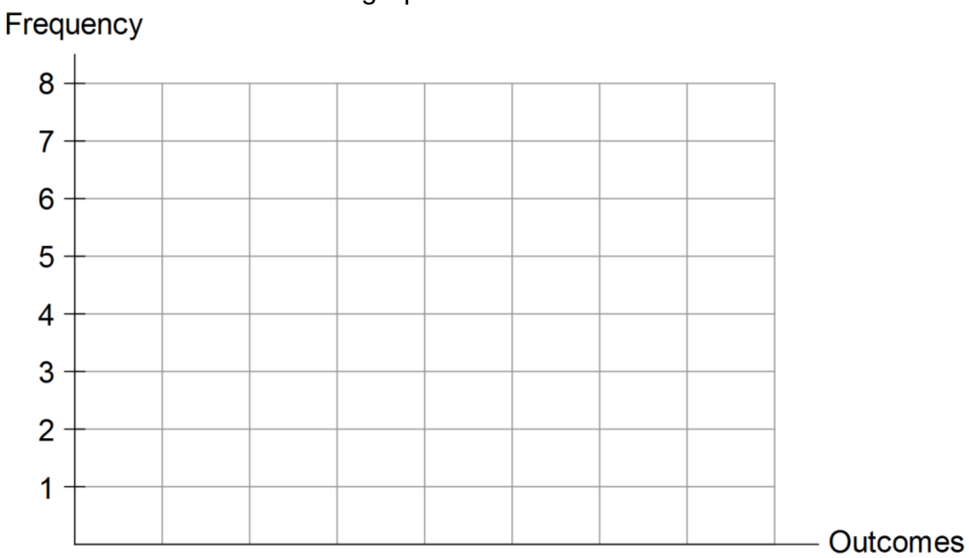
Outcome					
Theoretical probability					

Activity 3

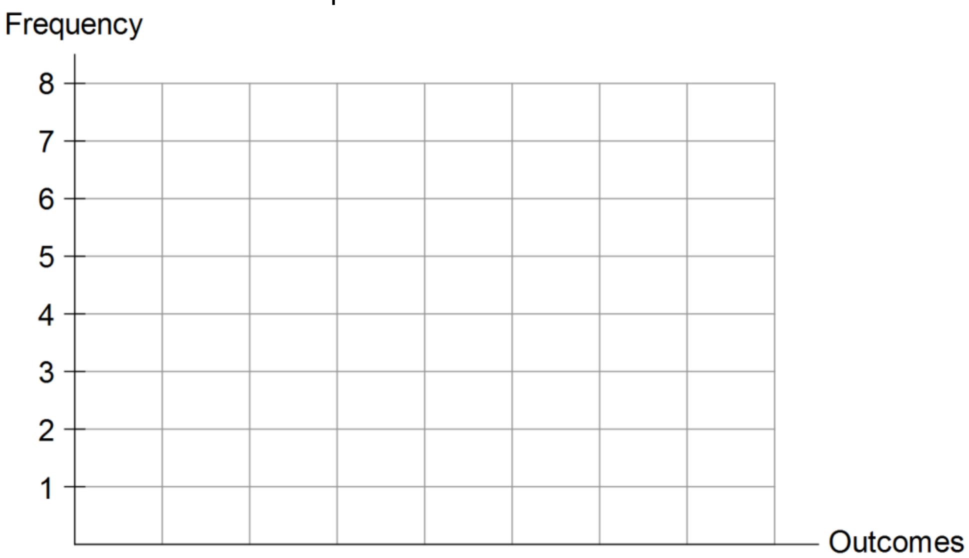
Spin your spinner 8 times and record your results in the table, and as two different graphs.

Outcome	Tally	Frequency

Column graph



Dot plot



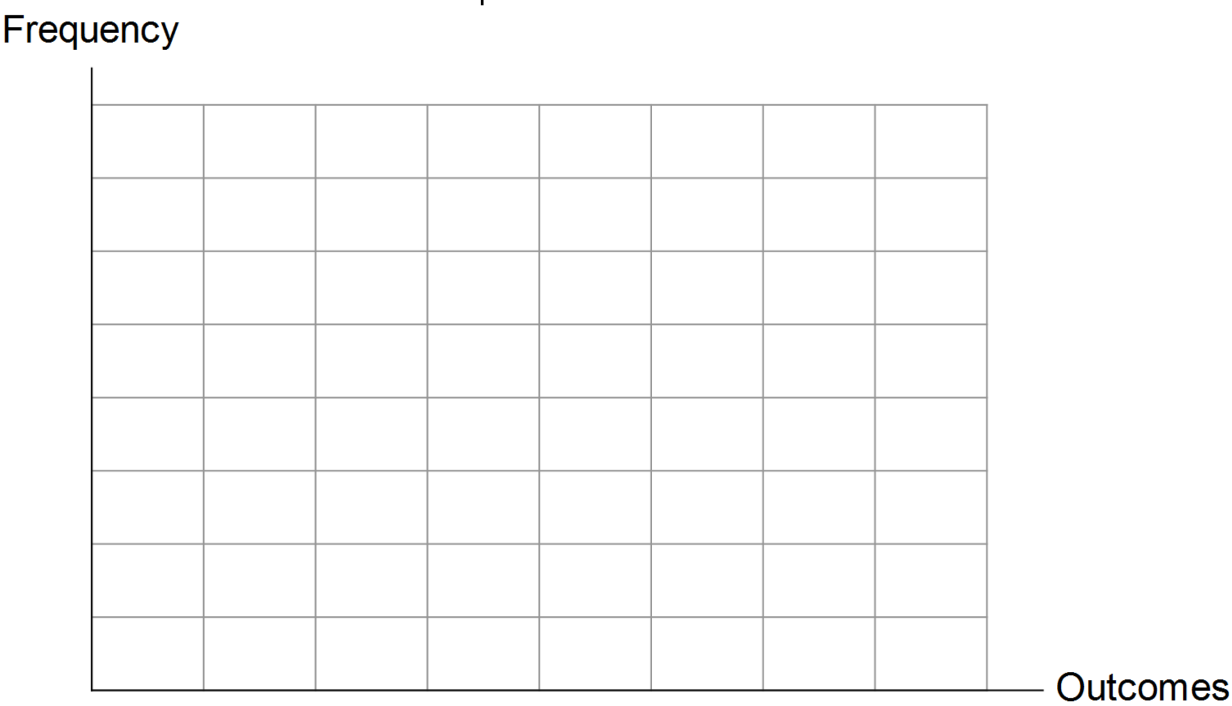
Activity 4

Spin your spinner 80 times, record your results in the table, and graph your results.

Outcome	Tally	Frequency

.

Column Graph



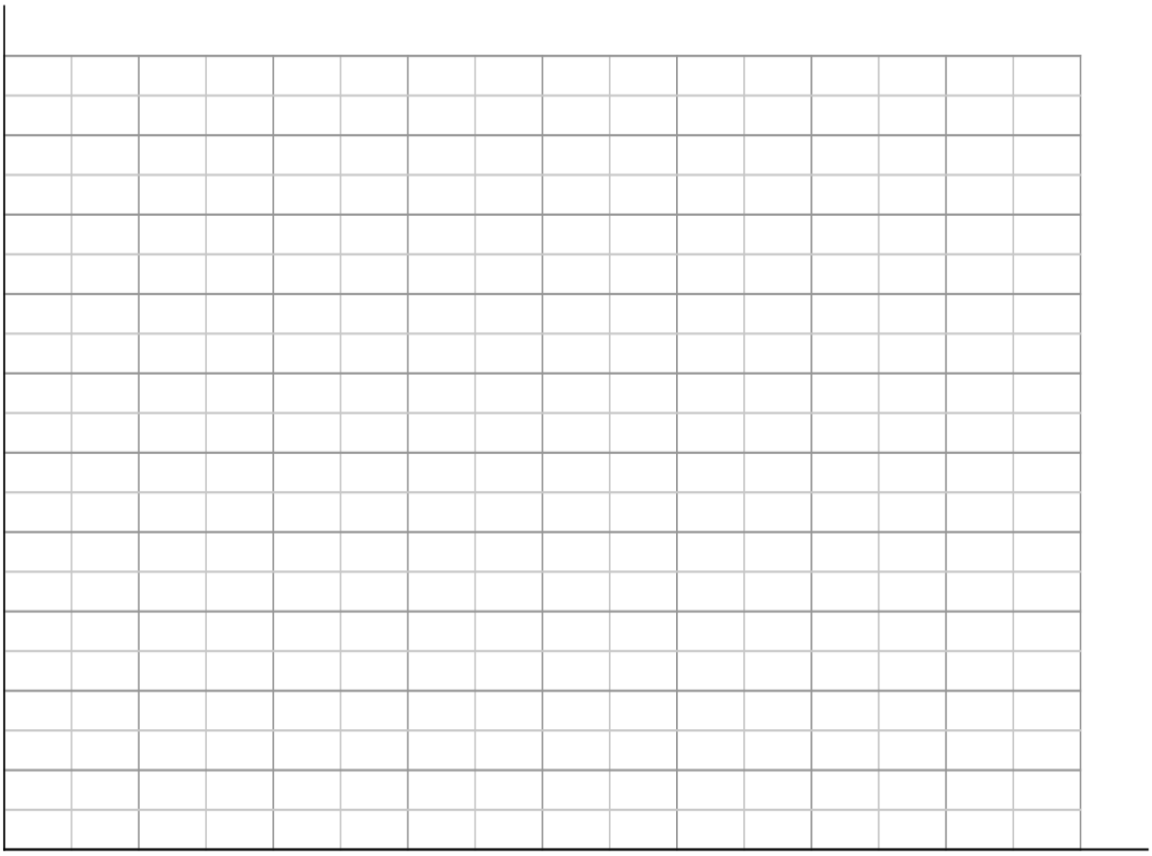
Activity 5

Summarise the results of your investigation in the table provided. Give the theoretical probability of each outcome as a fraction, decimal and percentage; and also the proportion for each outcome from your experiments as a percentage.

Outcome	Theoretical probability			Proportion for 8 spins	Proportion for 80 spins
	Fraction	Decimal	Percentage		

On one grid, draw a graph showing the proportions for each outcome for the 8 spins and the 80 spins. Make sure your graph has a legend and is clearly labelled.

Proportion



Activity 6

Prepare a poster showing the results of your experiments.

Examine the posters prepared by the other groups in your class.

Consider the following statements. State whether these statements are true according to the class results. If the class results do not support the statements provide the evidence and give a possible explanation.

1. *For the first 8 spins of the spinner the outcomes occurred in the same proportion as the theoretical probability that they should occur.*

2. *Spinning the spinners 80 times gave the same proportions as spinning the spinner 8 times.*

3. Which trial gave results closer to the theoretical probabilities. Why?

Department of Education

YEAR 7 MATHEMATICS

Multi-Strand Activity

Solving Equations

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 33: SOLVING EQUATIONS

Overview

This task could serve as part of an introduction to solving equations if students are familiar with the concepts of letters representing variables.

Students will not need any special equipment

Relevant content descriptions from the Western Australian Curriculum

- Solve simple linear equations (ACMNA179)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Students can demonstrate

- *fluency* when they
  - solve equations efficiently
- *understanding* when they
  - connect multiplication with division, and addition with subtraction
- *reasoning* when they
  - explain the relationship between operations
- *problem solving* when they
  - use mathematics to represent situations
  - create formulae to represent relationships

Review: *Solving an equation* means to find the value or values of the variables that make the equation true; also known as finding the solution.

Example	Equation	Solution
1	$h + 4 = 7$	$h = 3$
2	$20 – m = 15$	$m = 5$
3	$k + k + k = 21$	$k = 7$
4	$4 \times d = 8$	$d = 2$
5	$60 \div 10 = a$	$a = 6$
6	$w^2 = 9$	$w = 3$ or -3
7	$k + g = 7$	$k = 1, g = 6$ $k = 2, g = 5$ etc..
8	$20 \div a = 10$	$a = 2$

Note that for $k + g = 7$, the given examples are whole numbers, but there are infinitely many possible solutions, such as $k = 1.5$, $g = 5.5$; $k = 2.3$, $g = 4.7$; $k = -4.1$, $g = 11.1$.

If adding and subtracting are not too difficult and basic facts are well known, determining the solution often involves "knowing" the answer rather than working it out. A "guess and check" method can also be used. Thus, if not sure of the answer, try what it might be and then check in the equation.

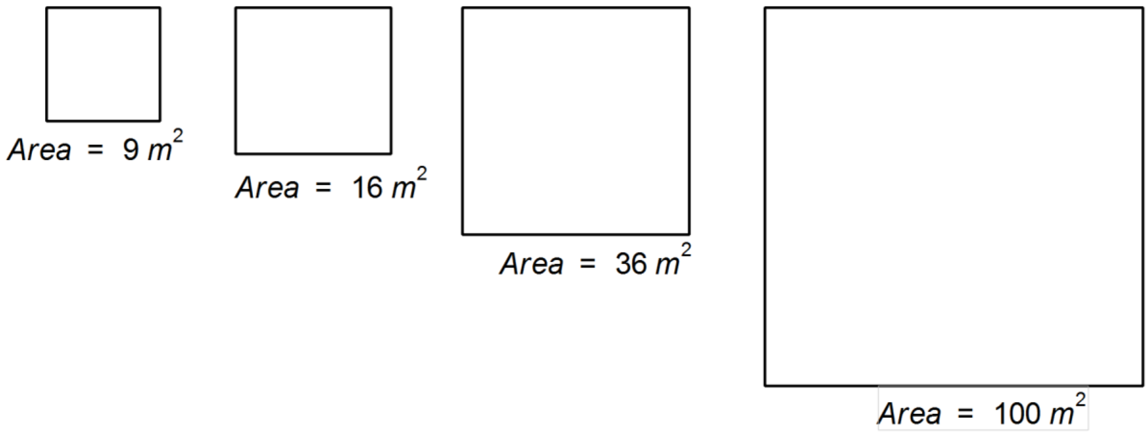
Activity 1

Use your knowledge of basic number facts or a guess and check method to determine the solutions to these equations.

