Department of Education

The Mathematical Association of Western Australia Inc.
MAΣWA

# Year 7 Mathematics

## Number & Algebra Tasks
## Set 3

## TASK LIST

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Activity

# Lazy Lines

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 107: LAZY LINES

## Overview

In this task, students will consider a mental strategy for multiplying large numbers. They will need to apply familiar ideas to develop new ideas that they can use efficiently. They will need to interpret, model and investigate to understand and apply the strategy. Students should reason about their finding using mathematical language to make connections between the known and unknown.

## Students will need

- calculators
- access to textbooks and the internet

## Relevant content descriptors from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)

## Students can demonstrate

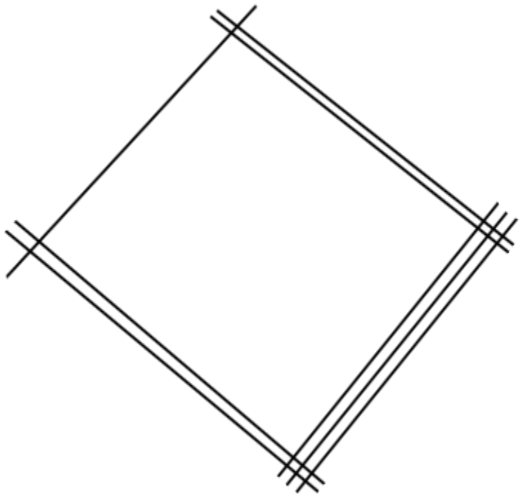
- *fluency* when they
    - recall a mental strategy for multiplying large numbers

  *understanding* when they
    - describe how the solution was achieved
    - use and extend the line strategy
- *reasoning* when they
    - identify constraints within the method
    - compare two strategies for similarities
- *problem solving* when they
    - identify possible ways to improve the diagram
    - attempt to overcome any constraints

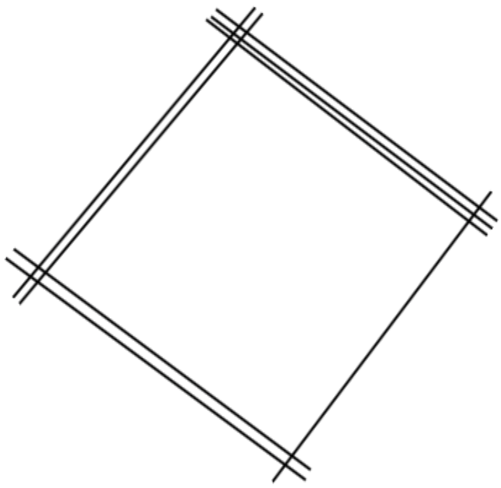
Activity 1

The Japanese have developed a visual strategy for multiplying large numbers.

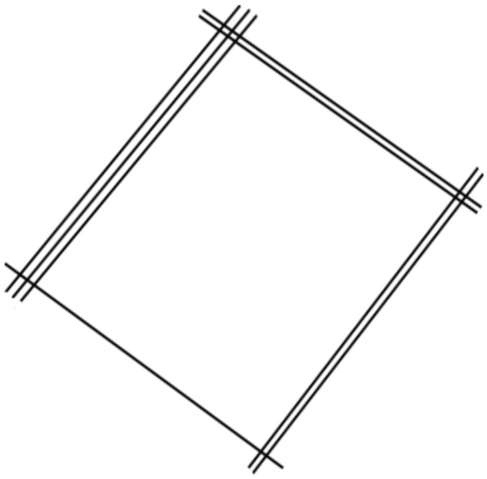
Let's look at some examples; if I wanted to find the solution to a large product, without a calculator, and using the strategy developed by the Japanese, we would end up with something like this:



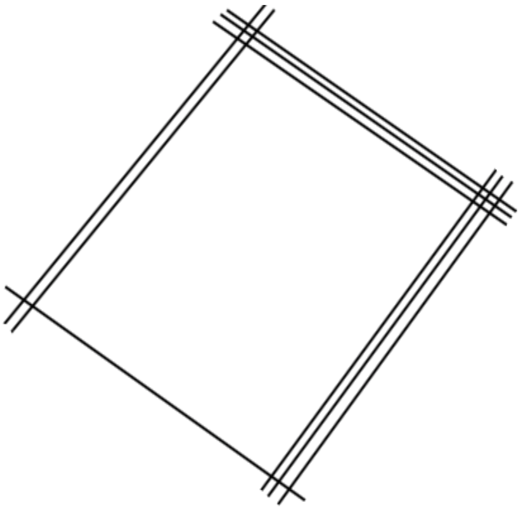
The solution to 13 x 22 is 286.



The solution to 21 x 23 is 483.



The solution to 32 x 12 is 384.



The solution to 23 x 13 is 299.

1. Explain how the lines are used to represent the multiplication.

   The first number is drawn sloping upwards from left to right. The first digit of the number is represented on the left and the second digit of the number is represented on the right.
   The second number is drawn sloping downwards from left to right. The first digit of the number is represented at bottom left and the second digit of the number is represented at top right.

2. Describe how the solutions were achieved using the diagrams above?

   The diagram is split into three zones, left, middle and right, where the lines cross. Starting at the right, count how many times the lines cross each other; this is the last digit of the number.
   The middle zone has two parts where the line cross each other; so again count how many times, and this is the middle digit.
   Lastly, to the left, count how many times the lines cross each other, and this becomes the first digit.

3. What information could you add to the diagram to make finding the solution easier?

   Circling the zones.
   Adding dots where the lines cross each other.

4. Can you identify any constraints with this method?

   When adding how many times the lines cross, what would happen if the number was greater that 9?

5. Thinking about the constraints you have identified, how could you overcome these so that the strategy will always work?

   If it adds up to more than 9, you could use the unit digit and carry over the tens digit.

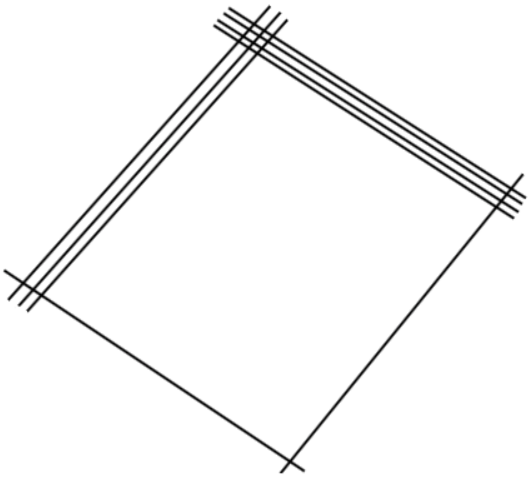
6. What strategy would you have used to solve this problem if the line strategy was not available?

   Long multiplication or use a calculator.

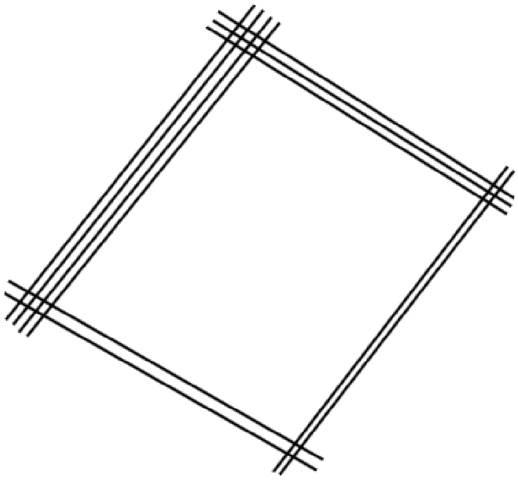
7. Looking at both your strategy and the one above, what makes these different strategies the same?

   Working from right to left
   Using the unit value and carrying over the tens value
   Visual
   Any other acceptable reason.

Activity 2



The solution to 31 x 14 is 434.



The solution to 42 x 23 is 966.

1. Describe how these solutions were achieved using the diagrams above?

   The same strategy as before; however, as the middle zones added to more than 9, we needed to use the units digit and ‘carrying’ the tens digit, then add this to the last zone to solve.

2. How were any constraints overcome in this problem?

   Following the strategy of carrying as discussed previous and used in long multiplication.

3. Use the line method to solve the following:

a. 42 x 36

   1512

b. 25 x 16

   400

c. 82 x 45

   3690

Activity 3: Extension

Consider the following larger numbers. Use and extend this strategy to calculate the solutions.

1. 142 x 23

   3266

2. 213 x 341

   72 633

Activity 1

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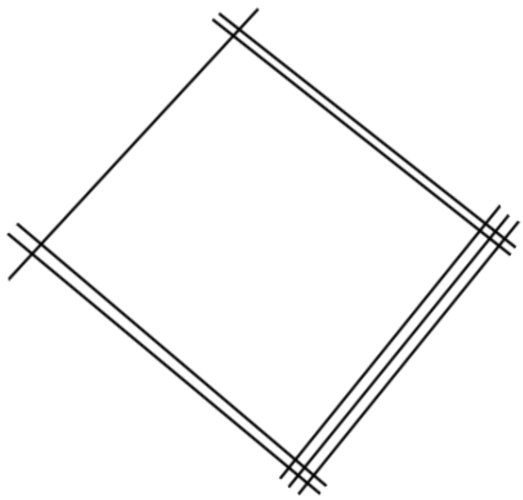

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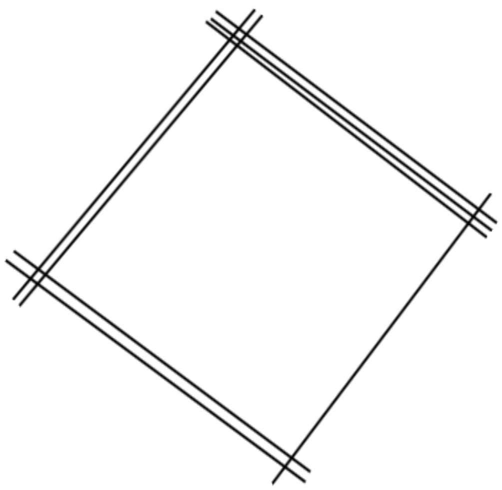

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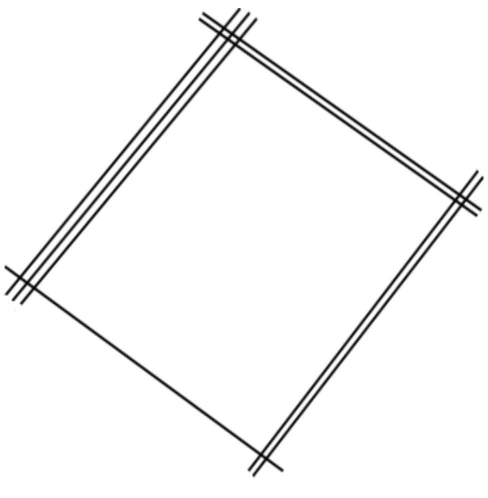

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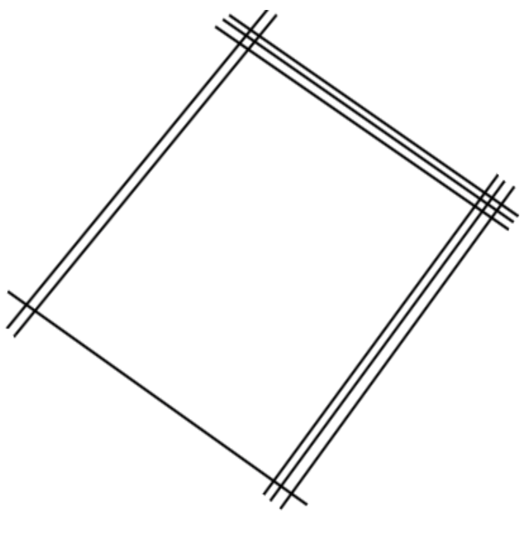

The solution to 23 x 13 is 299.

1. Explain how the lines are used to represent the multiplication.

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3. What information could you add to the diagram to make finding the solution easier?

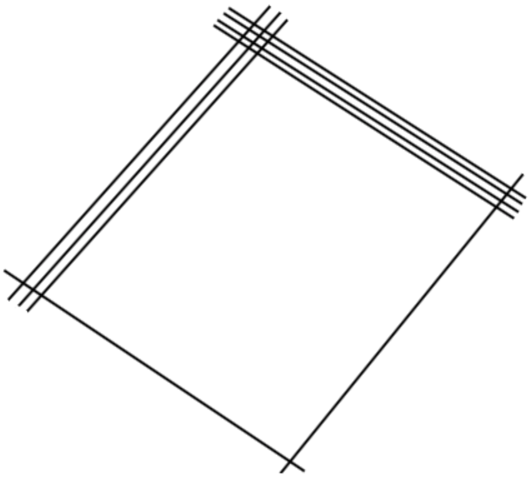
4. Can you identify any constraints with this method?

5. Thinking about the constraints you have identified, how could you overcome these so that the strategy will always work?