Example	Equation	Solution
1	$h + 9 = 9$	$h = 0$
2	$20 – m = -10$	$m = 30$
3	$k + k + k = 15$	$k = 5$
4	$3 \times x = 30$	$x = 10$
5	$49 \div 7 = a$	$a = 7$
6	$w^2 = 81$	$w = 9$ or -9
7	$10 + g = 7$	$g = -3$
8	$56 \div a = 7$	$a = 8$

Activity 2

1. For the four squares drawn below, write an equation connecting the **length of each side** with the size of the **area** and show the solution of the equation.



Let s represent the side length of a square.

Square	Area (m^2)	Equation	Solution
1	9	Area = 9 m^2 so $s^2 = 9$	$s = 3$
2	16	Area = 16 m^2 so $s^2 = 16$	$s = 4$
3	36	Area = 36 m^2 so $s^2 = 36$	$s = 6$
4	100	Area = 100 m^2 so $s^2 = 100$	$s = 10$
5	121	Area = 121 m^2 so $s^2 = 121$	$s = 11$
6	484	Area = 484 m^2 so $s^2 = 484$	$s = 22$
7	729	Area = 729 m^2 so $s^2 = 729$	$s = 29$
8	1089	Area = 1089 m^2 so $s^2 = 1089$	$s = 33$
9	4225	Area = 4225 m^2 so $s^2 = 4225$	$s = 65$
10	7310.25	Area = 7310.25 m^2 so $s^2 = 7310.25$	$s = 85.5$

2. When you did not *know* the answer, how did you solve the equation?
Find the square root of the area.

3. Write the symbolic form for solving the equation; i.e., length = $\sqrt{Area} = \sqrt{A}$

Activity 3

If the sum of two equal and unknown numbers is 8, most students would *know* the numbers must both be 4.

What if the numbers are not so easily recognised?
Example: If the sum of two equal and unknown numbers is 54.09 then a process to find each number is used.
The equation can be written as: 2 x w = 54.09 and w = half of 54.09 or 54.09 ÷ 2 = 27.045

For each of these descriptions, write equations and solutions as in the example provided.

Let the unknown number be represented by w.

Description	Equation	Solution
The sum of two equal and unknown numbers is 54.09	2 x w = 54.09	w = half of 54.09 = 54.09 ÷ 2 = 27.045
The sum of two equal and unknown numbers is 0.9065	2 x w = 0.9065	w = half of 0.9065 = 0.9065 ÷ 2 = 0.45325
The sum of four equal and unknown numbers is 54.09	4 x w = 54.09	w = quarter of 54.09 = 54.09 ÷ 4 = 13.525
The sum of two equal and unknown numbers is -45.573	2 x w = -45.573	w = half of -45.573 = -45.573 ÷ 2 = -22.7865
The sum of nine equal and unknown numbers is 42 354	9 x w = 42 354	w = one ninth of 42 354 = 42 354 ÷ 9 = 4706
The sum of five equal and unknown numbers is 3.02054	5 x w = 3.02054	w = one fifth of 3.02054 = 3.02054 ÷ 5 = 0.604108
The sum of three equal and unknown numbers is 159 042	3 x w = 159 042	w = a third of 54.09 = 159 042 ÷ 3 = 53 014

Identify the operation used in the equation.

Multiplication

Identify the operation used in determining the solution.

Division

Activity 4

The table below shows starting numbers, actions and finishing numbers. Your task is to identify the action required to go backwards from the finishing number to the starting number. These operations should be done mentally.

Starting number	Action	Finishing number	Opposite action
10	Add 5	15	Subtract 5 from 15
65.4	Add 5	70.4	Subtract 5
98.45	Subtract 2	96.45	Add 2
0.009	Subtract 0.001	0.008	Add 0.001
12 345	Divide by 5	2469	Multiply by 5
63	Divide by 9	7	Multiply by 9
80	Multiply by 4	320	Divide by 4
70	Square the number	4900	Find the square root
98	Multiply by 2	196	Divide by 2
2	Subtract 5	-3	Add 5
54	Halve the number	27	Double the number
23.81	Add 0.19	24	Subtract 0.19
–	Multiply by 4	2	Divide by 4
63	Divide by 9	7	Multiply by 9
-8	Add 8	0	Subtract 8
–	Add 7	7 –	Subtract 7
90	Subtract 0.7	89.3	Add 0.7
-10	Add 20	10	Subtract 20
13	Square the number	169	Find the square root
—	Multiply by 10	1	Divide by 10
0.25	Subtract 0.05	0.2	Add 0.05
0.5	Halve the number	0.25	Double the number
37	Add 39	76	Subtract 39

Activity 5

1. Write a summary of the "backwards" or "undoing" or opposite operations of *addition, subtraction, multiplication, division, squaring,* and *halving.*

When a total is formed by adding a second number to the first number, you can get back to the first number by subtracting that added number away from the total.

When a result is achieved by subtracting one number from another number, you can get back to the initial number by adding that subtracted number to the result.

When a result is achieved by multiplying one number by another number, you can get back to the first number by dividing the result by the number used in the multiplication.

When a result is achieved by dividing one number by another number, you can get back to the first number by multiplying the result by the number used in the division.

When a result is achieved by squaring a number, you can get back to the original number by taking the square root of the result.

When a result is achieved by halving a number, doubling the result will produce the original number that was halved.

2. Investigate the operations listed in part 1 above and the relationships between them. Record the results of your findings

The answers will vary.

This provides a good opportunity to discuss the relationship between multiplication and division, addition and subtraction, and squaring and square root.

Students should see that addition and subtraction are opposite operations, as one undoes the other. They might consider that all subtraction is a form of addition; e.g., $3 + 5 = 3 - -5$.

Students should see that multiplication and division are opposite operations, as one undoes the other. They might consider that all multiplication is simply division by the inverse; e.g., $3 \times 5 = 3 \div \frac{1}{5} = 3 \div 0.2$.

Students should see that squaring and square root are opposite operations, as one undoes the other; e.g., $7 \times 7 = 49$ and $\sqrt{49} = 7$.

Activity 6

For each of these problems:

- allocate a letter to the unknown value;
- write an equation linking values and the variable; and
- solve the equation.

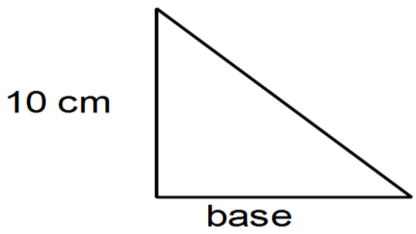
1. The area of a rectangle which is 6 cm wide is 24 cm^2. Determine the length.

   Let g represent the length of the rectangle.
   $6 \times g = 24$
   $g = 4$ cm

2. The area of a rectangular paddock is 86 400 m^2. If the paddock is 240 m wide, determine the length of the paddock.

   Let g represent the length of the rectangular paddock.
   $240 \times g = 86400$
   $g = 86400 \div 240$
   $= 360$ m

3. A triangle with a height of 10 cm has an area of 60 cm^2. Determine the length of the base.



   Let b represent the length of the base.
   Area = $0.5 \times h \times b$
   $0.5 \times h \times b = 60$
   $0.5 \times 10 \times b = 60$
   $5 \times b = 60$
   $b = 12$ cm

4. A sail which is triangular in shape has an area of 6.8 m^2. If the base is 4 m long, determine the height of the sail.

   Let h represent the height of the sail.
   Area = $0.5 \times h \times b$
   $0.5 \times h \times b = 6.8$
   $0.5 \times h \times 4 = 6.8$
   $2 \times h = 6.8$
   $h = 3.4$ m

5. The perimeter of a regular pentagon is 75 cm. How long is each side?

   Let s represent the length of each side.
   $5 \times s = 75$
   $s = 15$ cm

6. A hexagonal tile in a driveway is regular in shape and has a perimeter of 100.8 cm. What is the length of each side of the hexagon?

   Let s represent the length of each side.

   $6 \times s = 100.8$

   $s = 100.8 \div 6$

   $s = 16.8$ cm

7. The height of a cylindrical can of 4 tennis balls is 28 cm. Determine the width of each ball.

   Let w represent the width of each ball.

   $4 \times w = 28$

   $w = 7$ cm

8. The length of a tube of 6 table tennis balls is 21 cm. Determine the width of each ball.

   Let w represent the width of each ball.

   $6 \times w = 21$

   $w = 21 \div 6$

   $w = 3.5$ cm

9. A parallelogram with a perimeter of 82.2 cm has two sides that are twice as long as the other two sides. Determine the length of each side.

   Let s represent the length of the shortest sides.

   Then $2 \times s$ is the length of the other two sides

   $s + s + 2 \times s + 2 \times s = 82.2$

   $6 \times s = 82.2$

   $s = 82.2 \div 6$

   $s = 13.7$ cm

Review: *Solving an equation* means to find the value or values of the variables that make the equation true; also known as finding the solution.

Example	Equation	Solution
1	$h + 4 = 7$	$h = 3$
2	$20 – m = 15$	$m = 5$
3	$k + k + k = 21$	$k = 7$
4	$4 \text{ x } d = 8$	$d = 2$
5	$60 \div 10 = a$	$a = 6$
6	$w^2 = 9$	$w = 3$ or -3
7	$k + g = 7$	$k = 1, g = 6$ $k = 2, g = 5$ etc..
8	$20 \div a = 10$	$a = 2$

Note that for $k + g = 7$, the given examples are whole numbers, but there are infinitely many possible solutions, such as $k = 1.5$, $g = 5.5$; $k = 2.3$, $g = 4.7$; $k = -4.1$, $g = 11.1$

If adding and subtracting are not too difficult and basic facts are well known, determining the solution often involves "knowing" the answer rather than working it out. A "guess and check" method can also be used. Thus, if not sure of the answer, try what it might be and then check in the equation.

Activity 1

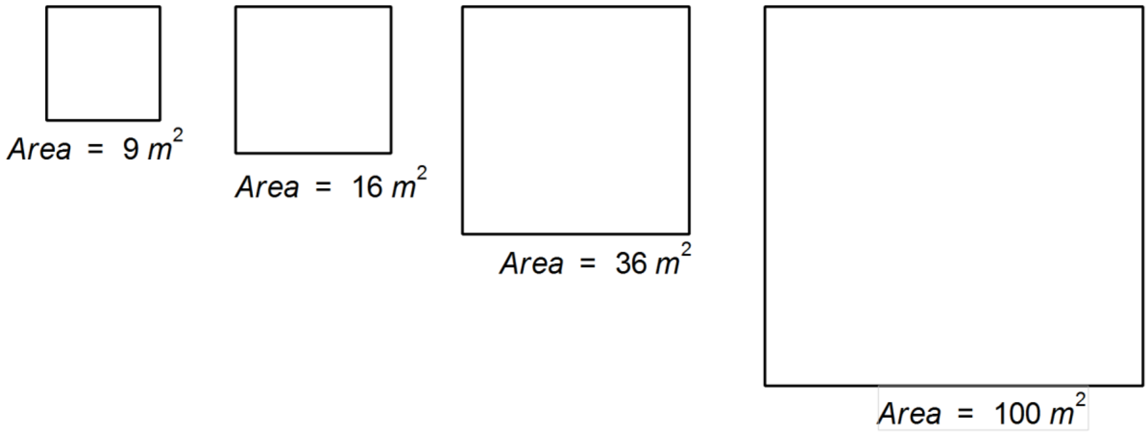
Use your knowledge of basic number facts or a guess and check method to determine the solutions to these equations.

Example	Equation	Solution
1	$h + 9 = 9$	$h =$
2	$20 – m = -10$	$m =$
3	$k + k + k = 15$	$k =$
4	$3 \text{ x } x = 30$	$x =$
5	$49 \div 7 = a$	$a =$
6	$w^2 = 81$	$w =$

7	$10 + g = 7$	$g =$
8	$56 \div a = 7$	$a =$

Activity 2

1. For the four squares drawn below, write an equation connecting the **length of each side** with the size of the **area** and show the solution of each equation in the table below. Then complete the table for more squares.



Square	Area (m^2)	Equation	Solution
1	9		
2	16		
3	36		
4	100		
5	121		
6	484		
7	729		
8	1089		
9	4225		
10	7310.25		

2. When you did not *know* the answer, how did you solve the equation?

3. Write the symbolic form for solving the equation; i.e., length = ______

Activity 3

If the sum of two equal and unknown numbers is 8, most students would *know* the numbers must both be 4.

What if the numbers are not so easily recognised?
Example: If the sum of two equal and unknown numbers is 54.09 then a process to find each number is used.
The equation can be written as: $2 \times w = 54.09$ and w = half of $54.09 = 54.09 \div 2 = 27.045$

For each of these descriptions, write equations and solutions as in the example provided.