6. What strategy would you have used to solve this problem if the line strategy was not available?

7. Looking at both your strategy and the one above, what makes these different strategies the same?

Activity 2



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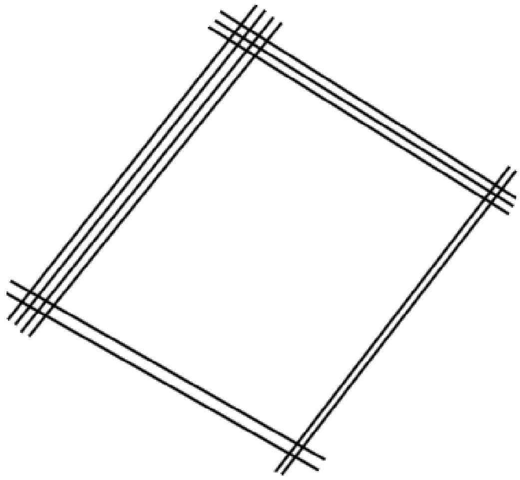

The solution to 42 x 23 is 966.

1. Describe how these solutions were achieved using the diagrams above?

2. How were any constraints overcome in this problem?

3. Use the line method to solve the following:

   a. 42 x 36

b. 25 x 16

c. 82 x 45

Activity 3: Extension

Consider the following larger numbers. Use and extend the line strategy to calculate the solutions.

1. 142 x 23

2. 213 x 341

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# Year 7 Mathematics

## Number & Algebra Activity

# Wedding Woes

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 112: WEDDING WOES

## Overview

In this task, students are required to demonstrate the skills of forming algebraic rules based on a set of information. The students had to investigate the most efficient method of setting up tables for a wedding. Students will represent numbers using variables. They will connect the laws and properties for numbers to algebra and interpret simple linear representations and model authentic information. To find a solution students must use problem-solving strategies to solve authentic problems and to apply reasoning and justification for their solutions.

## Relevant content descriptions from the Western Australian Curriculum

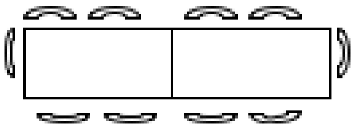
- Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

## Students can demonstrate

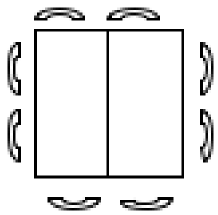
- *fluency* when they
  - calculate how many people can sit at two joined tables
  - use rules to calculate how many tables will be required/people seated
- *understanding* when they
  - correctly complete the table of values
  - create a rule to link the number of tables to the number of people seated
- *reasoning* when they
  - explain why joining the tables in certain ways affects the number of people that can be seated
- *problem solving* when they
  - design a table plan according to Laura’s and Jack’s families’ conditions

Laura and Jack are planning their wedding. One of the tasks is for them to arrange the seating at the reception. They have invited 150 guests and they expect all of them to attend. At the venue they have 30 rectangular tables; where one table can seat 6 guests. To seat more than 6 people, there are two ways in which the tables can join together.

Lengthways:



Widthways:



Activity 1

1. How many people can be seated around two table if they are joined together:
   a) Lengthways?
      10 people

   b) Widthways?
      8 people

2. Complete the following tables of values. Draw the tables and count the number of people around them if necessary.

Lengthways:

| Number of tables joined ($t$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of people seated ($p$) | 6 | 10 | 14 | 18 | 22 | 26 | 30 |

Widthways:

| Number of tables joined ($t$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of people seated ($p$) | 6 | 8 | 10 | 12 | 14 | 16 | 18 |

3. Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient (uses less tables)?
Lengthways = 2 tables
Widthways = 3 tables
Therefore lengthways is more efficient.

4. Is this type of arrangement always more efficient, explain your answer?

Yes. If I add one table lengthways, I can add 4 people but if I add one table widthways I can only add 2 people.

5. Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

When I add a table lengthways I block one seat but when I add a table widthways I block two seats.

6. For each of the two arrangements, write a rule that connects the number of tables ($t$), to the number of people that can be seated $p$).
Lengthways: $p = 4t + 2$
Widthways: $p = 2t + 4$

7. Using your rules, predict how many people could be seated at 50, 100 and 200 tables, **BOTH** lengthways **AND** widthways. Use a table to record your results.

| Lengthways | Widthways |
|---|---|
| $p = 4 \times 50 + 2 = 202$ people | $p = 2 \times 50 + 4 = 104$ people |
| $p = 4 \times 100 + 2 = 402$ people | $p = 2 \times 100 + 4 = 204$ people |
| $p = 4 \times 200 + 2 = 802$ people | $p = 2 \times 200 + 4 = 404$ people |

8. If all of the tables are joined lengthways, how many tables would be required to seat all of Laura and Jack's guests?
$150 = 4t + 2$
$148 = 4t$
$37 = t$
37 tables required

9. If all of the tables are joined widthways, how many tables would be required to seat all of Laura's and Jack's guests?
$150 = 2t + 4$
$146 = 2t$
$73 = t$
73 tables required

10. Will Laura and Jack have enough seating for all of their guests if they join all of the tables together?
No, if they join all of the tables, either lengthways or widthways, there will not be enough seating for all 150 guests.

## Activity 2

Laura and Jack have decided that they will not join all of the 30 tables together but they will group certain guests. Below are some conditions that their families have outlined.

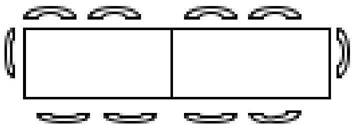
- The bridal party must sit together on a lengthways table at the top of the room. There are 14 people in the bridal party.
- Jack's cousins must all sit together. Jack has 25 cousins.
- Jack's friends would prefer to sit in their own group. There are 5 of them.
- Laura's mum's family cannot sit with her dad's family but they do not have to sit as one big group. There are 18 people in Laura's mum's family and 36 people in Laura's dad's family.
- Laura's friends would like to sit together but said it was OK if they could not. There are 15 of them.

Using these conditions, design a table plan for Laura and Jack for their wedding. Ensure that all tables are labelled correctly. Draw all tables the same size with your ruler.

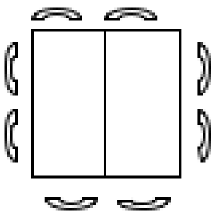
Answers will vary.
Students should refer back to the conditions.
Remind students they have 30 individual tables.

Laura and Jack are planning their wedding. One of the tasks is for them to arrange the seating at the reception. They have invited 150 guests and they expect all of them to attend. At the venue they have 30 rectangular tables; where one table can seat 6 guests. To seat more than 6 people, there are two ways in which the tables can join together.

Lengthways:



Widthways:



Activity 1

1. How many people can be seated around two table if they are joined together:

   (a) Lengthways?

   (b) Widthways?

2. Complete the following tables of values. Draw the tables and count the number of people around them if necessary.

Lengthways:

| Number of tables joined (t) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of people seated (p) | | | | | | | |

Widthways:

| Number of tables joined (t) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of people seated (p) | | | | | | | |

3. Consider the two ways of joining the tables in order to seat 10 people. Which type of arrangement (lengthways or widthways) is more efficient (uses less tables)?

4. Is this type of arrangement always more efficient, explain your answer?

5. Why does the way in which the tables are joined together (lengthways or widthways) affect the number of people that can be seated around them?

6. For each of the two arrangements, write a rule that connects the number of tables (*t*), to the number of people that can be seated *(p)*.

7. Using your rules, predict how many people could be seated at 50, 100 and 200 tables, **BOTH** lengthways **AND** widthways. Use a table to record your results.

8. If all of the tables are joined lengthways, how many tables would be required to seat all of Laura's and Jack's guests?

9. If all of the tables are joined widthways, how many tables would be required to seated all of Laura and Jack's guests?

10. Will Laura and Jack have enough seating for all of their guests if they join all of the tables together?

Activity 2

Laura and Jack have decided that they will not join all of the 30 tables together but they will group certain guests. Below are some conditions that their families have outlined.

- The bridal party must sit together on a lengthways table at the top of the room. There are 14 people in the bridal party.
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Using these conditions, design a table plan for Laura and Jack for their wedding. Ensure that all tables are labelled correctly. Draw all tables the same size with your ruler.

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# Year 7 Mathematics

## Number & Algebra Activity

# Make or Buy?

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 115: MAKE OR BUY?

## Overview

In this task, students will investigate whether it is more cost efficient to make or buy cupcakes to create a birthday cake. Students will be required to make connections between related concepts, choose appropriate procedures and calculate answers efficiently. They will need to make choices and use mathematics to formulate an argument for their chosen method. They will need to compare and contrast their findings to explain their choices.

## Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)
- Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)
- Recognise and solve problems involving simple ratios (ACMNA173)

## Students will need

- calculators

## Students can demonstrate

- *fluency* when they
    - calculate how many packs required
    - calculate the cost of buying cupcakes
- *understanding* when they
    - calculate the cost of buying 1 cupcake
    - calculate the cost of 1 cup and required amount
- *reasoning* when they
    - identify that they can only make one more batch of 12 cupcakes
- *problem solving* when they
    - present their findings to convince Jenny's mum to either buy or make

Jenny doesn't want a traditional cake for her birthday this year; she would like a cupcake cake!!! Her mother is concerned about the price of buying so many cupcakes so has asked Jenny to research whether making her own would be more cost effective. To fill the cupcake stand and make it look like a cake Jenny will need at least 24 cupcakes.

Activity 1

To buy a 4-pack of chocolate cupcakes from the supermarket will costs $3.50.

1. How many packs will Jenny need to buy so she has at least 24 cupcakes?
   24/4 = 6

2. How much will it cost to buy at least 24 cupcakes from the supermarket?
   6 x 3.50 = $21

3. What is the cost of buying 1 cupcake from the supermarket?
   21/24 = $0.875

Activity 2

To make a batch of 12 cupcakes you will need the following:

- 1/2 cup oil
- 2 cups self-raising flour
- 1/4 cup cocoa
- 3/4 cup caster sugar
- 1 egg lightly beaten
- 1 1/3 cups milk
- 3/4 cup white chocolate chopped
- 3/4 cup milk chocolate chopped

If Jenny decides to make the cupcakes she will need to buy all of the ingredients. The ingredients, price and amounts are outlined below:

| Ingredient | Price | Amount | Cost of 1 cup | Required amount | Cost of required amount |
|---|---|---|---|---|---|
| Oil | $2.80 | 2 cups | $1.40 | ½ cup | $0.70 |
| Self-raising flour | $0.80 | 4 cups | $0.20 | 2 cups | $0.40 |
| Cocoa | $4.20 | 1.5 cups | $2.80 | ¼ cup | $0.70 |
| Sugar | $0.80 | 4 cups | $0.20 | ¾ cup | $0.15 |
| Eggs | $3.60 | 6 eggs | $0.60 | 1 egg | $0.60 |
| Milk | $1.20 | 4 cups | $0.30 | $1^{1}/_{3}$ cups | $0.40 |
| Whits chocolate | $3.00 | 1.5 cups | $2.00 | ¾ cup | $1.50 |
| Milk chocolate | $3.00 | 1.5 cups | $2.00 | ¾ cup | $1.50 |

1. Fill in the table above with the cost of 1 cup.
2. Use the cost of one cup to calculate the cost of the required amount in each case.
3. How much will it cost to make a batch of 12 cupcakes?
   $5.95
4. How much will it cost to make 24 cupcakes?
   $11.90
5. How much cheaper is it to either buy or make?
   $9.10
6. What is this as a percentage of buying the cupcakes? Note: Per cent = hundredths
   Emphasise to students that per cent = hundredths. Thus need to use a calculator to undertake the fraction division: 9.1/21 = 0.4333 = 43 hundredths = 43%

7. What is this as a percentage of making the cupcakes?
   9.1/11.95 = 0.7615 = 76%

8. Will Jenny be able to make more than 24 cupcakes with her ingredients? Show mathematically whether she can or cannot make any more.

   Oil – 1 cup
   Self-raising flour – 0 cup
   Cocoa – 1 cup
   Sugar – 2.5 cups
   Eggs – 4 eggs
   Milk – 1 1/3 cups
   White chocolate – 0 cup
   Milk chocolate – 0 cup

   Jenny will not be able to make any more as there will not be any self-raising flour, white chocolate or milk chocolate left after she makes the 2 batches of cupcakes required.

<u>Activity 3</u>

Jenny needs to show her mum why it is or is not more cost effective to make the cupcakes herself. Present your findings in a way that Jenny's mum can easily understand and be convinced of your decision.

Answers will vary.
It's more cost effective to make the cupcakes than to buy them.