Description	Equation	Solution
The sum of two equal and unknown numbers is 54.09	$2 \times w = 54.09$	w = half of 54.09 $= 54.09 \div 2$ $= 27.045$
The sum of two equal and unknown numbers is 0.9065		
The sum of four equal and unknown numbers is 54.09		
The sum of two equal and unknown numbers is -45.573		
The sum of nine equal and unknown numbers is 42 354		
The sum of five equal and unknown numbers is 3.02054		
The sum of three equal and unknown numbers is 159 042		

Identify the operation used in the equation.

Identify the operation used in determining the solution.

Activity 4

The table below shows starting numbers, actions and finishing numbers. Your task is to identify the action required to go backwards from the finishing number to the starting number. These operations should be done mentally.

Starting number	Action	Finishing number	Opposite action
10	Add 5	15	Subtract 5 from 15
65.4	Add 5		
98.45	Subtract 2		
0.009	Subtract 0.001		
12 345	Divide by 5		
63	Divide by 9		
80	Multiply by 4		
70	Square the number		
98	Multiply by 2		
2	Subtract 5		
54	Halve the number		
23.81	Add 0.19		
–	Multiply by 4		
63	Divide by 9		
-8	Add 8		
–	Add 7		
90	Subtract 0.7		
-10	Add 20		
13	Square the number		
—	Multiply by 10		
0.25	Subtract 0.05		
0.5	Halve the number		
37	Add 39		

Activity 5

1. Write a summary of the “backwards” or “undoing” or opposite operations of *addition, subtraction, multiplication, division, squaring,* and *halving.*

2. Investigate the operations listed in part 1 above and the relationships between them. Record the results of your findings

Activity 6

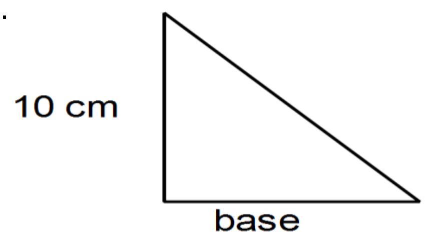
For each of these problems:

- allocate a letter to the unknown value;
- write an equation linking values and the variable; and
- solve the equation

1. The area of a rectangle, which is 6 cm wide, is 24 cm^2. Determine the length.

2. The area of a rectangular paddock is 86 400 m^2. If the paddock is 240 m wide, determine the length of the paddock.

3. A triangle with a height of 10 cm has an area of 60 cm^2. Determine the length of the base.



4. A sail which is triangular in shape has an area of 6.8 m^2. If the base is 4 m long, determine the height of the sail.

5. The perimeter of a regular pentagon is 75 cm. How long is each side?

6. A hexagonal tile in a driveway is regular in shape and has a perimeter of 100.8 cm. What is the length of each side of the hexagon?

7. The height of a cylindrical can of 4 tennis balls is 28 cm. Determine the width of each ball.

8. The length of a tube of 6 table tennis balls is 21 cm. Determine the width of each ball.

9. A parallelogram with a perimeter of 82.2 cm has two sides that are twice as long as the other two sides. Determine the length of each side.

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Moving Shapes

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 34: MOVING SHAPES

Overview

This task allows students to explore translations and reflection on the Cartesian plane. Students should be familiar with plotting and reading points on the Cartesian plane. Completion of the task on “moving points” would be good preparation for this activity.

Students will need

- scissors

Relevant content descriptions from the Western Australian Curriculum

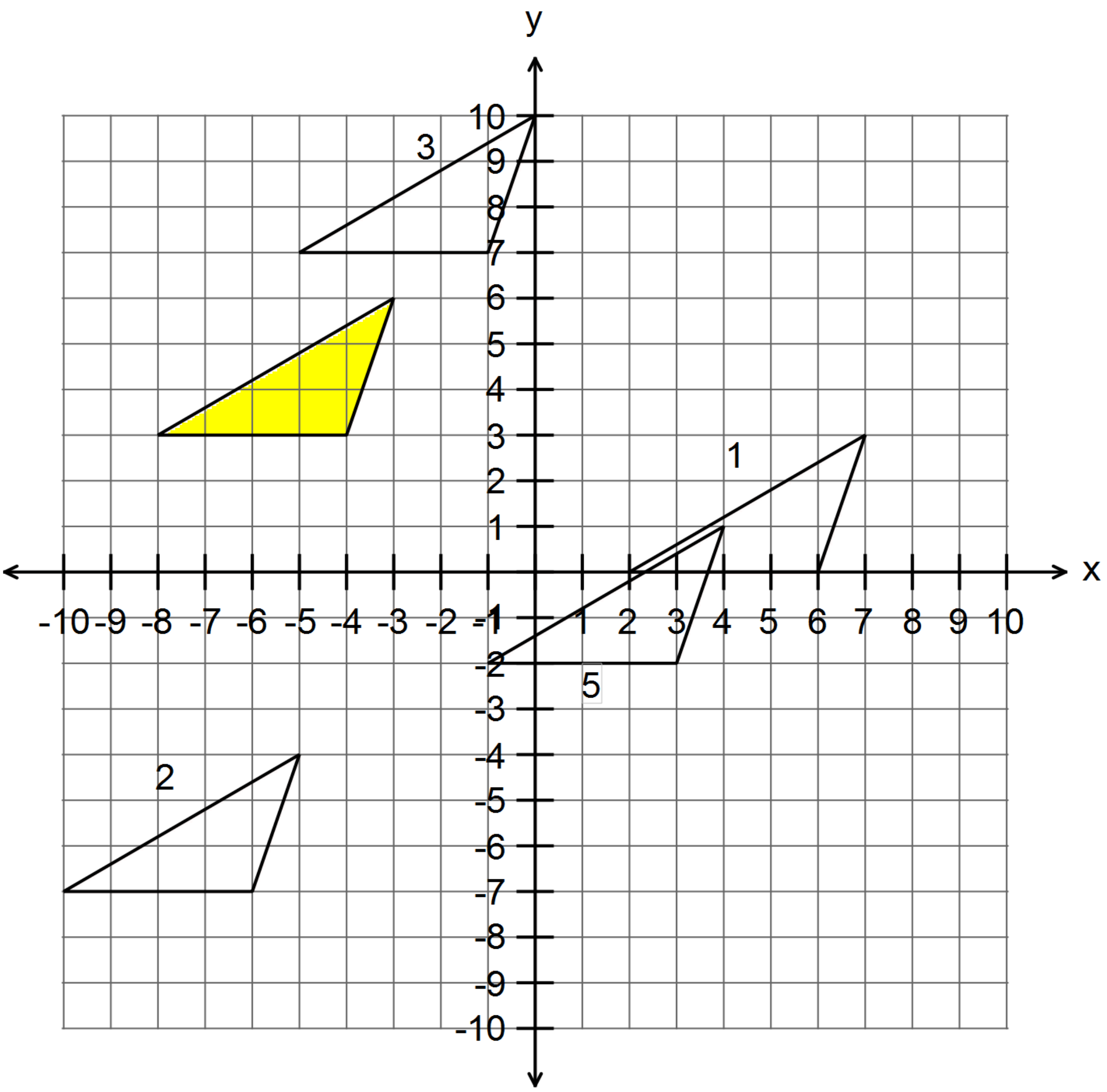
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)
- Describe translations, reflections in an axis, and rotations of multiples of 90º on the Cartesian plane using coordinates. Identify line and rotational symmetries (ACMMG181)

Students can demonstrate

- fluency when they
  - read and plot points on the Cartesian plane
- understanding when they
  - read and plot points on the Cartesian plane
  - connect the relationship between movements and changes to coordinates
- reasoning when they
  - explain the changes to the coordinates in terms of movement
- problem solving when they
  - work with transformations

Activity 1

Cut out a piece of paper the same shape and size as the triangle drawn on the grid.
Place your triangle over the one on the grid.



Move your triangle as follows and for each move, draw a new triangle and label it. Move your triangle back to the original position between moves.

Move 1: 10 units to the right and 3 units down

Move 2: 10 units down and two units to the left

Move 3: 4 units up and 3 units to the right

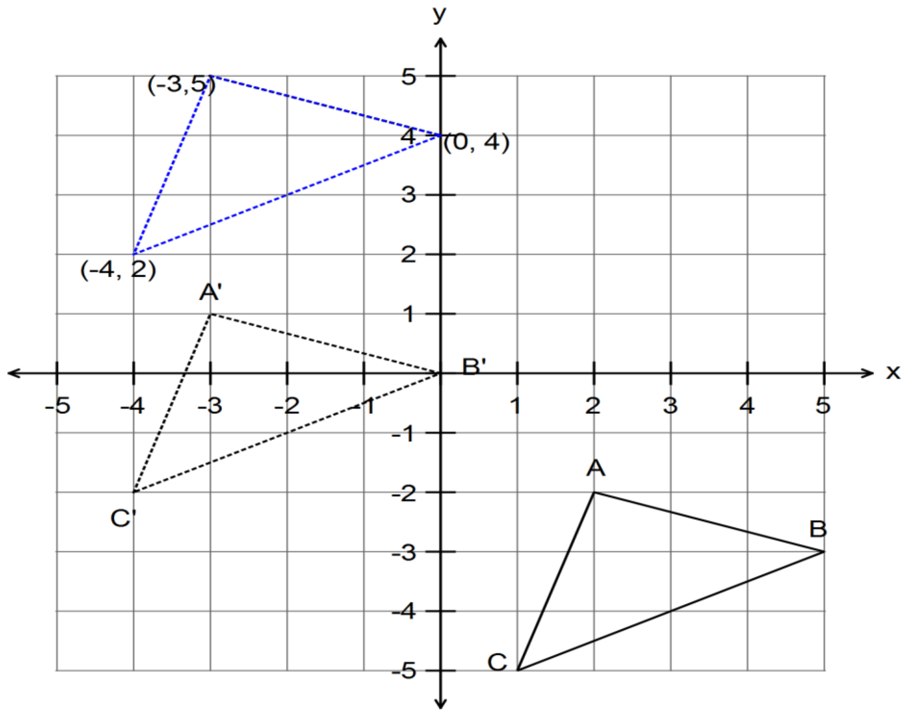
Move 4: 1 unit up, 7 units to the right and then 6 units down.

Translations are shown in the diagrams of triangles above.

In the following activities you are asked to "move" a shape. The shape may not actually move but rather an image of the original shape is formed. It is customary to have solid lines for the object that is moved and dotted lines for the image that is formed. The points at the vertices of the object are usually labelled with capital letters and the matching points on the image have the same letters but with a dash (') at the top.

Activity 2

1. Label the vertices of the object ABC. Label the vertices of the image.



2. Identify the coordinates of the vertices and complete the table.

Object			Image		
A	B	C	A'	B'	C'
(2, -2)	(5, -3)	(1, -5)	(-3,1)	(0, 0)	(-4, -2)

3. Describe the movement of the object that results in the formation of this image.

Up 3 units, left 5 units

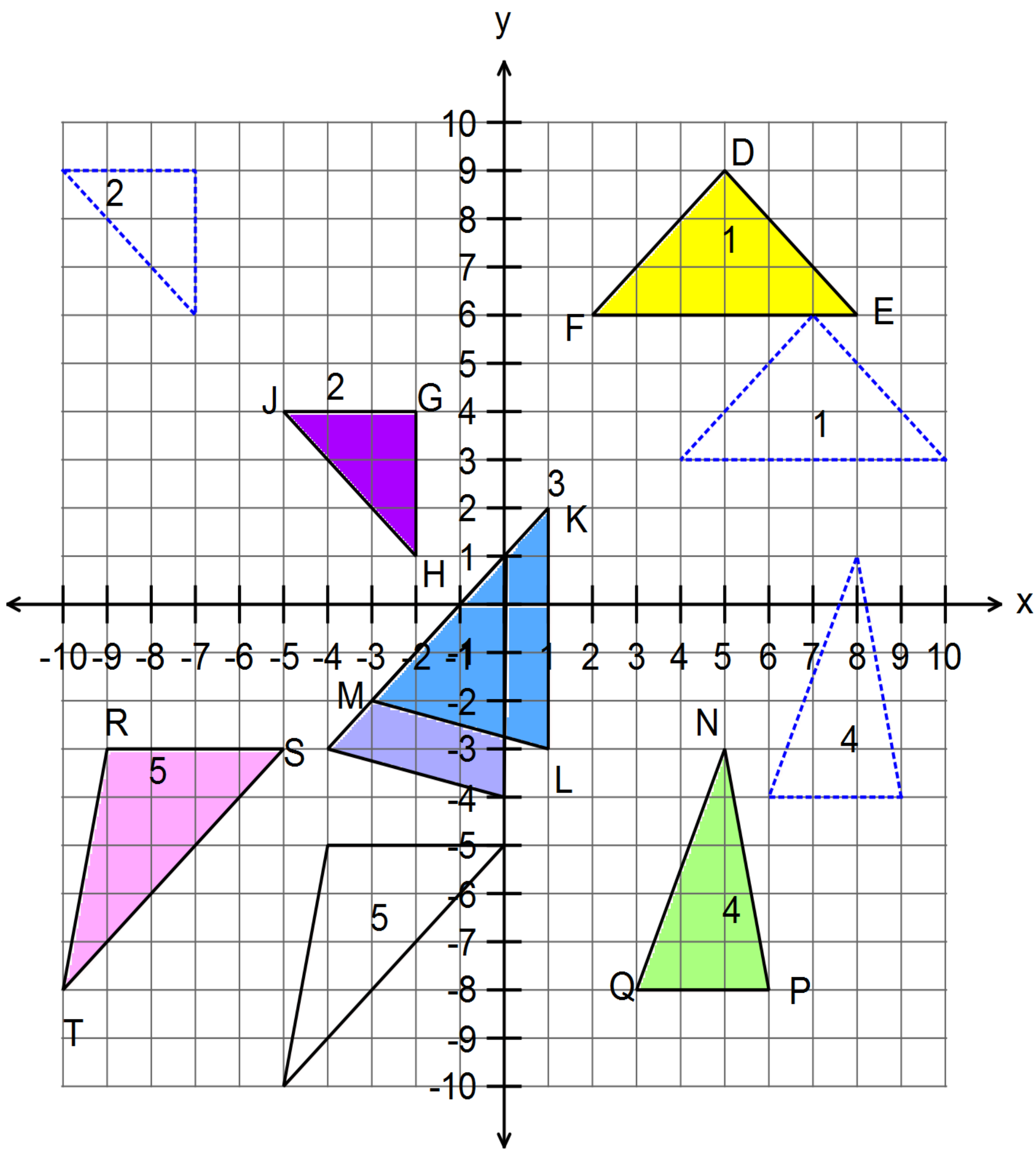
4. Investigate: What is the name given to this type of movement?