**STUDENT COPY**

**MAKE OR BUY?**

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Activity 1

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- 1/4 cup cocoa
- 3/4 cup caster sugar
- 1 egg lightly beaten
- 1 1/3 cups milk
- 3/4 cup white chocolate chopped
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| Self-raising flour | \$0.80 | 4 cups | | 2 cups | |
| Cocoa | \$4.20 | 1.5 cups | | ¼ cup | |
| Sugar | \$0.80 | 4 cups | | ¾ cup | |
| Eggs | \$3.60 | 6 eggs | | 1 egg | |
| Milk | \$1.20 | 4 cups | | $1^{1}/_{3}$ cups | |
| White chocolate | \$3.00 | 1.5 cups | | ¾ cup | |
| Milk chocolate | \$3.00 | 1.5 cups | | ¾ cup | |

1. Fill in the table above with the cost of 1 cup.
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3. How much will it cost to make a batch of 12 cupcakes?
4. How much will it cost to make 24 cupcakes?
5. How much cheaper is it to either buy or make?

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7. What is this as a percentage of making the cupcakes?

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Department of Education

# Year 7 Mathematics

## Number & Algebra Activity

# Coordinate Battleships

Produced by a Department of Education - MAWA partnership project
Written for the Year 7 Australian Curriculum

# TASK 120: COORDINATE BATTLESHIPS

### Overview

In this task, students will play against another student and attempt to ‘sink’ all of their opponent’s battleships.

- Students should cut out their battle ships and stick them onto the ‘my battleships worksheet’. They must ensure that each point covers a coordinate.
- The first student will call out a coordinate on their opponent’s battleship worksheet. If they ‘hit’ a battleship, they should record it on their copy of the opponent’s battleship worksheet. They will continue to call out coordinates until they have sunk their opponent’s ship.
- The second student then has a turn. If they do not hit a battleship, this is the end of their turn.
- The students will continue to play until all battleships of one student have been sunk.
- The student with the remaining battleships is the winner.

There are two versions of this game, using one quadrant and using four quadrants. Choose the appropriate worksheets for your class or even differentiate within the class group.

### Students will need

- my battleships worksheet
- opponent’s battleships worksheet
- scissors

### Relevant content descriptions from the Western Australian Curriculum

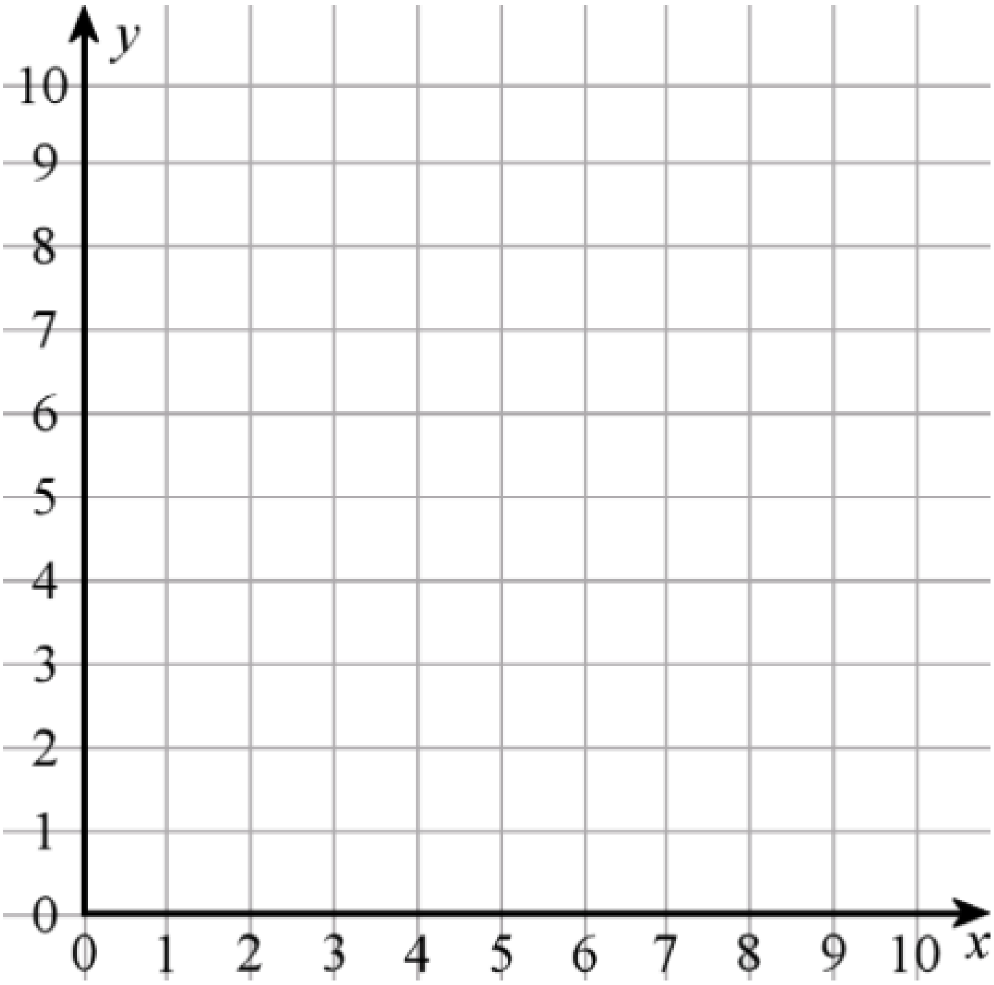
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

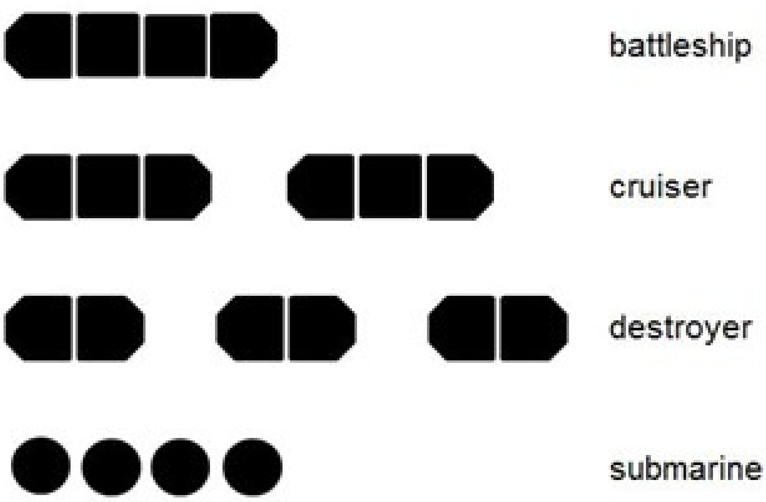
### Students can demonstrate

- *fluency* when they
  - place battleships correctly
  - identify and plot coordinates
- *understanding* when they
  - determine whether or not their opponent has sunk their battleship
- *problem solving* when they
  - strategically sink their opponent’s battleship

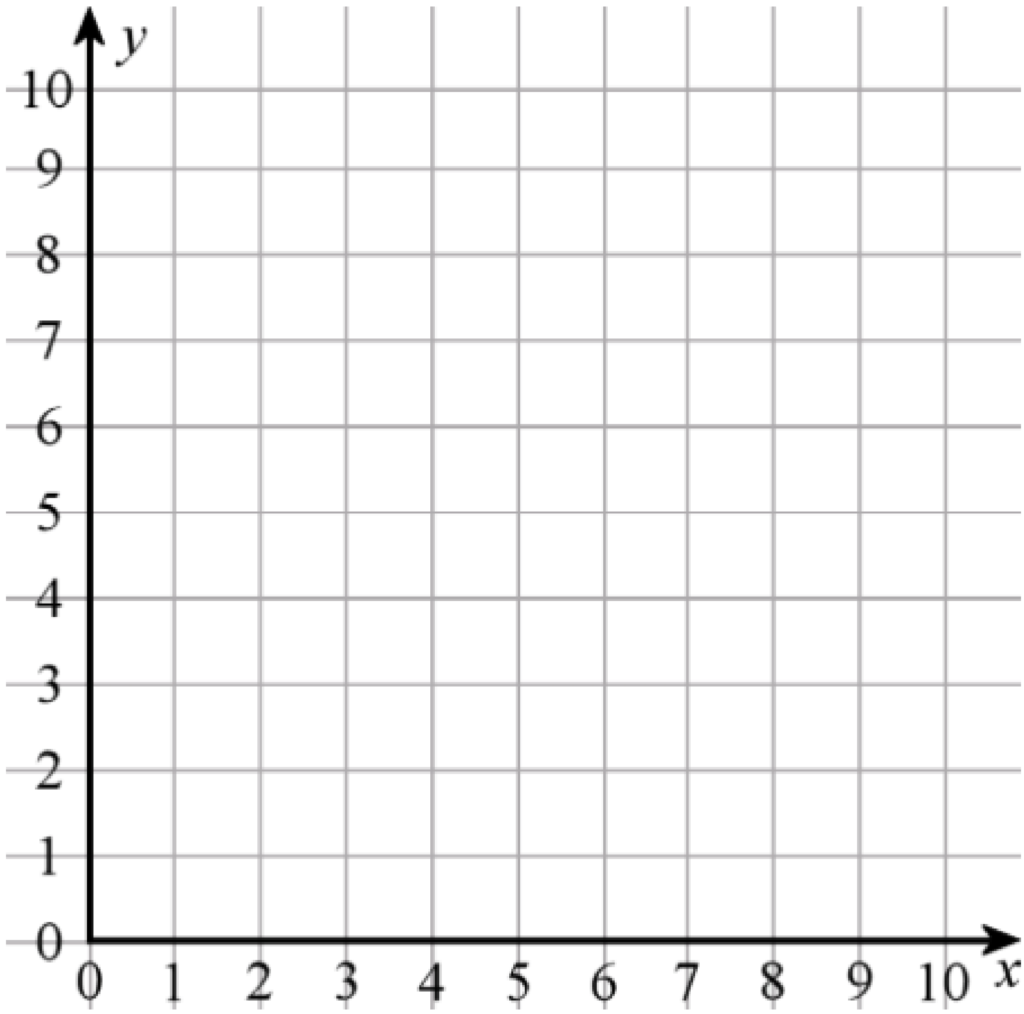
There are two versions of this game – using one quadrant, and using four quadrants. Choose the appropriate worksheets for your class or even differentiate within the class group. Direct students through this activity as appropriate.