Slide or Translation

5. Move the object triangle 7 units vertically upwards and 5 units to the left. Write the coordinates of the points of this new image at its vertices.

Activity 3

1. There are 5 triangles drawn on the grid. Translate the triangles as shown in the table and draw the images formed. Determine the coordinates of the vertices of the object triangles and their images and complete the tables.



Triangle 1

Movement	Object			Image		
	D	E	F	D’	E’	F’
down 3 units right 2 units	(5, 9)	(8, 6)	(2, 6)	(7, 6)	(10, 3)	(4, 3)

Triangle 2

Movement	Object			Image		
	G	H	J	G'	H'	J'
left 5 units up 5 units	(-2, 4)	(-2, 1)	(-5, 4)	(-7, 9)	(-7, 6)	(-10, 9)

Triangle 3

Movement	Object			Image		
	K	L	M	K'	L'	M'
down 1 unit left 1 unit	(1, 2)	(1, -3)	(-3, -2)	(0, 1)	(0, -4)	(-4, -3)

Triangle 4

Movement	Object			Image		
	N	P	Q	N'	P'	Q'
right 3 units up 4 units	(5, -3)	(6, -8)	(3, -8)	(8, 1)	(9, -4)	(6, -4)

Triangle 5

Movement	Object			Image		
	R	S	T	R'	S'	T'
down 2 units right 5 units	(-9, -3)	(-5, -3)	(-10, -8)	(-4, -5)	(0, -5)	(-5, -10)

2. Examine the patterns in these tables and identify ways of determining the coordinates of an image after a translation without the need to draw the image. Explain your findings.

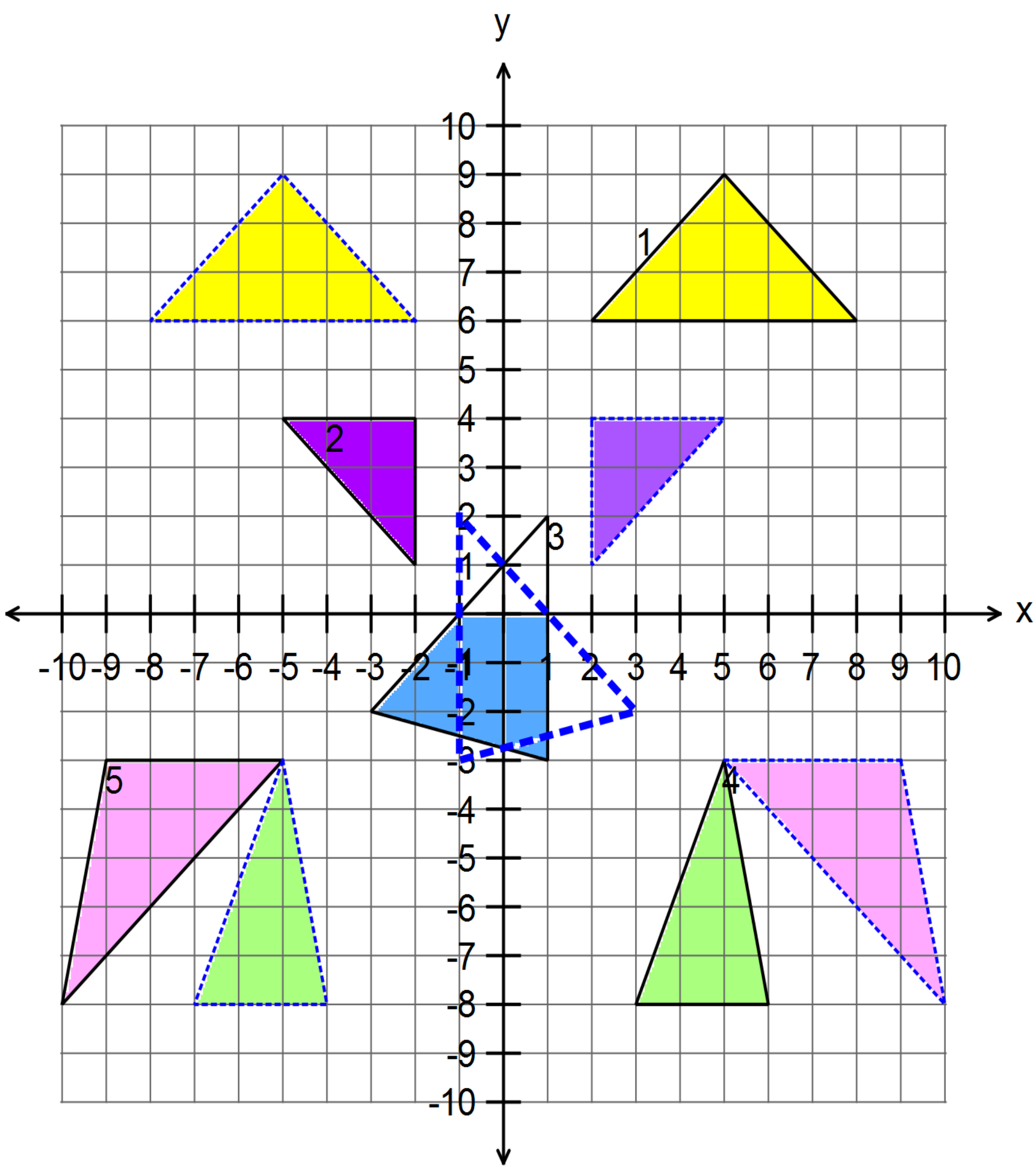
To determine the new coordinates after a translation:
Consider the coordinates as (first number, second number); i.e., (x, y).

If the translation is -
Up – you add the number of units to the second number.
Down - you subtract the number of units from the second number.
Right – you add the number of units to the first number.
Left – you subtract the number of units from the first number.

Using algebra:
Say the coordinates are (x, y)
Up by k units → $(x, y + k)$
Down by k units → $(x, y – k)$
Right by k units → $(x + k, y)$
Left by k units → $(x - k, y)$

Activity 4

1. Before commencing this activity check your understanding of a reflection over the y-axis. The grid from the previous activity is reproduced below. Reflect the triangles over the y-axis and draw the images formed. Determine the coordinates of the vertices of the object triangles and their images. Coordinates will vary but values in the table will be correct.



Triangle	Object			Image		
	D	E	F	D'	E'	F'
1	(5, 9)	(8, 6)	(2, 6)	(-5, 9)	(-8, 6)	(-2, 6)

Triangle	Object			Image		
	G	H	J	G'	H'	J'
2	(-2, 4)	(-2, 1)	(-5, 4)	(2, 4)	(2, 1)	(5, 4)

Triangle	Object			Image		
	K	L	M	K'	L'	M'
3	(1, 2)	(1, -3)	(-3, -2)	(-1, 2)	(-1,-3)	(3, -2)

Triangle	Object			Image		
	N	P	Q	N'	P'	Q'
4	(5, -3)	(6, -8)	(3, -8)	(-5, -3)	(-6, -8)	(-3, -8)

Triangle	Object			Image		
	R	S	T	R'	S'	T'
5	(-9, -3)	(-5, -3)	(-10, -8)	(9, -3)	(5, -3)	(10, -8)

2. Examine the patterns in these tables and identify ways of determining the coordinates of an image after a reflection over the y-axis, without the need to draw the image.

The coordinates of the points are changed in that the first number changes sign and the second number is not altered.
Algebraically, (x, y) becomes $(-x, y)$

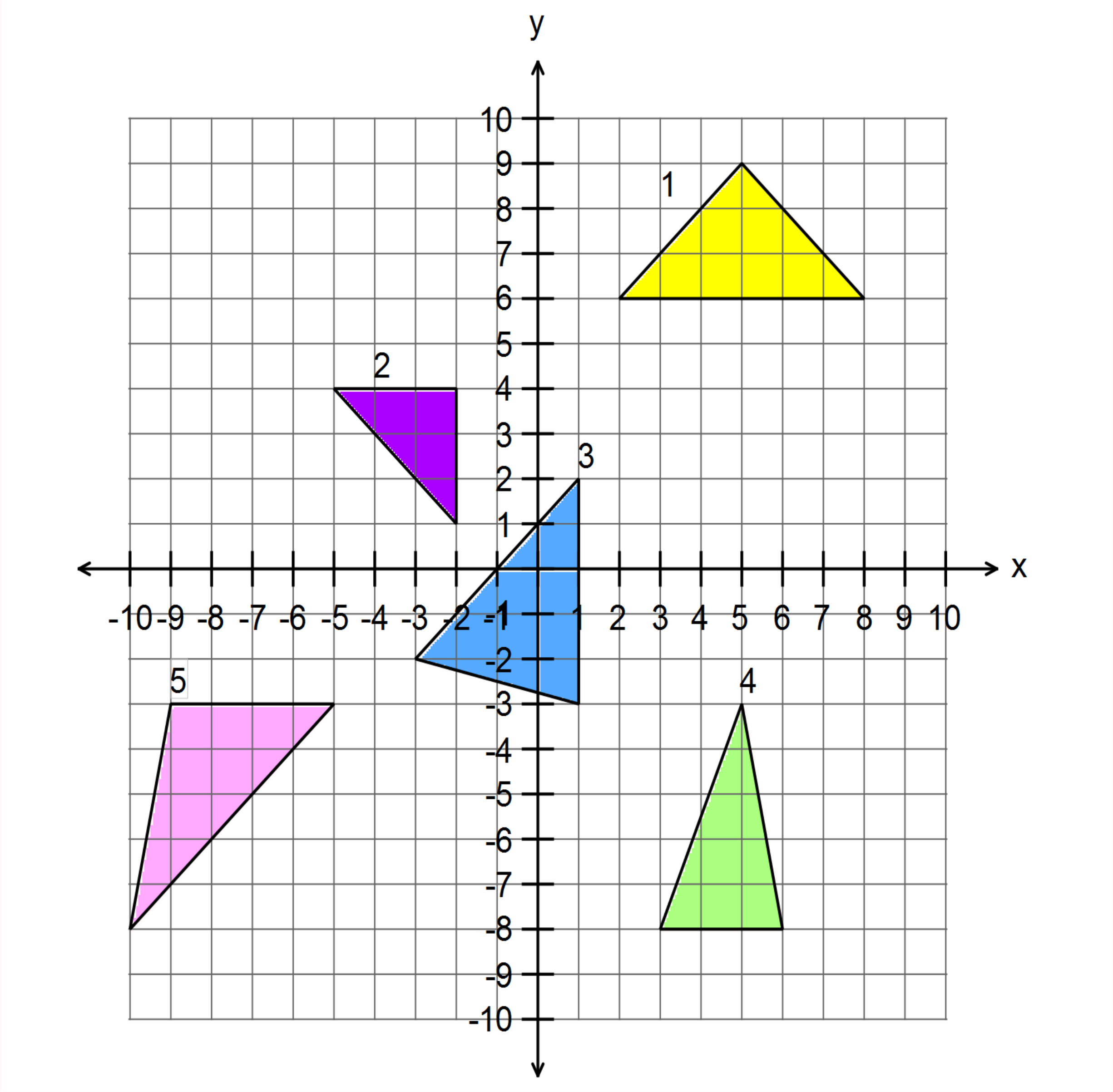
3. What can you predict about changes to the coordinates after a reflection over the x-axis? Justify your predictions.

One would expect that the students who understand what is happening will predict that the first number is unchanged and the second number changes sign.
Algebraically, (x, y) becomes $(x, -y)$
The points are not changing how far they are along the x-axis so their x-values do not change. The y-values simply change signs.

Students may need to check their predictions. The grid that follows provides material for checking.