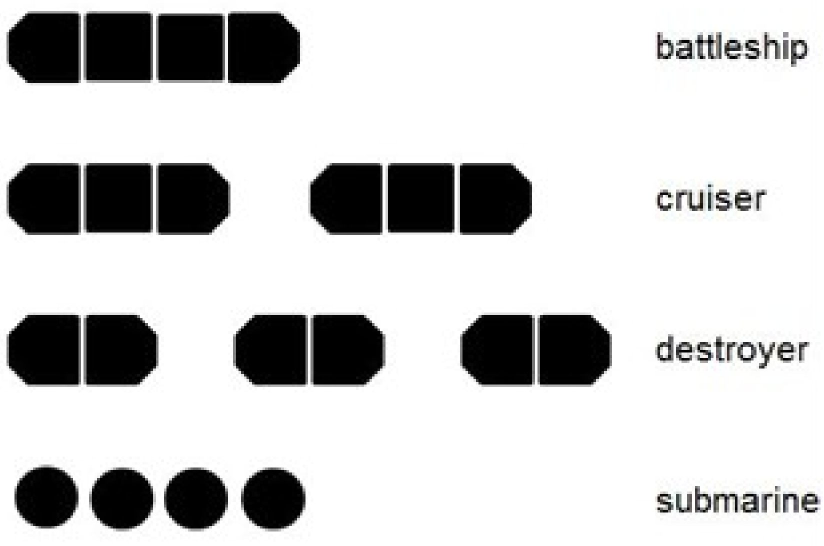
My Battleships



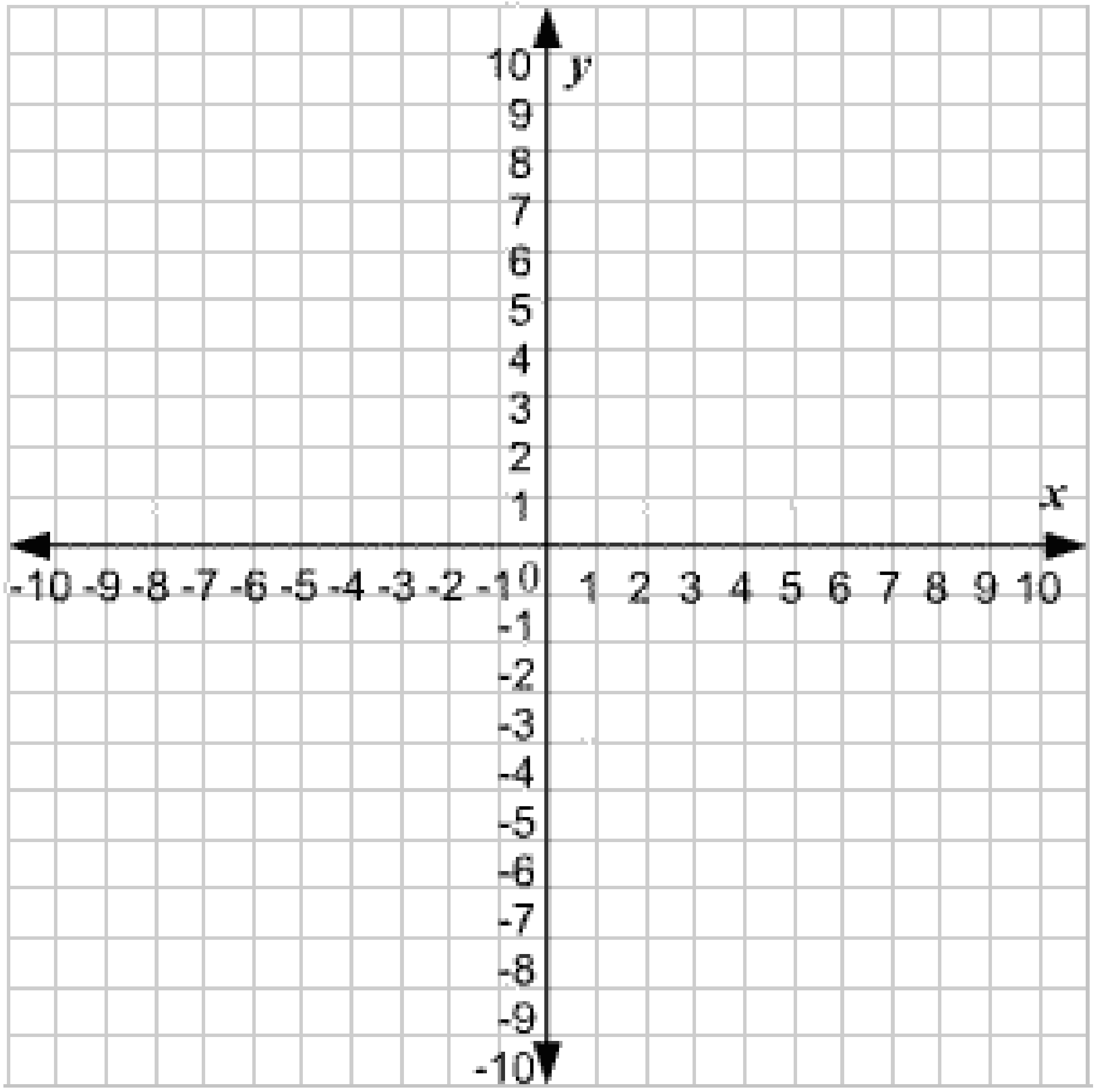



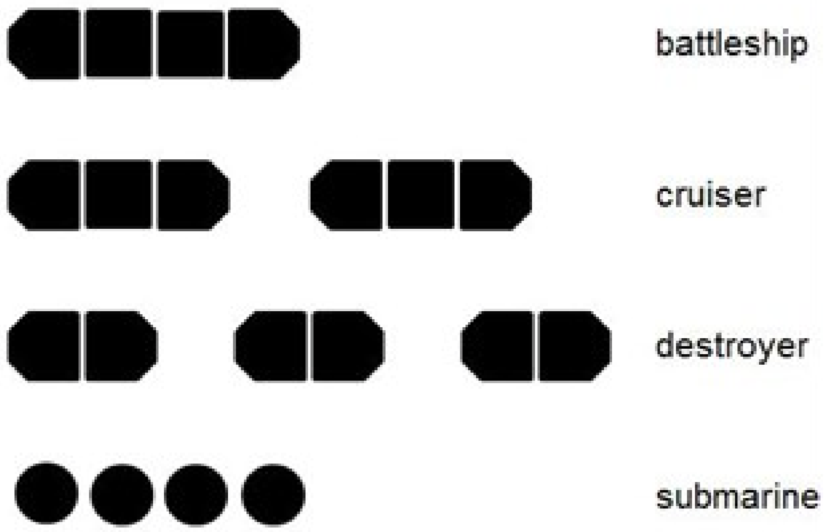
Opponent's Battleships



My Battleships



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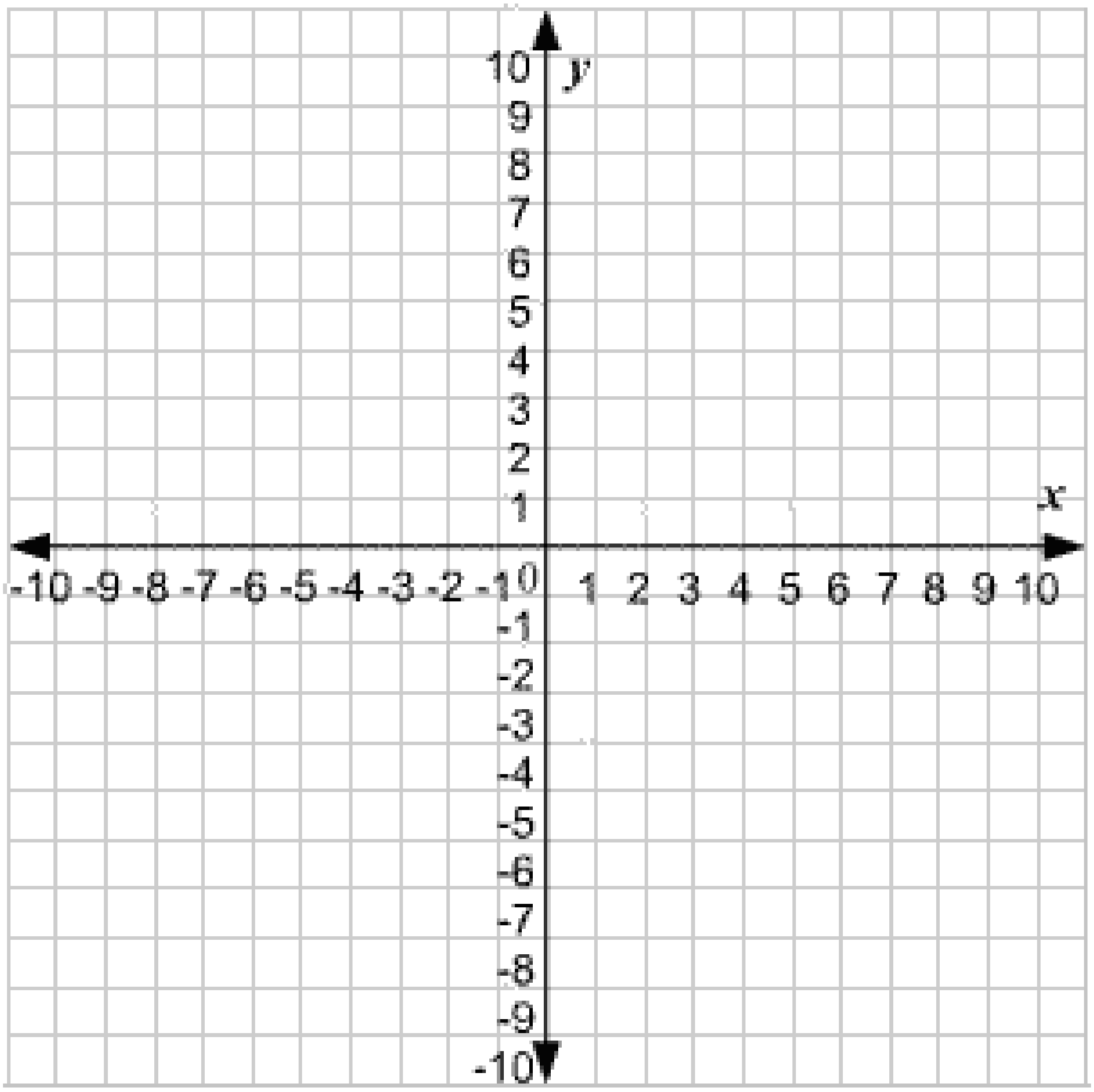
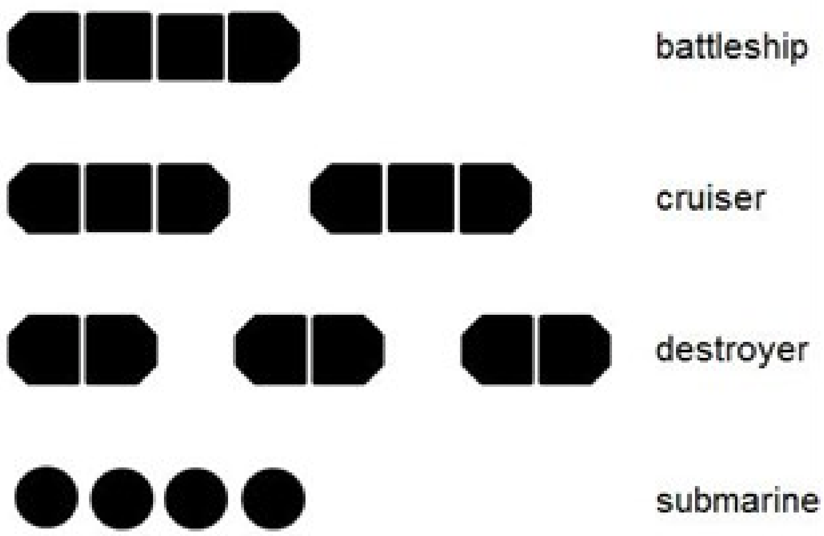
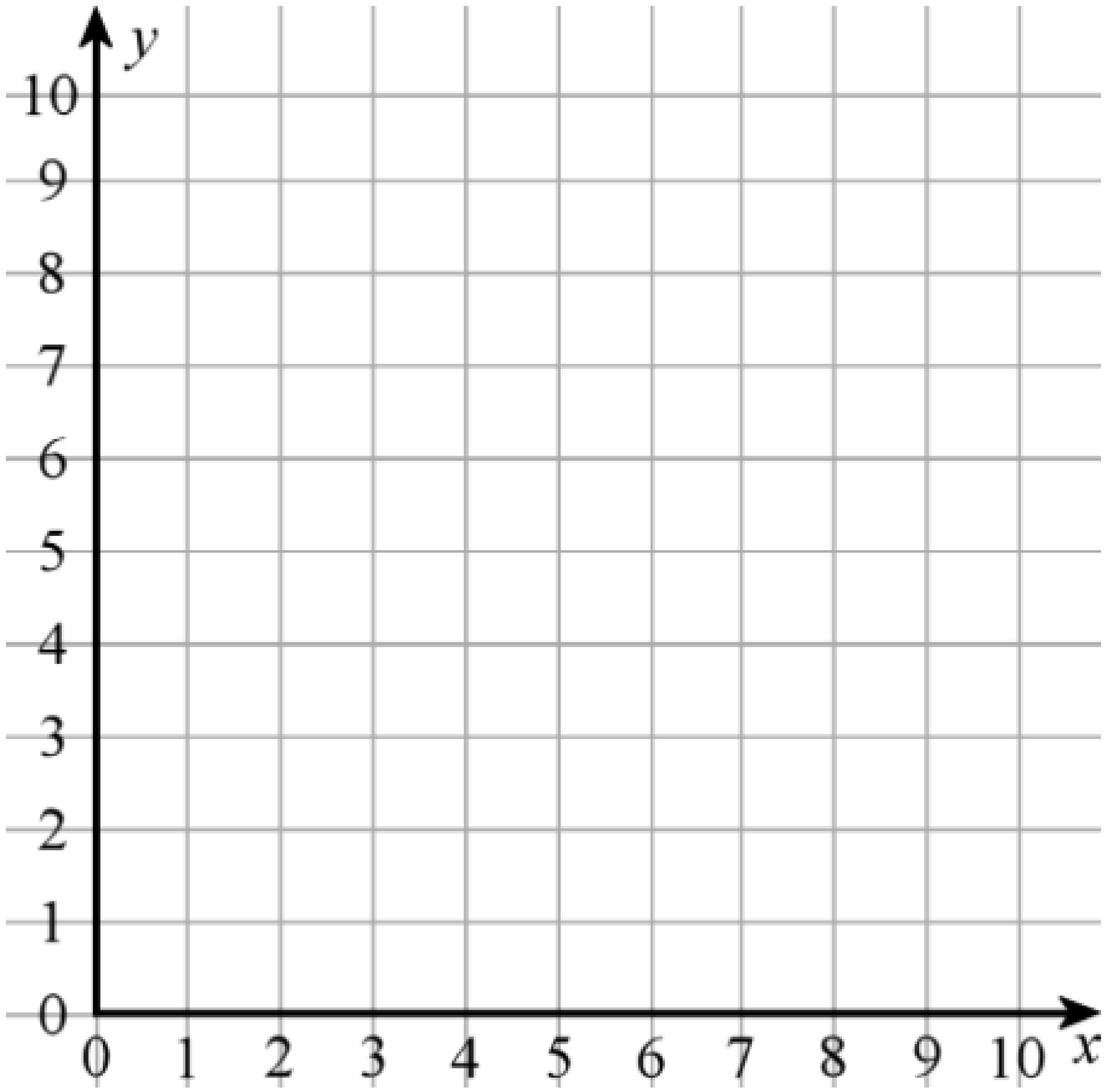
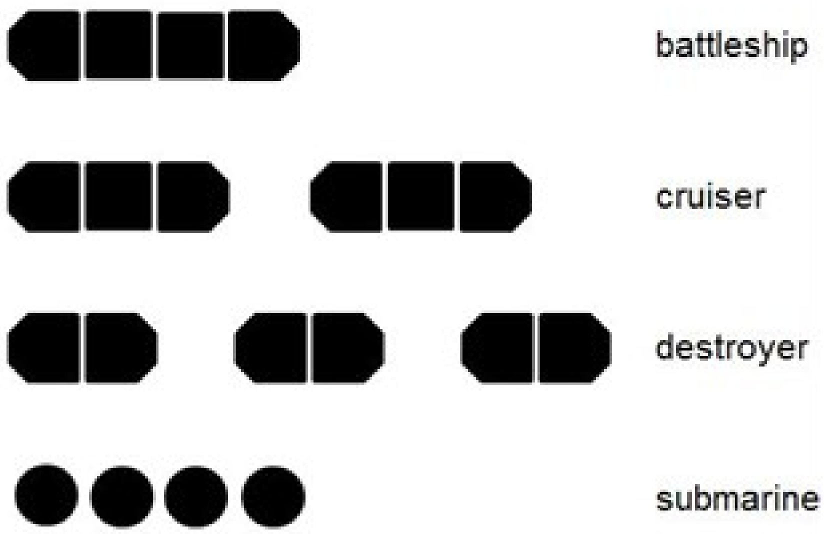