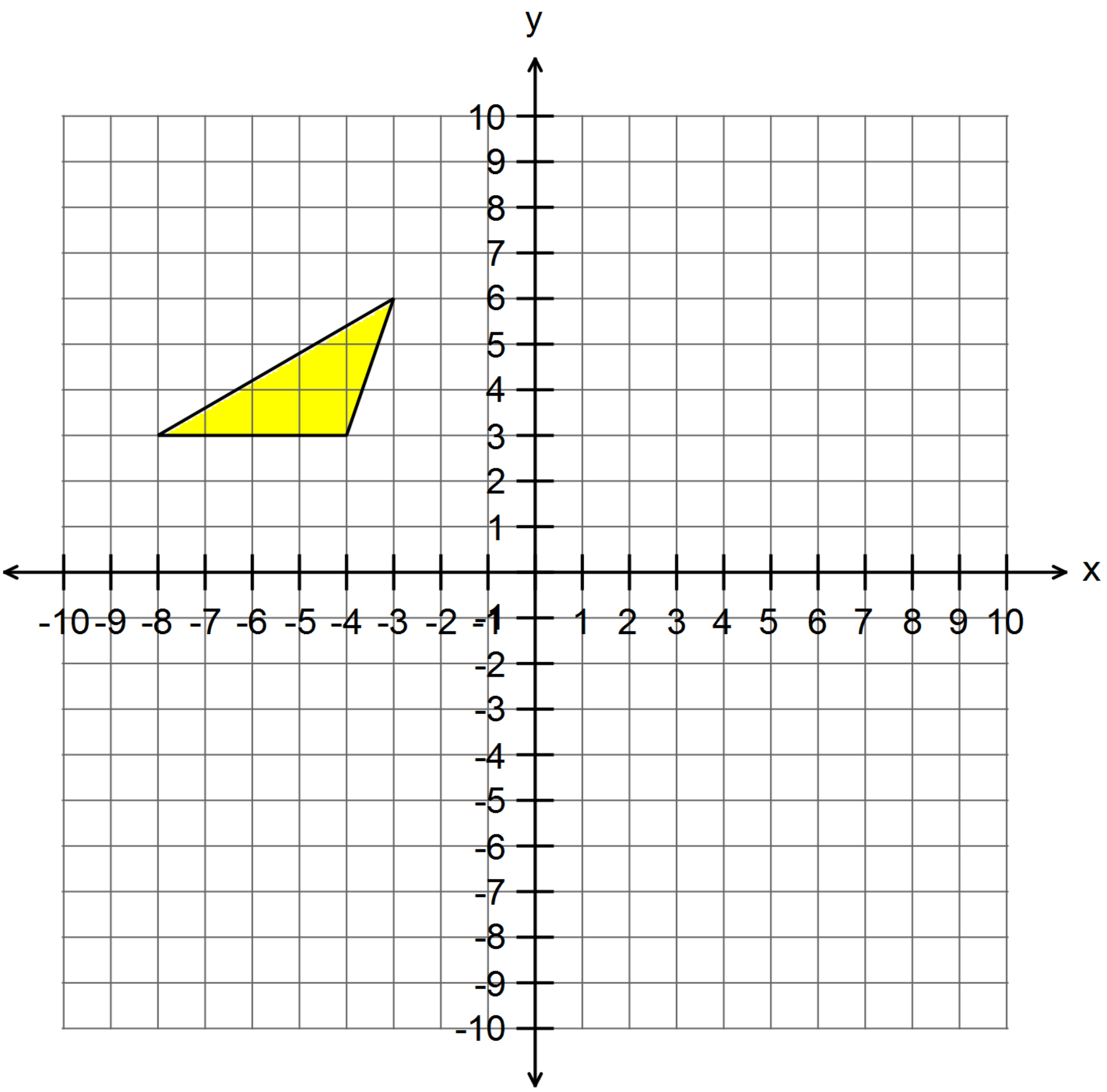
Activity 5

Use the grid below to test the predictions you made in the previous activity.



Activity 1

Cut out a piece of paper the same shape and size as the triangle drawn on the grid.
Place your triangle over the one on the grid.



Move your triangle as follows and for each move, draw a new triangle and label it. Move your triangle back to the original position between moves.

Move 1: 10 units to the right and 3 units down

Move 2: 10 units down and two units to the left

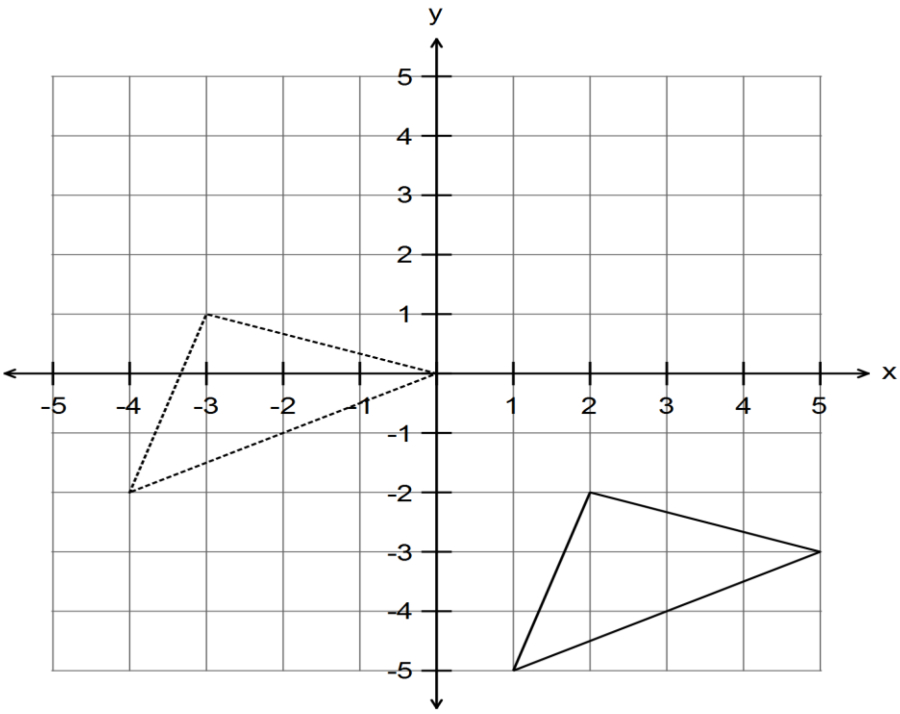
Move 3: 4 units up and 3 units to the right

Move 4: 1 unit up, 7 units to the right and then 6 units down

In the following activities you are asked to “move” a shape. The shape may not actually move but rather an image of the original shape is formed. It is customary to have solid lines for the object that is moved and dotted lines for the image that is formed. The points at the vertices of the object are usually labelled with capital letters and the matching points on the image have the same letters but with a dash (‘) at the top.

Activity 2

1. Label the vertices of the object ABC. Label the vertices of the image.



2. Identify the coordinates of the vertices and complete the table.

Object			Image		
A	B	C	A’	B’	C’

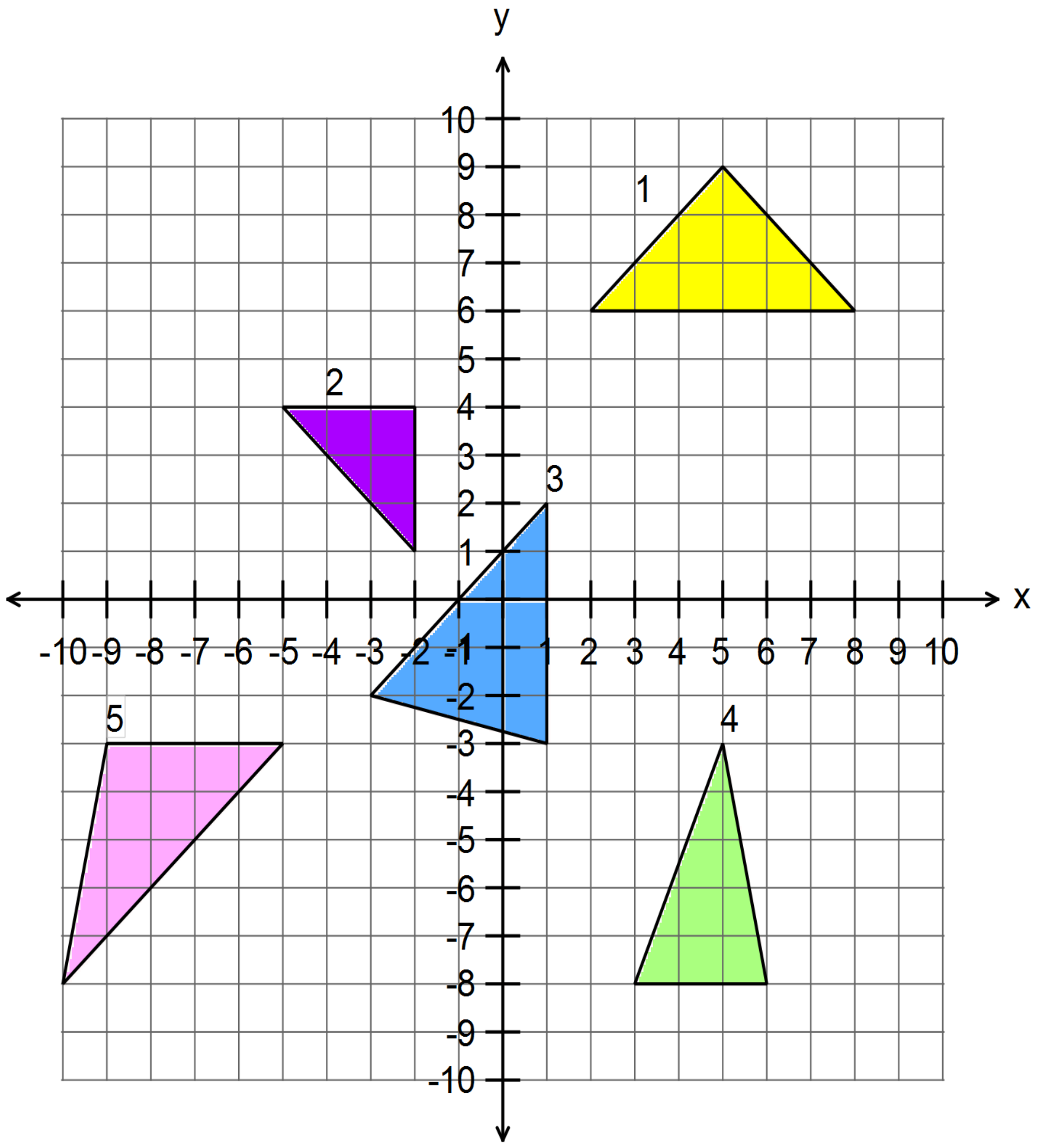
3. Describe the movement of the object that results in the formation of this image.

4. Investigate: What is the name given to this type of movement?

5. Move the object triangle 7 units vertically upwards and 5 units to the left. Write the coordinates of the points of this new image at its vertices

Activity 3

1. There are 5 triangles drawn on the grid. Translate the triangles as shown in the table and draw the images formed. Determine the coordinates of the vertices of the object triangles and their images and complete the tables.



Triangle 1

Movement	Object			Image		
	D	E	F			
down 3 units right 2 units						

Triangle 2

Movement	Object			Image		
	G	H	J			
left 5 units up 5 units						

Triangle 3

Movement	Object			Image		
	K	L	M			
down 1 unit left 1 unit						

Triangle 4

Movement	Object			Image		
	N	P	Q			
right 3 units up 4 units						

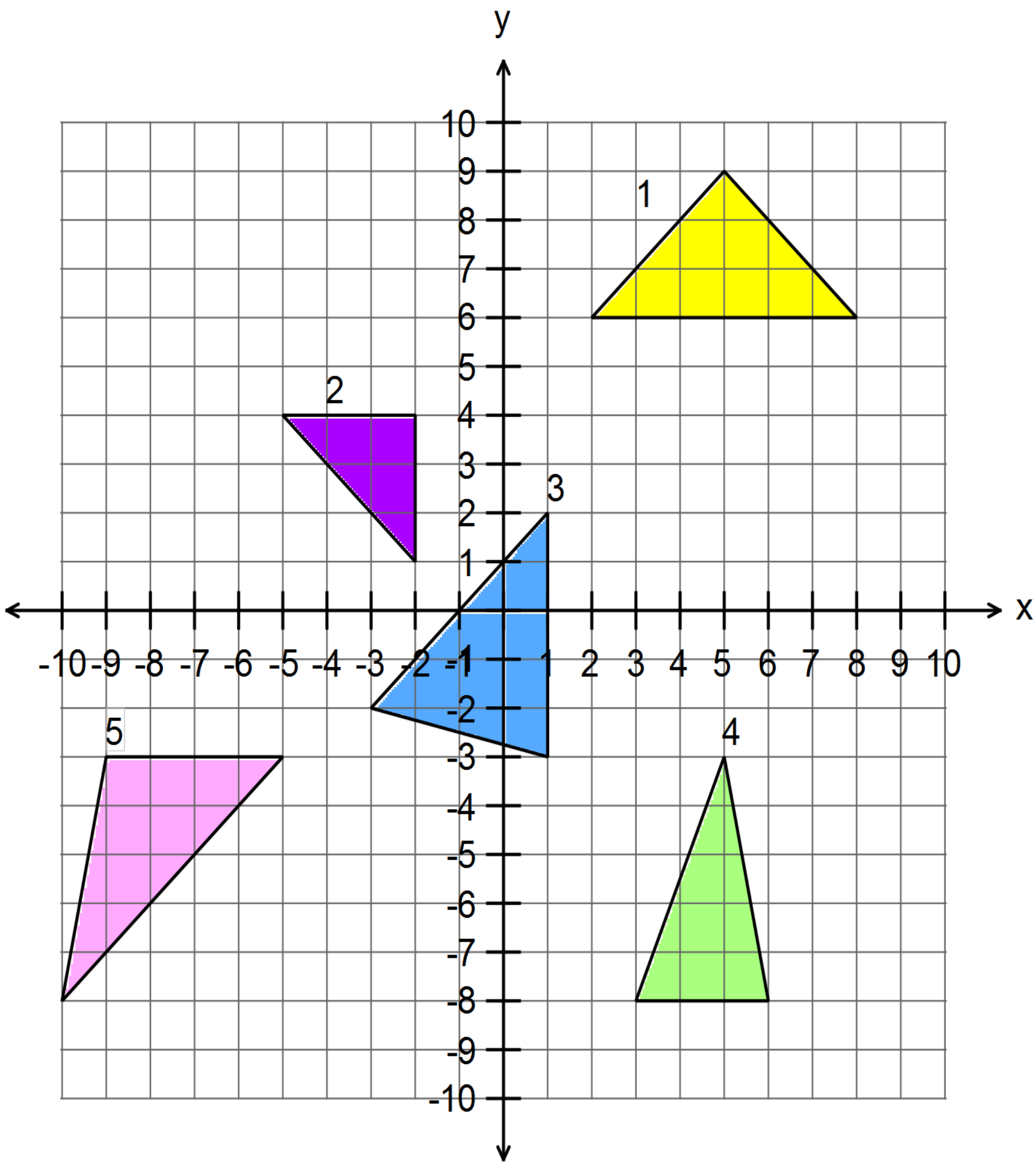
Triangle 5

Movement	Object			Image		
	R	S	T			
down 2 units right 5 units						

2. Examine the patterns in these tables and identify ways of determining the coordinates of an image after a translation without the need to draw the image. Explain your findings.

Activity 4

1. Before commencing this activity check your understanding of a reflection over the y-axis. The grid from the previous activity is reproduced below. Reflect the triangles over the y -axis and draw the images formed. Determine the coordinates of the vertices of the object triangles and their images.



Triangle	Object			Image		
	D	E	F			
1						

Triangle	Object			Image		
	G	H	J			
2						

Triangle	Object			Image		
	K	L	M			
3						

Triangle	Object			Image		
	N	P	Q			
4						

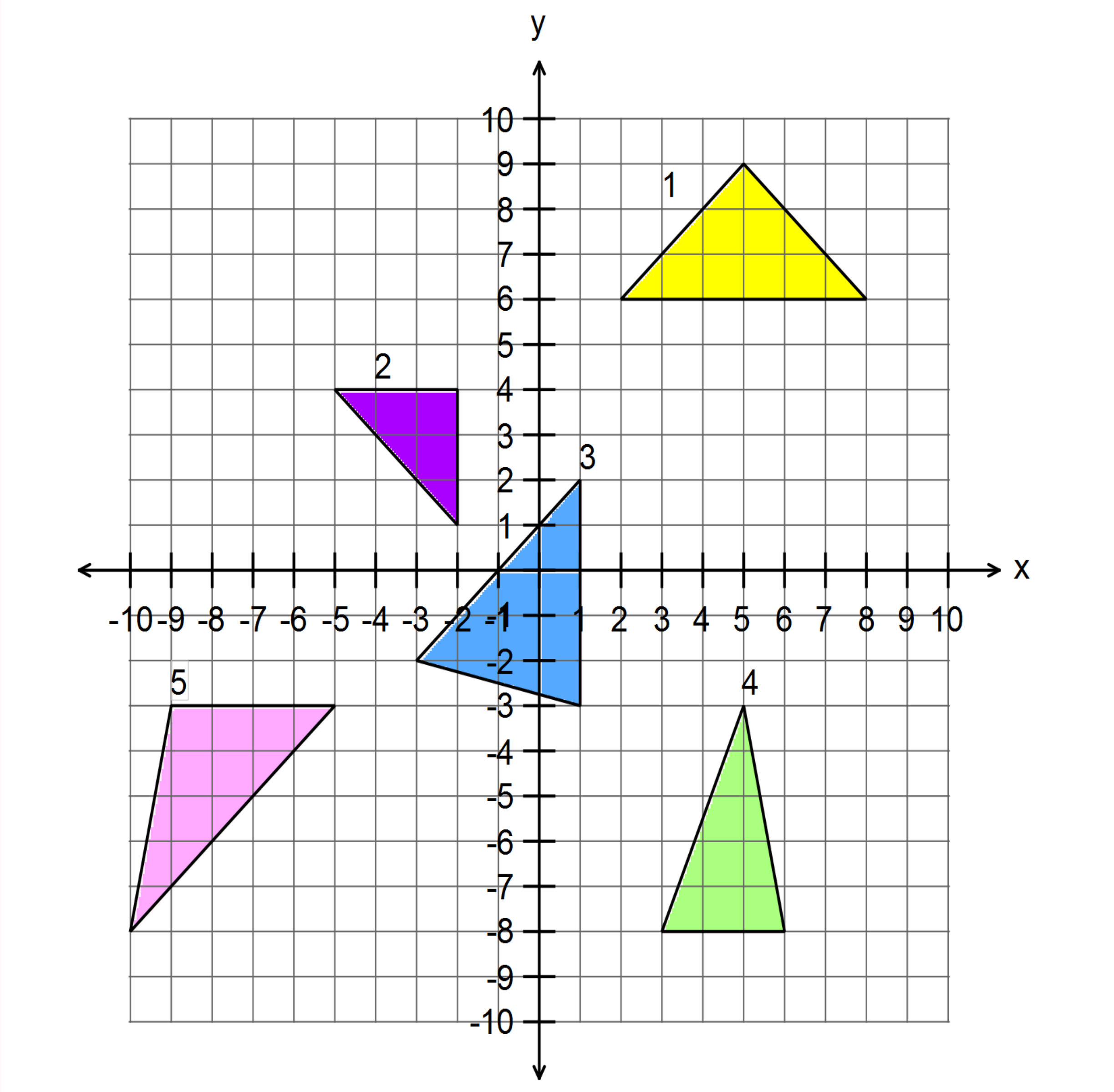
Triangle	Object			Image		
	R	S	T			
5						

2. Examine the patterns in these tables and identify ways of determining the coordinates of an image after a reflection over the y-axis, without the need to draw the image.

3. What can you predict about changes to the coordinates after a reflection over the x-axis? Justify your predictions.

Activity 5

Use the grid below to test the predictions you made in the previous activity.



Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Volume and Area

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 36: VOLUME AND AREA

Overview

For this task it is assumed that the students have used the rule (and possibly the formula) to calculate the volume of a rectangular prism. They should be familiar with the features of a cube and have been introduced to the concept of using letters to represent variables.

Students will need

- measuring tape or ruler (metre lengths)
- access to the internet

Relevant content descriptions from the Western Australian Curriculum

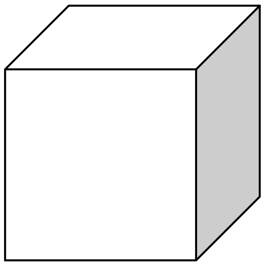
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Solve simple linear equations (ACMNA179)
- Calculate volumes of rectangular prisms (ACMMG160)

Students can demonstrate

- *fluency* when they
    - calculate volumes of prisms
    - calculate total surface area
- *understanding* when they
    - express calculations in various ways
    - distinguish between area and volume
    - determine possible dimensions for unfamiliar volumes
- *problem solving* when they
    - determine the processes to solve the problems in Activity 4
    - determine formulae to represent connections between dimensions

Activity 1

1. The following diagram represents a cube with all sides equal to 1 m.



(a) State the length of each side in cm.
100 cm

(b) State the length of each side in mm. 1000 mm

(c) Determine the volume of the cube in -

(i) m^3 $1\ m \times 1\ m \times 1\ m = 1\ m^3$

(ii) cm^3 $100\ cm \times 100\ cm \times 100\ cm = 1\ 000\ 000\ cm^3$

(iii) mm^3 $1000\ mm \times 1000\ mm \times 1000\ mm = 1\ 000\ 000\ 000\ mm^3$

2. Write a statement of equality relating m^3, cm^3 and mm^3.

$1\ m^3 = 1\ 000\ 000\ cm^3 = 1\ 000\ 000\ 000\ mm^3$

3. Create a space of $1\ m^3$ in the classroom or outside.
How many students can fit inside the $1\ m^3$?

Answers will vary

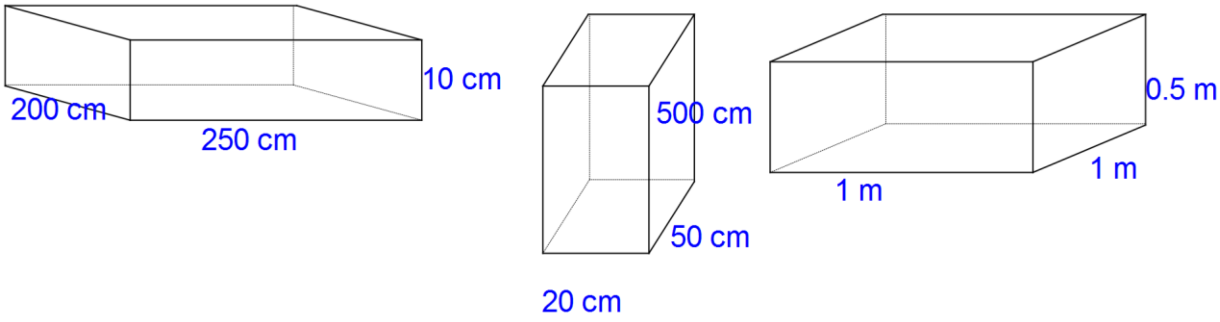
4. Compare the measures of volume and area. Describe the differences.

- Volume measures space occupied by a 3-D object whereas area measures coverage of a 2-D shape.
- Units for volume are cubic units; e.g., cm^3; while units of area are square units; e.g., cm^2.

Activity 2

Consider a rectangular prism that has a volume of 0.5 m^3.
Identify and list at least three possible dimensions for the prism. Draw labelled diagrams to display your answers.

Not to scale



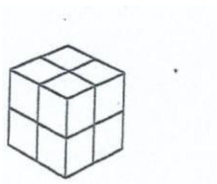
Activity 3

1. Describe the features of all cubes.
   - All faces have identical area
   - All faces are squares
   - All edges equal
   - 6 faces
   - 12 edges
   - 8 vertices
   - Angles at all vertices are right angles

2. Are cubes rectangular prisms? Explain.

Cubes are rectangular prisms as they have all the features of rectangular prisms. The faces of cubes are all squares which are also rectangles - one requirement for a rectangular prism.

3. For this activity, consider each cube as consisting of numbers of smaller cubes, each of which is 1 cm x 1 cm x 1 cm.



(a) How many smaller cubes are there? 8

(b) What is the length of each side of this cube? 2 cm

(c) What is another name given to the "side" of a cube's face? Edge

(d) What is the total length of all the sides of the faces of this cube? 12 cm

(e) Calculate the area of each face. 4 cm^2

(f) Calculate the total area of all the faces. 24 cm^2
[This total area of all the outside faces is called the *Total Surface Area*, shortened to TSA]

4. Enter the results for the above cube into the table below. Determine similar measurements for other cubes made out of cubes with sides of 1 cm and complete the table.

	Units	Cubes					
Length of each side	Small cubes	1	2	3	4	5	6
Length of each side	cm	1	2	3	4	5	6
Total length of all sides	cm	12	24	36	48	60	72
Area of each face	cm^2	1	4	9	16	25	36
TSA	cm^2	6	24	54	96	150	216
Volume	cm^3	1	8	27	64	125	216
Total number of smaller cubes		1	8	27	64	125	216

5. Describe the patterns you see in the sets of numbers in each row.

- The number of small cubes along each side equals the side length.
- The total length of all sides is a multiple of 12 and is 12 x number in the previous row.
- The area of each face is a square number.
- TSA is a multiple of 6 and is 6 x area of each face.
- Volume = the cube of the side length.

Using the letters provided to represent the given dimensions, write algebraic statements of equality linking dimensions to each other.

Length of each side: s; Area of each face: $A = s \times s = s^2$.
Total surface area: TSA = $6 \times A = 6 \times s \times s = 6s^2$ Volume: $V = s \times s \times s = s^3$

Activity 4

1. The total surface area of a cubic block of concrete is 7.26 m^2. Write an equation linking the length of each side of the cube and the TSA. Solve the equation.

$TSA = 6\,s^2$
$7.26 = 6\,s^2$
$1.21 = s^2$
$1.1 = s$

2. A cubic-shaped wooden frame has a total edge length of 18 m. Write an equation linking the length of each side of the frame and the total edge length. Solve the equation.

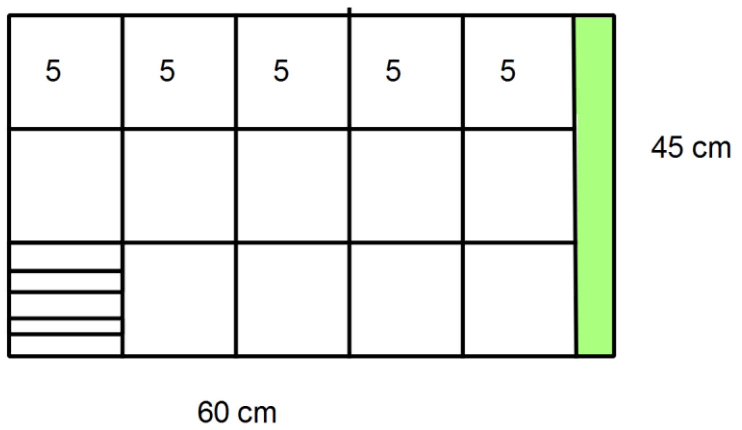
Total edge length = 12 x length of each side
18 = 12 x s
1.5 = s

3. Tim is preparing a quote to cover the floor of the shed with concrete. The shed is 6.3 m wide and 3.1 m long. The floor will be 80 mm thick. To lay the concrete it will cost $1000 per cubic metre. Determine the cost of the concrete floor.