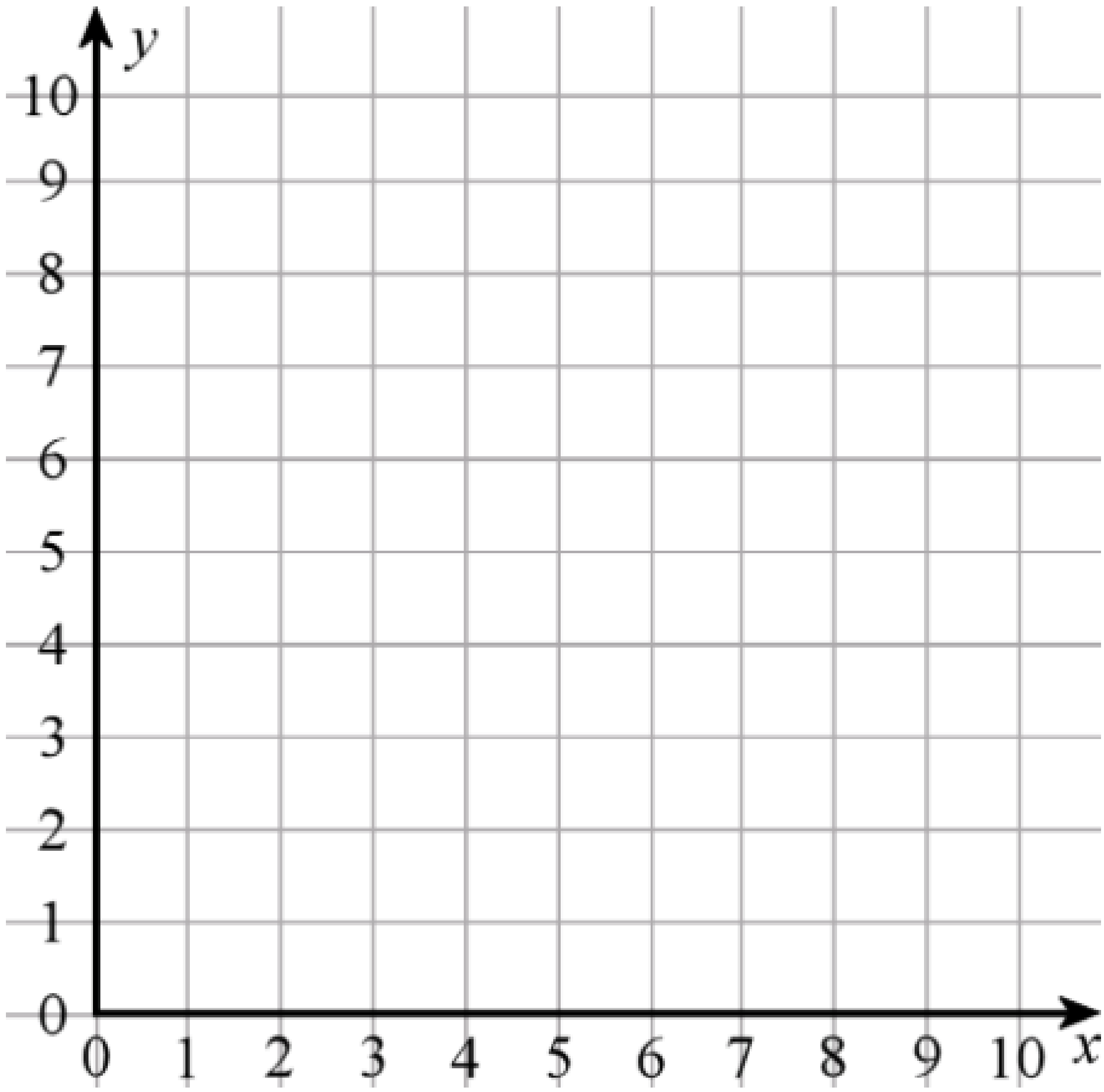



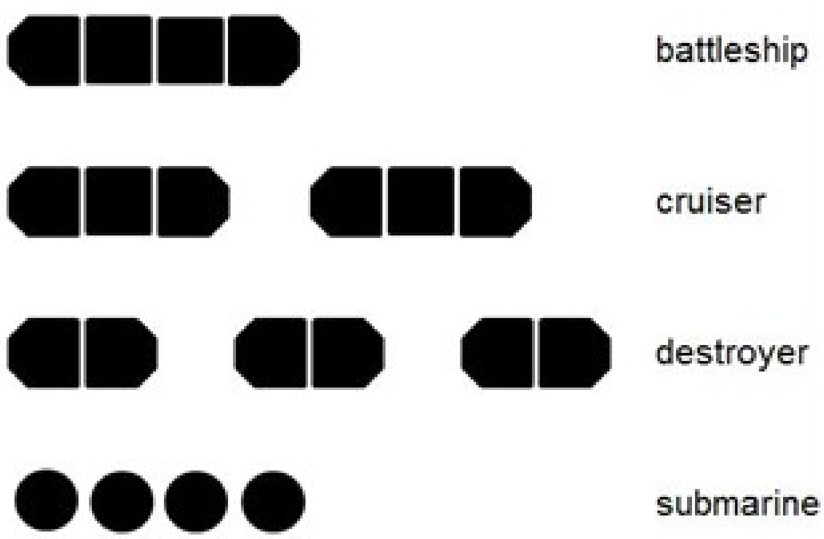
My Battleships



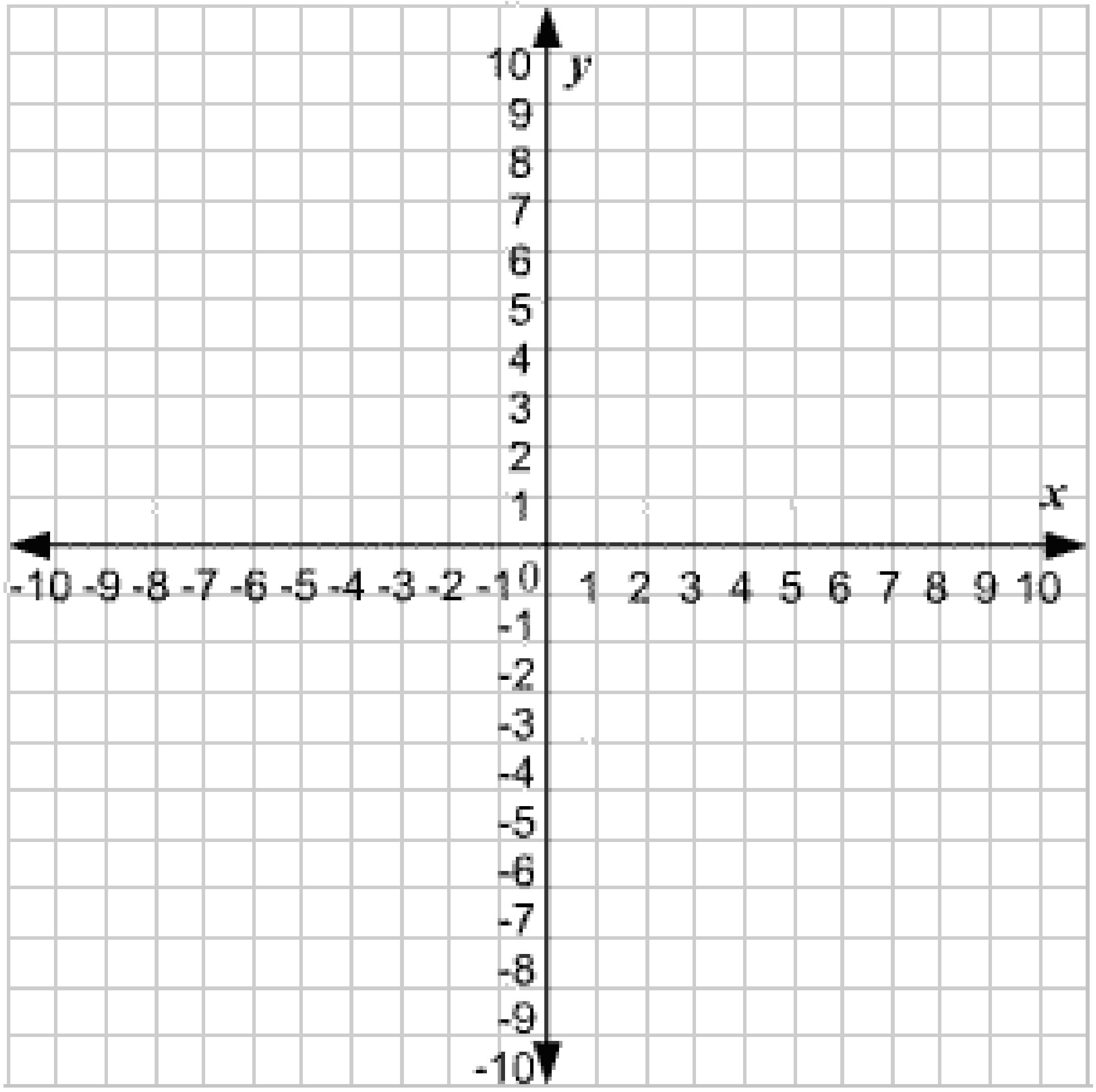



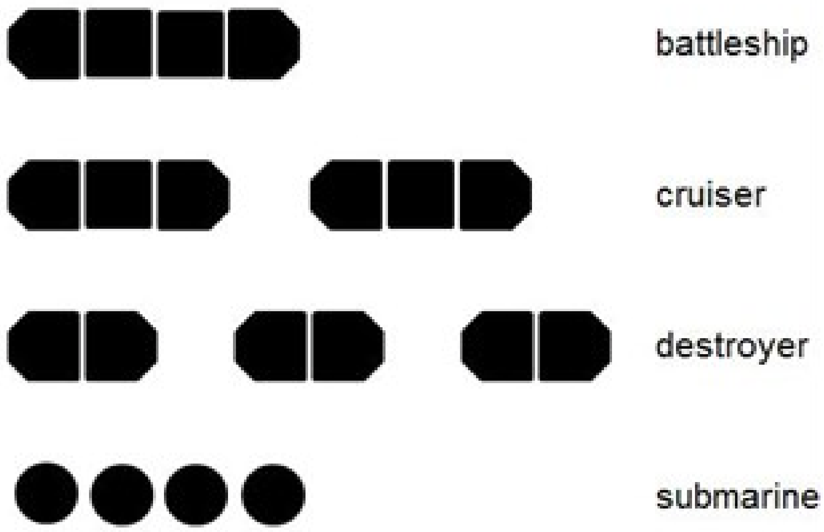
Opponent's Battleships



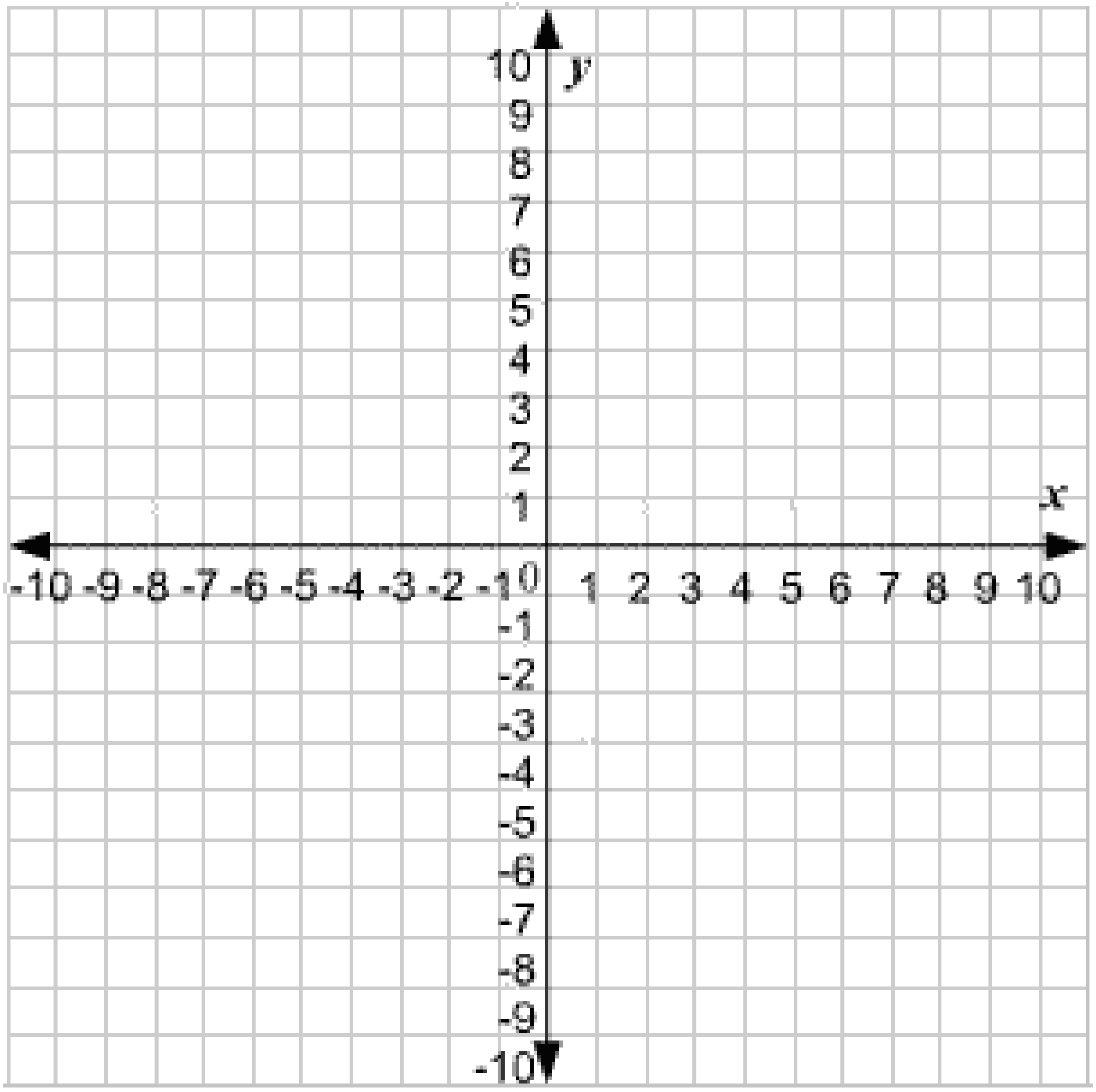



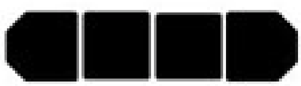
My Battleships

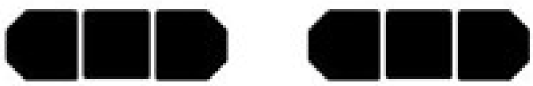


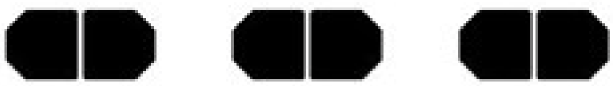



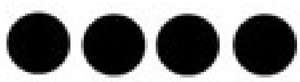
<u>Opponent's Battleships</u>





battleship


cruiser


destroyer


submarine

Department of Education

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# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Travel Match

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 121: TRAVEL MATCH

## Overview

In this task, students are asked to interpret a travel graph. They will need to use these skills to make choices and investigate problem situations. After matching the cards they should verify their answers and ensure they are reasonable. Students will reason mathematically to explain their choices.

## Students will need

- graph card set
- table card set
- description card set
- scissors

## Relevant content descriptions from the Western Australian Curriculum

- Investigate, interpret and analyse graphs from authentic data (ACMNA180)
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

## Students can demonstrate

- *fluency* when they
  - read information from a travel graph
- *understanding* when they
  - describe, using a story, the possible events on a travel graph
- *reasoning* when they
  - explain, using mathematics, why they have matched the cards
- *problem solving* when they
  - match the card sets of graphs, tables and descriptions

Activity 1

Sam's car has broken down, which means his only form of transport is walking. Below is a graph showing Sam's activities for Saturday morning.
Use the graph to answer the questions that follow.


1. If Sam left home at 8 a.m., what time did he return home?
   1.30 p.m.

2. What do the horizontal sections of the graph indicate?

   Sam has stopped. Time is passing but no distance is being travelled.

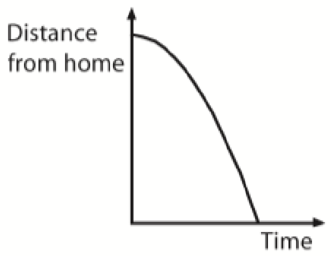
3. Use the graph to help you write a description of what may have happened on Sam's Saturday morning outing. Include information regarding how fast/slow he was walking.

Various stories will suffice, but the key elements of movement, speed and time need to match the graph. An example is given below.

Sam left home at 8 a.m. travelling at a steady speed of 2 km/hr. He walked 2 km in the first hour. He arrived at the shops at 9 a.m. where her spent 30 minutes getting some shopping. He then travelled another 2 km over the next hour to his sister's house. He stayed 1 hour at his sister's having tea and cake and catching up. He left his sister's house at 11.30 a.m. and started the long walk home. He walked at a steady speed of 2 km/h and arrived home at 1.30 p.m. He was very tired after his long morning walk.