Volume = length x width x height
= 3.1 x 6.3 x 0.08 = 1.5624 m^3
Cost = 1.5624 m^3 x $1000 = $1562.40

4. Muesli bars are sold in boxes of eight. The boxes are packed into cartons to be delivered to the supermarket. A box is 12 cm long, 10 cm wide and 3 cm high. A carton is 60 cm long, 45 cm wide and 30 cm high. How many boxes can fit into each carton?

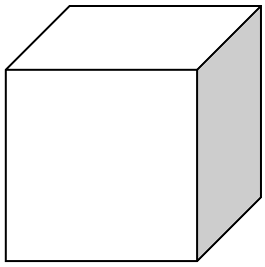
Volume of carton = 60 x 45 x 30 = 81 000 cm^2 Volume of a box = 12 x 3 x 10 = 360 cm^2
81 000 ÷ 360 = 225, but can we stack them?



Use the base of the carton as 60 cm x 45 cm. Each 12 x 15 block can fit 5 boxes so there are 75 boxes in the bottom layer. The boxes are now 10 cm high and the carton 30 cm high, so yes we can fit 75 x 3 = 225 boxes.

Activity 1

1. The following diagram represents a cube with all sides equal to 1 m.



(a) State the length of each side in cm.

(b) State the length of each side in mm.

(c) Determine the volume of the cube in -

(i) m^3

(ii) cm^3

(iii) mm^3

2. Write a statement of equality relating m^3, cm^3 and mm^3

3. Create a space of 1 m^3 in the classroom or outside.
How many students can fit inside the 1 m^3?

4. Compare the measures of volume and area. Describe the differences.

Activity 2

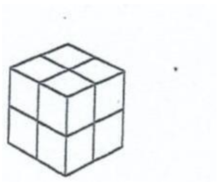
Consider a rectangular prism that has a volume of 0.5 m^3.
Identify and list at least three possible dimensions for the prism. Draw labelled diagrams to display your answers.

Activity 3

1. Describe the features of all cubes.

2. Are cubes rectangular prisms? Explain.

3. For this activity, consider each cube as consisting of numbers of smaller cubes, each of which is 1 cm x 1 cm x 1 cm.



(a) How many smaller cubes are there?

(b) What is the length of each side of this cube?

(c) What is another name given to the "side" of a cube's face?

(d) What is the total length of all the sides of the faces of this cube?

(e) Calculate the area of each face.

(f) Calculate the total area of all the faces.
[This total area of all the outside faces is called the *Total Surface Area*, shortened to TSA]

4. Enter the results for the above cube into the table below. Determine similar measurements for other cubes made out of cubes with sides of 1 cm and complete the table.

	Units	Cubes					
Length of each side	Small cubes	1	2	3			
Length of each side		1	2	3	4	5	6
Total length of all sides							
Area of each face							
TSA							
Volume							
Total number of smaller cubes							

5. Describe the patterns you see in the sets of numbers in each row.

6. Using the letters provided to represent the given dimensions, write algebraic statements of equality linking dimensions to each other.

Length of each side: s; Area of each face: $A =$ ……….....
Total surface area: TSA = ………………………..Volume: V =……………………..

Activity 4

1. The total surface area of a cubic block of concrete is 7.26 m^2. Write an equation linking the length of each side of the cube and the TSA. Solve the equation.

2. A cubic-shaped wooden frame has a total edge length of 18 m. Write an equation linking the length of each side of the frame and the total edge length. Solve the equation.

3. Tim is preparing a quote to cover the floor of the shed with concrete. The shed is 6.3 m wide and 3.1 m long. The floor will be 80 mm thick. To lay the concrete it will cost $1000 per cubic metre. Determine the cost of the concrete floor.

4. Muesli bars are sold in boxes of eight. The boxes are packed into cartons to be delivered to the supermarket. A box is 12 cm long, 10 cm wide and 3 cm high. A carton is 60 cm long, 45 cm wide and 30 cm high. How many boxes can fit into each carton?

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Muesli Muddle

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 104: MUESLI MUDDLE

Overview

In this task, students are required to determine which product is the best value for money. They will need to choose appropriate procedures and adapt their knowledge of mathematical concepts to select the product that is the best value for money. They will investigate the effect different information may have on their solution and explain the conclusions reached.

Students will need

- calculators

Relevant content descriptors from the Western Australian Curriculum

- Calculate volumes of rectangular prisms (ACMMG160)
- Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
- Round decimals to a specified number of decimal places (ACMNA156)
- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

Students can demonstrate

- *fluency* when they
  - draw scale diagrams of each product
  - order the products from the best value for money to the least value for money
- *understanding* when they
  - choose the product that is the best value for money
- *reasoning* when they
  - explain how they can use the information to decide which product is the best value for money
- *problem solving* when they
  - describe the effect the different information has on the product that is the best value for money
  - reorder the products from the best value for money to the least value for money based on the additional information

To help reduce your weekly shopping bill, you must compare products to ensure you are getting the best value for your money. To begin with, you decide to look at the different brands of your favourite muesli bars.

Activity 1

All the five possible boxes are rectangular prisms and are all different sizes and prices. The information is given below.

Brand	Length	Width	Height	Price
Brand 1	5.2 cm	2.2 cm	1.5 cm	$1.10
Brand 2	65 mm	18 mm	11 mm	$1.54
Brand 3	6 cm	3 cm	2 cm	$1.98
Brand 4	5 cm	2 cm	15 mm	$1.17
Brand 5	4.5 cm	2 cm	1 cm	$1.80

1. Draw a labelled scale diagram of each product to help you visualise them.

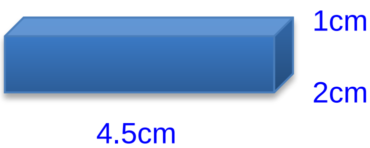
Brand 1

Brand 2

Brand 3

Brand 4

Brand 5



2. Explain how you could use the above information to help you decide which muesli bar is the best value for your money.

   Find the volume of each box in cm^3.
   Find the price per cm^3.
   The lowest price is the best value.

3. Use the above information to decide which muesli bar is the best value for your money.

   1. 17.16 cm^3 -> $0.064 per cm^3
   2. 12.87 cm^3 -> $0.120 per cm^3
   3. 36 cm^3 -> $0.055 per cm^3
   4. 15 cm^3 -> $0.078 per cm^3
   5. 9 cm^3 -> $0.2 per cm^3

   Brand 3 is the best value for money.

4. Order each of the products from the best value for money to the least value for money.

   Brand 3
   Brand 1
   Brand 4
   Brand 2
   Brand 5

Activity 2

Here are the same products as in Activity 1; however, we have now been given the mass of each product instead of the dimensions of each product.

Brand	Mass	Price
Brand 1	50 g	$1.10
Brand 2	75 g	$1.54
Brand 3	80 g	$1.98
Brand 4	65 g	$1.17
Brand 5	70 g	$1.80

1. What effect does this have on the product that you have stated in Activity 1 as the best value for money?

   1. $0.022 per g
   2. $0.021 per g
   3. $0.025 per g
   4. $0.018 per g
   5. $0.026 per g

   Brand 4 is now the best value for money.

2. What effect does this have on the order of the products from the best value for money to the least value for money?

   Brand 4
   Brand 2
   Brand 1
   Brand 3
   Brand 5

   Brand 1 is now less value for money.
   Brand 2 is now better value for money.
   Brand 3 has gone from best value for money to second worst value for money.
   Brand 4 is now the best value for money.
   Brand 5 remains the least value for money.

Activity 3: Extension

We have considered the dimensions and the mass of the muesli boxes to determine which is the best value for money. Are there any other factors that may be useful to consider when calculating which muesli bar is the best value for money. Provide reasons for you answer.

Answers will vary.
Use this as a plenary/feedback session.

To help reduce your weekly shopping bill, you must compare products to ensure you are getting the best value for your money. To begin with, you decide to look at the different brands of your favourite muesli bars.

Activity 1

All the five possible boxes are rectangular prisms and are all different sizes and prices. The information is given below.

Brand	Length	Width	Height	Price
Brand 1	5.2 cm	2.2 cm	1.5 cm	$1.10
Brand 2	65 mm	18 mm	11 mm	$1.54
Brand 3	6 cm	3 cm	2 cm	$1.98
Brand 4	5 cm	2 cm	15 mm	$1.17
Brand 5	4.5 cm	2 cm	1 cm	$1.80

1. Draw a labelled scale diagram of each product to help you visualise them.

2. Explain how you could use the above information to help you decide which muesli bar is the best value for your money.

3. Use the above information to decide which muesli bar is the best value for your money.

4. Order each of the products from the best value for money to the least value for money.

Activity 2

Here are the same products as in Activity 1. However, we have now been given the mass of each product instead of the dimensions of each product.

Brand	Mass	Price
Brand 1	50 g	$1.10
Brand 2	75 g	$1.54
Brand 3	80 g	$1.98
Brand 4	65 g	$1.17
Brand 5	70 g	$1.80

1. What effect does this have on the product that you have stated in Activity 1 as the best value for money?

2. What effect does this have on the order of the products from the best value for money to the least value for money?

Activity 3: Extension

We have considered the dimensions and the mass of the muesli boxes to determine which is the best value for money. Are there any other factors that may be useful to consider when calculating which muesli bar is the best value for money. Provide reasons for you answer.

Department of Education

The Mathematical Association of Western Australia Inc.

Year 7 Mathematics

Multi-Strand Activity

Squ-Area

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 109: SQU-AREA

Overview

In this task, students will investigate how the area of a square relates to square numbers. The activities are scaffolded to allow students to focus on one topic independently. They are then required to compare solutions and make links between their answers.

Students will need

- calculators
- access to textbooks and the internet

Relevant content descriptors from the Western Australian Curriculum

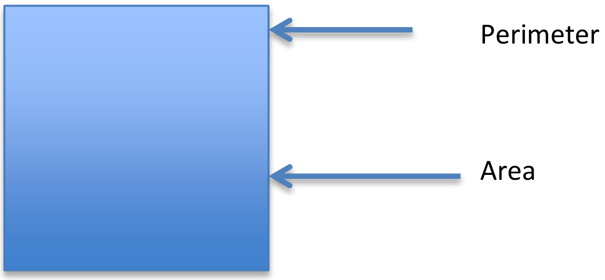
- Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)
- Investigate and use square roots of perfect square numbers (ACMNA150)
- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Students can demonstrate

- *fluency* when they
  - calculate the area of a square
  - multiply without assistance

  understanding when they
  - define perimeter and area
  - explain how perimeter and area are found
  - explain the difference between perimeter and area
- *reasoning* when they
  - can compare the two sets of solutions to develop a rule
- *problem solving* when they
  - attempt to apply their method to different shapes

Activity 1



1. Name the shape above.

   Square

2. What is perimeter? How is it found?

   Perimeter is the distance around the outside of a shape. It is found by adding the length of all four sides together.

3. What is area? How is it found?

   Area measures the space inside a shape. It is found by multiplying the length by the width.

4. State the main difference when presenting your solution for perimeter and your solution for area.

   Length is given in mm, cm, m or km whereas area is given in mm^2, cm^2, m^2 or km^2.

Activity 2

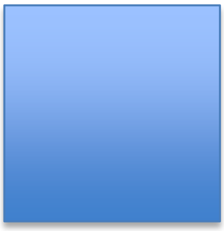
Find the area of the following shapes.

1 cm

$1 \times 1 = 1\ cm^2$

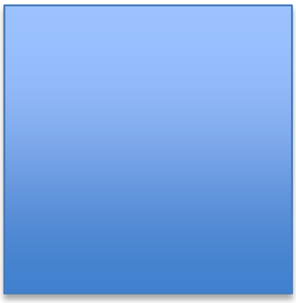
2 cm

$2 \times 2 = 4\ cm^2$



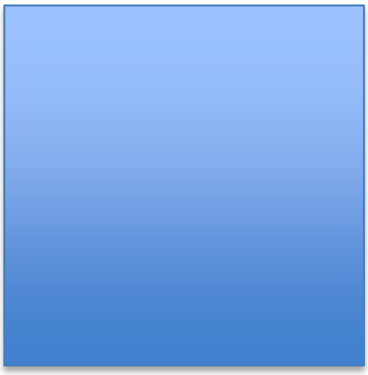
3 cm

$3 \times 3 = 9\ cm^2$



4 cm

$4 \times 4 = 16\ cm^2$



5 cm

$5 \times 5 = 25\ cm^2$

Activity 3

Complete the following table.

Problem	Solution
1 x 1	1
2 x 2	4
3 x 3	9
4 x 4	16
5 x 5	25

When numbers are multiplied as above, is there a special name for this? If you don't know, use textbooks or the Internet to find out.

Squaring the number.

Activity 4

1. Looking at the work you did in Activity 2 and Activity 3, do they have anything in common?

   The solutions are the same.