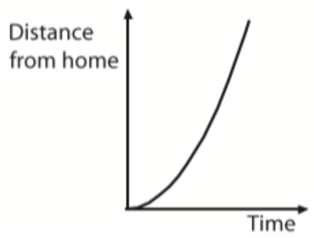
## Activity 2: Card Match

There are 3 sets of cards, each containing 10 separate cards. There is a set of travel graphs, a set of table of values, and a set of descriptions. Match the cards so you have 10 sets; each containing one graph, one table of values, and one description.



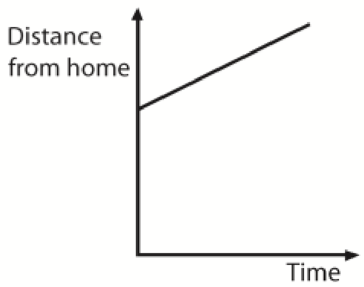

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 150 | 140 | 100 | 40 | 0 | 0 |

Travels towards home slowly then more quickly.



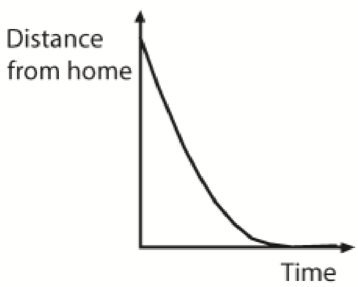

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 0 | 5 | 20 | 40 | 100 | 150 |

Travels away from home slowly then more quickly.



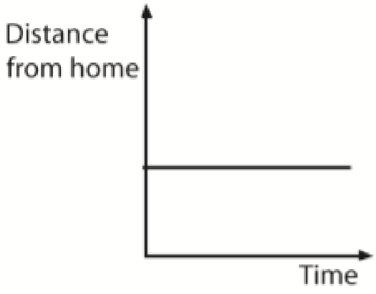

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 70 | 80 | 90 | 100 | 110 | 120 |

Is away from home and slowly travels further away.



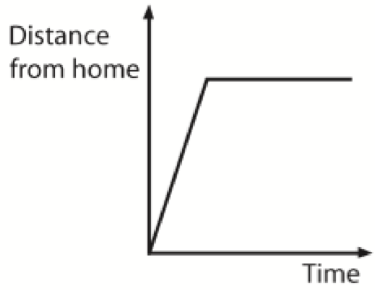

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 150 | 100 | 40 | 20 | 0 | 0 |

Travels towards home quickly then more slowly.



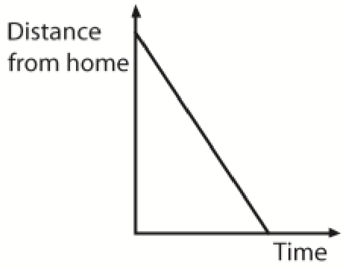

| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | 40 | 40 | 40 | 40 | 40 | 40 |

Is away from home but not moving.



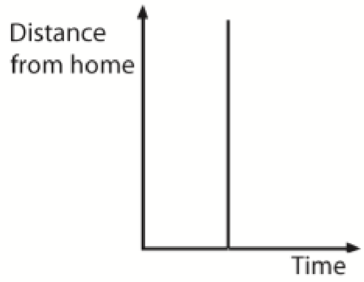

| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | 0 | 40 | 80 | 80 | 80 | 80 |

Travelling quickly away from home then stops suddenly.



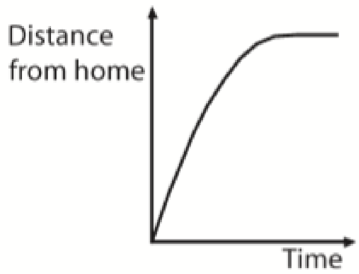

| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | 100 | 80 | 60 | 40 | 20 | 0 |

Travels home at a steady pace.



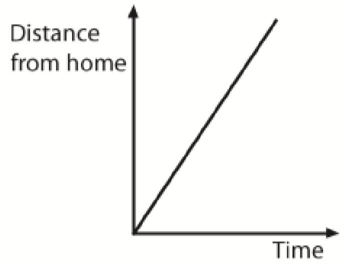

| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | | | | | | |

Impossible journey.




| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | 0 | 30 | 80 | 100 | 120 | 120 |

Travels away from home quickly then slowly stops.




| | | | | | | |
|---|---|---|---|---|---|---|
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
| Distance | 0 | 20 | 40 | 60 | 80 | 100 |

Travels away from home at a steady pace.

Activity 3:

Explain how and why you have matched each of the ten sets.
Answers will vary.

Activity 1

Sam's car has broken down, which means his only form of transport is walking. Below is a graph showing Sam's activities for Saturday morning.
Use the graph to answer the questions that follow.


1. If Sam left home at 8 a.m., what time did he return home?

2. What do the horizontal sections of the graph indicate?

3. Use the graph to help you write a description of what may have happened on Sam's Saturday morning outing. Include information regarding how fast/slow he was walking.

Activity 2: Card Match

There are 3 sets of cards, each set containing 10 separate cards. There is a set of travel graphs, a set of table of values, and a set of descriptions. Match the cards so you have 10 sets; each containing one graph, one table of values, and one description.

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 40 | 40 | 40 | 40 | 40 | 40 |

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 0 | 30 | 80 | 100 | 120 | 120 |

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 0 | 5 | 20 | 40 | 100 | 150 |

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 150 | 140 | 100 | 40 | 0 | 0 |

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 0 | 40 | 80 | 80 | 80 | 80 |

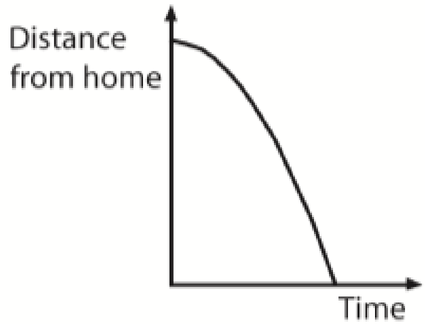
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | | | | | | |

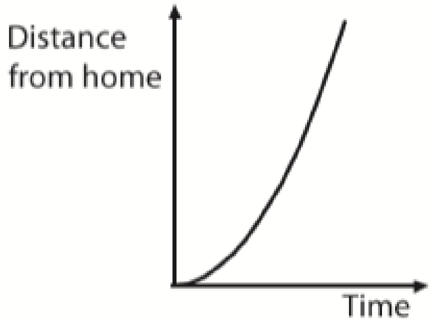
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 150 | 100 | 40 | 20 | 0 | 0 |

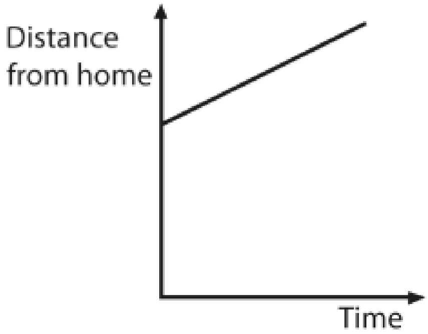
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 100 | 80 | 60 | 40 | 20 | 0 |

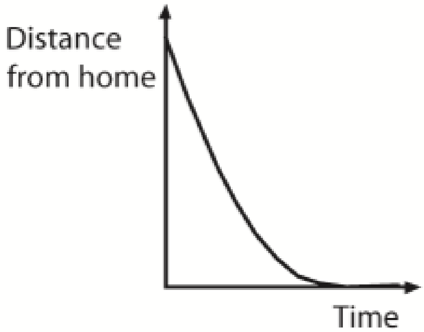
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 0 | 20 | 40 | 60 | 80 | 100 |

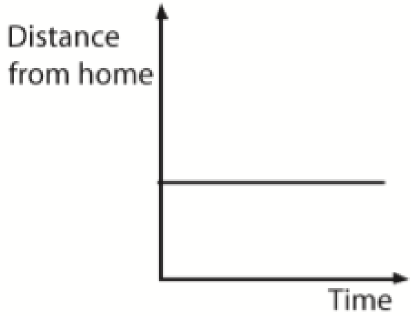
| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance | 70 | 80 | 90 | 100 | 110 | 120 |

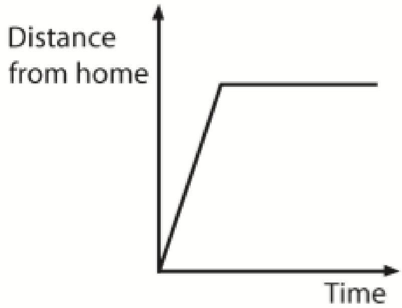

Distance
from home
Time

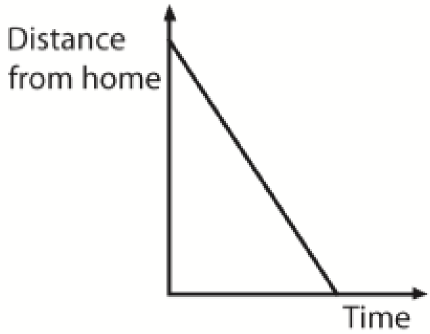

Distance
from home
Time

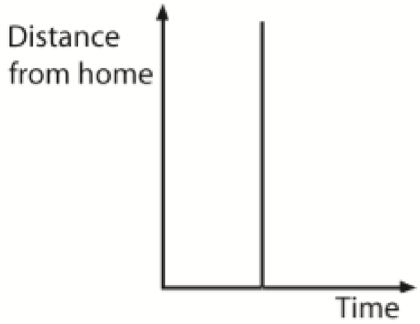

Distance
from home
Time

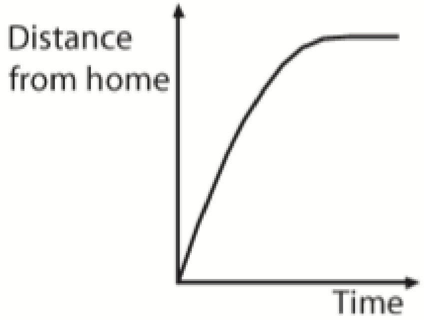

Distance
from home
Time

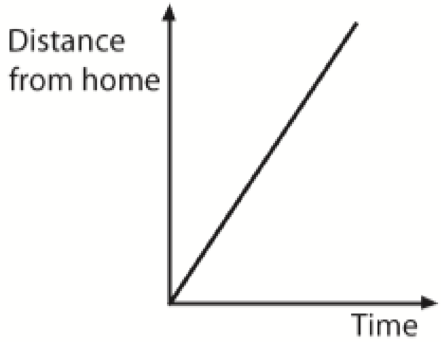

Distance
from home
Time


Distance
from home
Time


Distance
from home
Time


Distance
from home
Time


Distance
from home
Time


Distance
from home
Time

| | |
|---|---|
| Travels away from home quickly then slowly stops | Travels away from home slowly then more quickly |
| Impossible journey | Is away from home but not moving |
| Travels towards home quickly then more slowly | Travels away from home at a steady pace |
| Travels towards home slowly then more quickly | Travelling quickly away from home then stops then suddenly |
| Travels home at a steady pace | Is away from home and slowly travels further away |

Activity 3:

Explain how and why you have matched each of the ten sets.

Department of Education

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# Year 7 Mathematics

## Number & Algebra Activity

# Missing Button

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 123: MISSING BUTTON

## Overview

In this task, students will represent concepts in different ways to find solutions. They are required to make choices, formulate and investigate alternative procedures and communicate their solutions effectively.

## Students will need

- Calculator

## Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)
- Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Solve simple linear equations (ACMNA179)

## Students can demonstrate

- *fluency* when they
  - calculate correct solutions
- *understanding* when they
  - develop alternative ways to perform the calculations
- *reasoning* when they
  - explain why their alternative calculations are correct
- *problem solving* when they
  - investigate possible problems for the given solution

Activity 1

Liam has an important Mathematics test this morning and has come to school prepared, or so he thinks. When he retrieves his calculator from his bag, the number 6 button is missing. He doesn't have time to get a replacement. Explain how Liam could solve the following problems using his broken calculator.

Answers may vary. Below are some possible solutions.

| | |
|---|---|
| 1. 28846 + 4287<br><br>(14423 x 2) + 4287 = 33 133 | 5. 6.39 x 217.4<br><br>(2.13 x 3) x 217.4 = 1389.186 |
| 2. 853 – 356<br><br>853 – 178 – 178 = 497 | 6. $45^6$<br><br>45 x 45 x 45 x 45 x 45 x 45 = 8 303 765 625 |
| 3. 98 x 56<br><br>98 x (28 x 2) = 5488 | 7. 1/6 x 312<br><br>(312 x 1/3) ÷ 2 = 52 |
| 4. 20% of 640<br><br>40% of 320 = 128 | 8. $4^5 + 7^6$<br><br>$4^5 + (7^3)^2$ = 118 673 |

Activity 2

On the next set of problems, Liam is required to perform a multiplication and then a division. He is not paying attention to his work and ends up with a solution of 694 on his calculator display. Remember, he does not have the use of the number 6 button.

1. What could the problem have been?

   Answers will vary. Some possible solutions:
   347 x 4 ÷ 2 = 694
   1041 x 2 ÷ 3 = 694
   347 x 8 ÷ 4 = 694

2. How many possible problems can you find?

   Answers will vary

Activity 1

Liam has an important Mathematics test this morning and has come to school prepared, or so he thinks. When he retrieves his calculator from his bag, the number 6 button is missing. He doesn't have time to get a replacement. Explain how Liam could solve the following problems using his broken calculator.

| | |
|---|---|
| 5. 28846 + 4287 | 5. 6.39 x 217.4 |
| 6. 853 – 356 | 6. $45^6$ |
| 7. 98 x 56 | 7. 1/6 x 312 |
| 8. 20% of 640 | 8. $4^5 + 7^6$ |

Activity 2

On the next set of problems, Liam is required to perform a multiplication and then a division. He is not paying attention to his work and ends up with a solution of 694 on his calculator display. Remember, he does not have the use of the number 6 button.

3. What could the problem have been?

4. How many possible problems can you find?

Department of Education

The Mathematical Association of Western Australia Inc.

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Operation 100

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 124:  OPERATION 100

## Overview

In this task, students are required to use the digits from 1 to 9 and the operations of addition, subtraction, multiplication, and division to create an expression with a solution of 100. They should design and plan their approach and apply existing strategies to seek solutions. Students should verify that their answers are reasonable.

## Students will need

- Calculator

## Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)
- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Solve simple linear equations (ACMNA179)

## Students can demonstrate

- *fluency* when they
    - calculate correct solutions
- *understanding* when they
    - use the digits to calculate in various ways
- *reasoning* when they
    - verify their solutions
- *problem solving* when they
    - attempt to find multiple solutions

1. Using all of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and any combination of the operations +, -, x, ÷, create an expression with a solution of 100.

Answers will vary. Some examples:

$(1+2+3+4)\times5+67-(8+9)=100$

$(1+2+3-4)+5+6+78+9=100$

$1+2+3+4+5+6+7+(8\times9)=100$

$(1+9)(2+8)((7-3)\div4)(6-5)=100$

$((1+2+3+4)\times(5+6))+7-8-9=100$

2. How many possible expressions can you find?

Answers will vary.

3. Decide on another target number, then apply the rules as above to create this number.

Answers will vary.

4. Using all of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 and any combination of the operations +, -, x, ÷, create an expression with a solution of 100.

5. Create as many possible expressions can you find?

6. Decide on another target number, then apply the rules as above to create this number.

Department of Education

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Fraction Help

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT
WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 125: FRACTION HELP

## Overview

In this task, students are required to develop a strategy to order both positive and negative fractions, and mixed numbers. They will be building their understanding of the relationship between related ideas when they represent the concept in different ways. They will apply existing strategies to seek solutions and explain their thinking.

## Students will need

- Calculator

## Relevant content descriptions from the Western Australian Curriculum

- Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line (ACMNA152)
- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

## Students can demonstrate

- *fluency* when they
  - correctly order positive and negative fractions and mixed numbers.
- *understanding* when they
  - present their information in a logical manner
- *reasoning* when they
  - reflect on their work to improve it
- *problem solving* when they
  - explain their solutions to aid understanding
  - find alternative methods to help students understand the concept
  - extend their method to include fractions, decimals and percentages

A student in your class is struggling with ordering sets of numbers that include fractions. Since you have easily mastered this skill the teacher has asked that you help. You decide to draw up number lines and use diagrams to explain the concept.

Talk through the problems with the students to clarify any uncertainties. The direction and assistance given to students will impact on the level of independent learning and solutions achieved.

Activity 1

You decide to first draw up a number line with diagrams for numbers between 0 and 1.

Before beginning the task, consider the following:

- What information needs to be included?
- How many fractions will you need to include?
- Will you include fractions of different denominators?
- What assumptions do you need to make?

You may want to use these points as a class discussion and set guidelines before they begin. Fractions should be placed in ascending order.

Draw up your number line diagram and include an explanation to help the students in your class understand the ordering of sets of numbers that include fractions.

Answers will vary. Students may need assistance with scale and diagram types. Ensure diagrams are drawn with equal parts.

Activity 2

1. Have you included enough information in your number line and diagram to help the students with their understanding? List the key components.

Straight line; numbers marked 0 to 1; marks at equal distances apart to show fractions of 1

2. What worked really well for you?

   Answers will vary.

3. What didn't work so well? What would you change and how would you change it?

   Answers will vary.

4. If the number line and diagrams are not enough to help other students, how else could you explain the ordering of fractions?

   Answers will vary.

Activity 3

After you have helped the above student and reviewed your process to improve it, other students are now looking to you for help and advice. In particular, students are having problems ordering mixed numbers. As your previous method worked well you decide to draw up a number line with diagrams for numbers between 0 and 3.

Again, before beginning the task, consider the following:

- What information needs to be included?
- How many fractions will you need to include?
- Will you include fractions of different denominators?
- What assumptions do you need to make?

Draw up your number line, with a detailed diagram and include an explanation to help the students in your class understand ordering sets of numbers that include mixed numbers.

Answers will vary.

Activity 4

While you are very pleased with your efforts and the help you provided for you class mates today, you are still struggling with the concept of negative fractions. You begin to ponder whether you could employ the same method as you did earlier. Investigate whether this method would be suitable to help order fractions between -2 and 2.

Answers will vary.

Activity 5: Extension

Use your method as above to explain how fractions, decimals and percentages can be ordered.

Answers will vary.

A student in your class is struggling with ordering sets of numbers that include fractions. Since you have easily mastered this skill the teacher has asked that you help. You decide to draw up number lines and use diagrams to explain the concept.

Activity 1

You decide to first draw up a number line with diagrams for numbers between 0 and 1.

Before beginning the task, consider the following:

- What information needs to be included?
- How many fractions will you need to include?
- Will you include fractions of different denominators?
- What assumptions do you need to make?

Draw up your number line diagram and include an explanation to help the students in your class understand the ordering of sets of numbers that include fractions.

Activity 2

5. Have you included enough information in your number line and diagram to help the students with their understanding? List the key components.

6. What worked really well for you?

7. What didn't work so well? What would you change and how would you change it?

8. If the number line and diagrams are not enough to help other students, how else could you explain the ordering of fractions?

Activity 3

After you have helped the above student and reviewed your process to improve it, other students are now looking to you for help and advice. In particular, students are having problems ordering mixed numbers. As your previous method worked well you decide to draw up a number line with diagrams for numbers between 0 and 3.

Again, before beginning the task, consider the following:

- What information needs to be included?
- How many fractions will you need to include?
- Will you include fractions of different denominators?
- What assumptions do you need to make?