2. Can you find a pattern in the set of numbers? What is this pattern? Explain it in different ways.

   1. Answers may vary.
   2. 1, 4, 9, 16, 25

      $^{+}3$, $^{+}5$ $^{+}7$ $^{+}9$
   3. Adding increasing odd numbers.
   4. 1st number is 1 multiplied by itself. 2nd number is 2 multiplied by itself.

      3rd number is 3 multiplied by itself and so on.

3. Can you come up with a rule or short cut to find any number in this pattern? For example, how could you find the 20th or 100th number?

   To find any number multiply it by itself.

   For example, the 20th number would be 20 x 20 = 400

4. What can you conclude about finding the area of this shape and multiplying numbers as above?

   Finding the area of a square and squaring a number is the same thing. You must multiply the number by itself in both cases.

5. If given the area and asked to find the side length, describe how you could do this.

   Try to find the number that when multiplied by itself gives you that area as an answer. This will be the side length.

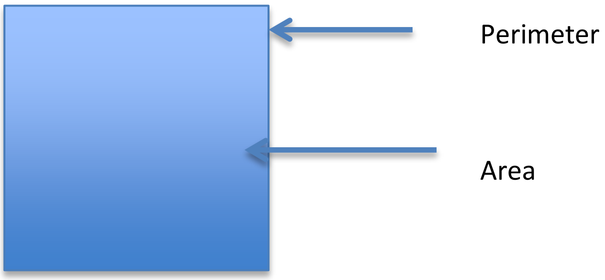
   The square root. You can use the square-root symbol on your calculator ($\sqrt{\ }$) which was derived from the letter **r** for **root**.

Activity 5: Further investigation

Can you use your rule for different shapes?

Answers will vary.

Activity 1



1. Name the shape above.

2. What is perimeter? How is it found?

3. What is area? How is it found?

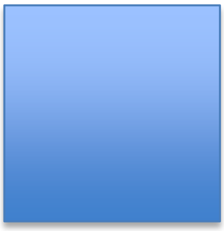
4. State the main difference when presenting your solution for perimeter and your solution for area.

Activity 2

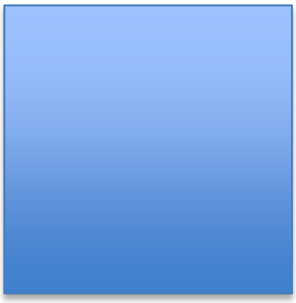
Find the area of the following shapes.

1 cm

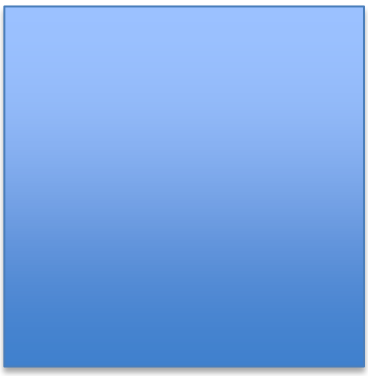
2 cm



3 cm



4 cm



5 cm

Activity 3

Complete the following table.

Problem	Solution
1 x 1	
2 x 2	
3 x 3	
4 x 4	
5 x 5	

When numbers are multiplied as above, is there a special name for this? If you don't know, use textbooks or the Internet to find out.

Activity 4

1. Looking at the work you did in Activity 2 and activity 3, do they have anything in common?

2. Can you find a pattern is the numbers? What is the pattern? Explain it in different ways.

3. Can you come up with a rule or short cut to find any number is this pattern? For example, how could you find the 20^{th} or 100^{th} number?

4. What can you conclude about finding the area of this shape and multiplying numbers as above?

5. If given the area and asked to find the side length, describe how you could do this.

Activity 5: Further investigation

Can you use your rule for different shapes?

Department of Education

Year 7 Mathematics

Multi-Strand Activity

Tax A Million

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

TASK 113: TAX A MILLION

Overview

In this task, students will investigate the amount of tax paid by the average Australian. Students will be required to make connections between related ideas and interpret mathematical information. They will choose appropriate procedures to assist in the investigation of a meaningful situation. They will need to analyse their findings to reach a conclusion.

Students will need

- calculators
- access to the internet

Relevant content descriptors from the Western Australian Curriculum

- Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)
- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)
- Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)

Students can demonstrate

- *fluency* when they
    - identify average amounts of tax for a typical Australian
    - calculate appropriate percentages
- *understanding* when they
    - present their information in a mathematical way
    - recognise average amounts as a tool for investigation
- *reasoning* when they
    - explain why they are (not) surprised by their results
- *problem solving* when they
    - successfully collect, present and analyse the required information

Every Australian pays tax on the money they earn, whether it's from a job or income generated by a business they own. How much they earn will determine how much tax they pay. Is this the only tax we pay? How much of our earnings really go to the taxman?

To explore this idea, let's look at an average Australian and their income and expenditure.

Some things you may want to consider before you start:

- What is the average earning (before tax)?
- Is there a difference between male and female?
- What percentage is income Tax?
- What percentage is GST (Goods and Services Tax) on expenditure?
- Is tax on fuel the same as on other goods?
- Can we consider rates as taxes?
- Can we consider car registration as taxes?
- Any other taxes we might need to consider?

You may want students to use the ATO website. Alternatively you could give them set rates you would like them to use. Below are the 2015-16 rates. You may or may not want them to consider the Medicare and Budget Repair Levies.

The following rates for 2015–16 apply from 1 July 2015.

Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $37,000	19c for each $1 over $18,200
$37,001 – $80,000	$3,572 plus 32.5c for each $1 over $37,000
$80,001 – $180,000	$17,547 plus 37c for each $1 over $80,000
$180,001 and over	$54,547 plus 45c for each $1 over $180,000

The above rates **do not** include the:

- Medicare levy of 2%
- Temporary Budget Repair Levy; this levy is payable at a rate of 2% for taxable incomes over $180,000.

- Add to this list as required.
- Remove any dot points you do not wish students to use.
- Tax on fuel, excise tax, is about 22% in Australia.
- Rates would need to be researched for a particular area.
- Car registration is based on the type of car.
- GST is paid on most other expenditure.
- There would not be any GST on donations or on certain foods.

Activity 1

Use the Internet or other reliable resources to determine the following for an average Australian:

1. Monthly/weekly income (before tax)
2. Monthly/weekly savings
3. Monthly/weekly bills
4. Monthly/weekly expenditure

You could draw up a list of bills and expenditure you would like the students to research and put this on the whiteboard. Bear in mind that all students will more than likely have slightly different values.

Activity 2

Draw up a monthly budget, allocating all of the average Australian's monthly/weekly income to saving, bills or expenditure and identify any taxes paid. You will need to include:

1. income tax
2. rates
3. car registration
4. GST
5. tax paid on fuel
6. any other taxes

Students could use Excel or similar to draw up their budget.
Students may need a lot of guidance to develop a template.
You could allow students to research budget templates on the Internet.

Activity 3

1. What is the total amount of tax paid?
2. What percentage is this of the average Australian's income?
3. Are these results as you expected? Provide reasons why you are or are not surprised.

Answer these questions as a group; use them as a Q&A session. Allow students to give their feedback, and maybe draw up a response sheet on the whiteboard. Create a class discussion on the pros and cons of paying tax.

Every Australian pays tax on the money they earn, whether it's from a job or income generated by a business they own. How much they earn will determine how much tax they pay. Is this the only tax we pay? How much of our earnings really go to the taxman?

To explore this idea, let's look at an average Australian and their income and expenditure.

Some things you may want to consider before you start:

- What is the average earning (before tax)?
- Is there a difference between male and female?
- What percentage is income tax?
- What percentage is GST (Goods and Services Tax) on expenditure?
- Is tax on fuel the same as on other goods?
- Can we consider rates as taxes?
- Can we consider car registration as taxes?
- Any other taxes we might need to consider?

Activity 1

Use the Internet or other reliable resources to determine the following for an average Australian:

1. Monthly/weekly income (before tax)

2. Monthly/weekly savings

3. Monthly/weekly bills

4. Monthly/weekly expenditure

Activity 2

Draw up a monthly budget, allocating all of the average Australian's monthly/weekly income to saving, bills or expenditure and identify any taxes paid. You will need to include:

1. income tax
2. rates
3. car registration
4. GST
5. tax paid on fuel
6. any other taxes

Activity 3

1. What is the total amount of tax paid?

2. What percentage is this of the average Australian's income?

3. Are these results as you expected? Provide reasons why you are or are not surprised.

Department of Education

The Mathematical Association of Western Australia Inc.

YEAR 7 MATHEMATICS

Multi-Strand Activity

Box Enough

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

TASK 135: BOX ENOUGH

Overview

In this task, students will investigate whether a shipping container that a company currently using is the most efficient container to use. They will need to calculate answers efficiently, recall factual knowledge and choose appropriate methods to seek solutions. They are required to reason mathematically and justify their conclusions.

Students will need

- Calculator
- Isometric dot paper (optional)

Relevant content descriptions from the Western Australian Curriculum

- Calculate volumes of rectangular prisms (ACMMG160)
- Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)
- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

Students can demonstrate

- *fluency* when they
  - draw appropriate diagrams of a rectangular prism
  - calculate the volume of a rectangular prism
- *understanding* when they
  - label their diagrams appropriately
  - calculate how many boxes should fit into the shipping container considering dimensions and mass
- *reasoning* when they
  - decide, using mathematical reasoning, which is a better container to use
- *problem solving* when they
  - check whether the stated number of boxes fit into the container

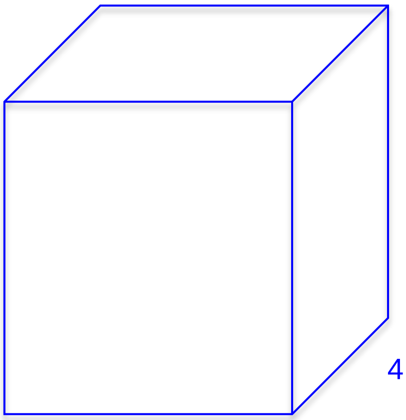
BOX ENOUGH

Solutions and Notes for Teachers

An electrical supply company produces a transformer box that has measurements of 4 cm wide, 5 cm long and 6 cm high and each transformer weighs 300 g. They currently use shipping containers that are 12 cm wide, 15 cm long and 16 cm high and each container can hold up to 15 kg.

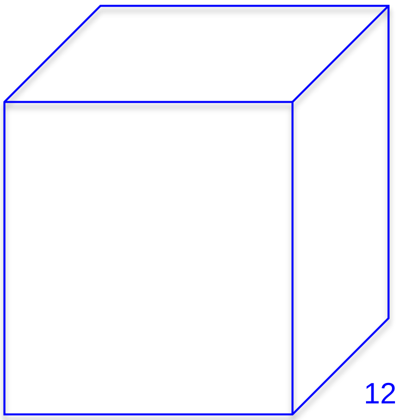
<u>Activity 1</u>

1. Draw a labelled scale diagram of the transformer box.



2. Calculate the volume of a transformer box.
   $4 \times 5 \times 6 = 120\ cm^3$

3. Draw a labelled diagram, not to scale, of the shipping container.



4. Calculate the volume of the shipping container.
   $15 \times 12 \times 16 = 2880\ cm^3$

5. Considering the volumes you have found, how many transformer boxes should fit in the shipping container?
   $2880 \div 120 = 24$ boxes

6. Considering the dimensions of the boxes and containers, check whether the above number of transformer boxes will actually fit in the shipping container.
   $3 \times 3 \times 2 = 18$ boxes

7. Considering the mass capacities of the boxes and containers, how many transformer boxes can the shipping container hold?
   $30\,000 \div 300 = 100$ boxes

Activity 2

Considering all of the information in Activity 1, and that the company is charged $0.15 per cm^3 for the shipping container. Use mathematical reasoning to decide if these are the most efficient containers to use? If not, suggest a more efficient shipping container that they could use.

- Charged: 2880 x 0.15 = $432
  Using: 120 x 18 = 2160; 2160 x 0.15 = $324
  Over charged: 432 – 324 = $108
  432 ÷ 18 = $24 per box
  The containers hold 15 kg, but only 7.2 kg is required.

- Suggested: 12 x 15 x 18 that hold 7.5 kg
  Charged 2 x 15 x 18 = 3240 x 0.15 = $486
  Holds 27 boxes
  486 ÷ 27 = $18 per box
  Mass is 8.1 kg
  If they used a lighter box it may cost less but using this option, even with the 15 kg mass limit, it is cheaper.

An electrical supply company produces a transformer box that has measurements of 4 cm wide, 5 cm long and 6 cm high and each transformer weighs 300 g. They currently use shipping containers that are 12 cm wide, 15 cm long and 16 cm high, and each container can hold up to 15 kg.

Activity 1

1. Draw and label a scale diagram of the transformer box.

2. Calculate the volume of the transformer box.

3. Draw a labelled diagram, not necessarily to scale, of the shipping container.

4. Calculate the volume of the shipping container.

5. Considering the volumes you have found, how many transformer boxes could fit in the shipping container?

6. Considering the dimensions of the boxes and containers, check whether the above number of transformer boxes will actually fit in the shipping container.

7. Considering the mass capacities of the boxes and containers, how many transformer boxes can the shipping container hold?

Activity 2

Considering all of the information in Activity 1, and that the company is charged \$0.15 per cm^3 for the shipping container, use mathematical reasoning to decide if these are the most efficient containers to use. If not, suggest a more efficient shipping container that they could use.