Draw up your number line, with a detailed diagram and include an explanation to help the students in your class understand ordering sets of numbers that include mixed numbers.

Activity 4

While you are very pleased with your efforts and the help you provided for you class mates today, you are still struggling with the concept of negative fractions. You begin to ponder whether you could employ the same method as you did earlier. Investigate whether this method would be suitable to help order fractions between -2 and 2.

Activity 5: Extension

Use your method as above to explain how fractions, decimals and percentages can be ordered.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Which Cleaner?

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 126: WHICH CLEANER?

## Overview

In this task, students are required to investigate the fees charged by three cleaning companies to determine which would be the most cost effective. Students will need to recall factual knowledge to calculate answers fluently and manipulate equations to find solutions. They are required to analyse the information, seek a solution and justify their choices.

## Students will need

- Calculator

## Relevant content descriptions from the Western Australian Curriculum

- Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)
- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)
- Investigate, interpret and analyse graphs from authentic data (ACMNA180)

## Students can demonstrate

- *fluency* when they
    - create algebraic rules to describe cleaning cost
    - use these rules to complete table of values and calculate costs
    - use table of values to plot graphs
- *understanding* when they
    - read the graphs to find solutions
- *reasoning* when they
    - choose a company that is most cost effective for Mary, based on their findings

Mary is looking for a cleaner to come to her house one day per week. She would like the most cost effective company available. She has found three local cleaning companies that have the following charges:

a) Sparkle charges a $20 call-out fee and then $20 for every hour, or part of, for cleaning.
b) Gleaming charges a $10 call-out fee and then $25 for every hour, or part of, for cleaning.
c) Clean Mites does not have a call-out fee but they charge $30 for every hour, or part of, for cleaning.

1. Write a rule for each company to describe the cost, ($c$), of cleaning a property for ($h$) hours.

a) $c = 20 + 20h$
b) $c = 10 + 25h$
c) $c = 30h$

2. Use your rules to complete the following tables.

a)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | 40 | 60 | 80 | 100 | 120 |

b)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | 35 | 60 | 85 | 110 | 135 |

c)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | 30 | 60 | 90 | 120 | 150 |

3. Using your rules, calculate how much each company charges for:

i) 1 hour of cleaning?

a) $40
b) $35
c) $30

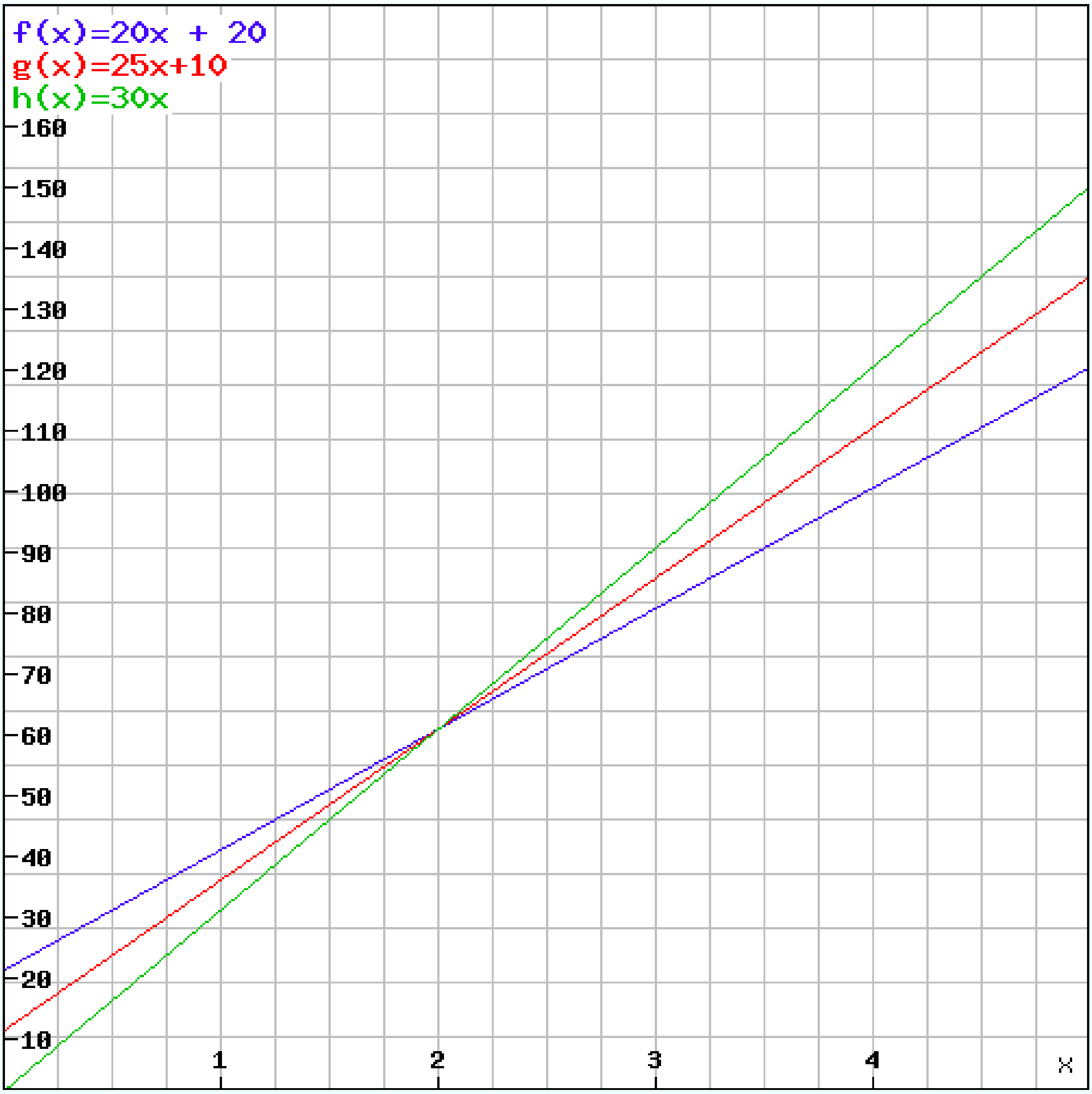
ii) 3 hours of cleaning?

a) $80
b) $85
c) $90

iii) 10 hours of cleaning?

a) $220
b) $260
c) $300

4. Use your tables to plot the cost of cleaning a property on the graph below.




5. Use your graph to answer the following questions.

(a) At which hour is the cost of all three companies the same?

2 hours

(b) Which company is cheapest for 1 hour?

Company C – Clean Mites

6. Which company is cheapest for 5 hours?

Company A – Sparkle

7. Which company would be the most cost effective for Mary if she wants 2 hours of cleaning each week? Although, some weeks she may need more.

All companies charge the same for 2 hours but if she occasionally wants more she should choose company A – Sparkle.

Mary is looking for a cleaner to come to her house one day per week. She would like the most cost effective company available. She has found three local cleaning companies that have the following charges:

d) Sparkle charges a $20 call-out fee and then $20 for every hour, or part of, for cleaning.
e) Gleaming charges a $10 call-out fee and then $25 for every hour, or part of, for cleaning.
f) Clean Mites does not have a call-out fee but they charge $30 for every hour, or part of, for cleaning.

1. Write a rule for each company to describe the cost, (*C*), of cleaning a property for (*h*) hours.

2. Use your rules to complete the following tables.

a)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | | | | | |

b)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | | | | | |

c)

| Hours of Cleaning | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cleaning Cost | | | | | |

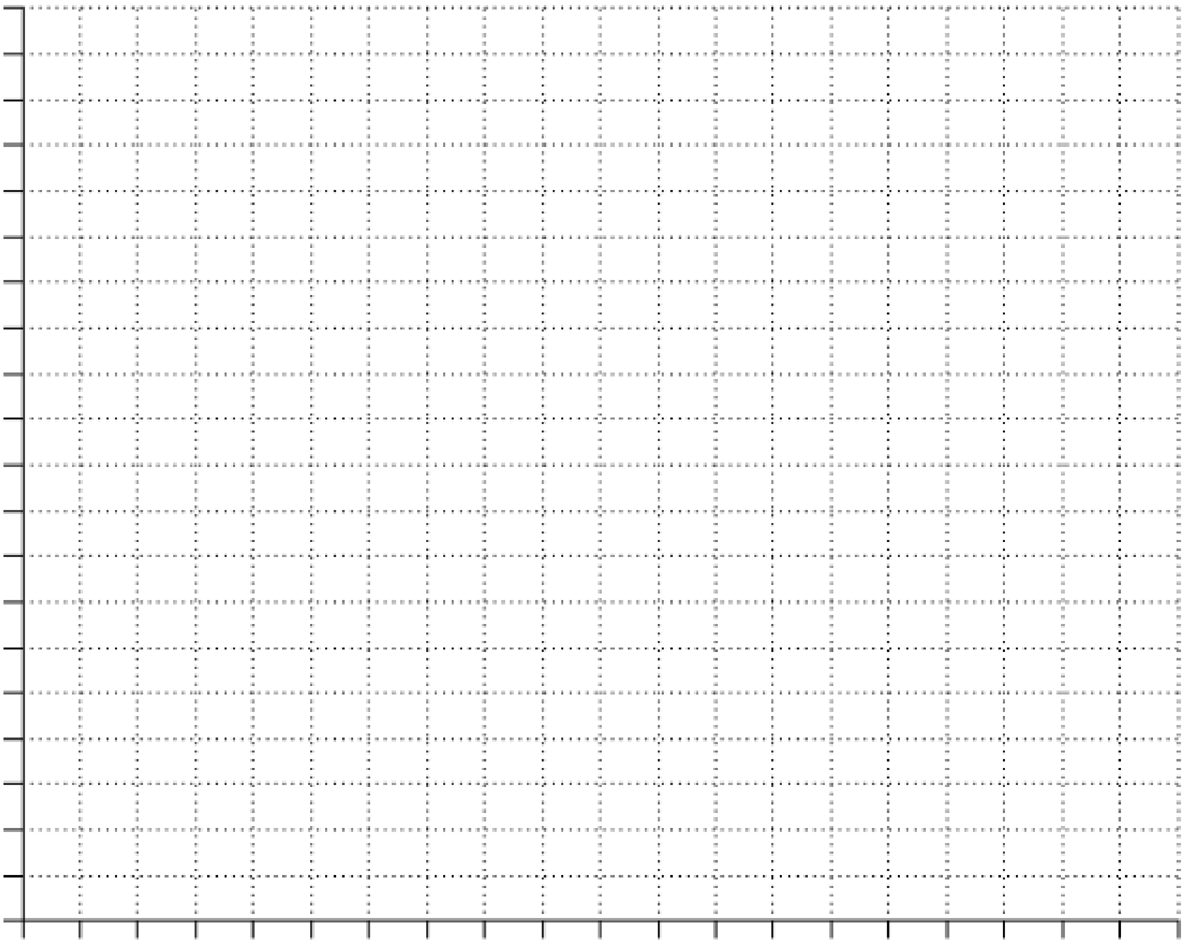
3. Using your rules, calculate how much each company charges for:

i) 1 hour of cleaning?

ii) 3 hours of cleaning?

iii) 10 hours of cleaning?

4. Use your tables to plot the cost of cleaning a property on the graph below.



5. Use your graph to answer the following questions.

(a) At which hour is the cost of all three companies the same?

(b) Which company is cheapest for 1 hour?

(c) Which company is cheapest for 5 hours?

(d) Which company would be the most cost effective for Mary if she wants 2 hours of cleaning each week? Although, some weeks she may need more.

Department of Education

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# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Negative Square

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 130: NEGATIVE SQUARE

## Overview

In this task, students will investigate square numbers and their square roots, taking negative numbers into consideration. They are required to represent their answers in various ways and explain why they may not be able to do this all of the time. They will be required to use strategies and work backwards to find all possible solutions. They will need to develop strategies to seek solutions and reflect on the efficiency of those strategies.

## Students will need

- calculator

## Relevant content descriptions from the Western Australian Curriculum

- Investigate and use square roots of perfect square numbers (ACMNA150)

## Students can demonstrate

- *fluency* when they
  - find the first 10 square numbers
  - find the positive square root of the first 10 square numbers
- *understanding* when they
  - represent the first 10 square numbers in a diagram
  - find the first 10 square numbers using negatives
  - find the positive and negative square root of the first 10 square numbers
- *reasoning* when they
  - explain why they cannot represent the negatives in a diagram
  - reflect on their method for calculating a square root without the use of a calculator and explain how to improve it
- *problem solving* when they
  - devise a method to calculate the square root without the use of a calculator

Activity 1

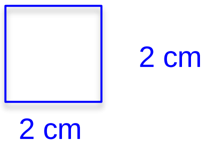
When we multiply a number by itself, we say that we are squaring the number. The result is then called a square number. For example, 2 x 2 = 4, here we are squaring the number 2, the solution 4, is a square number.

1. Write the first 10 square numbers.

   1, 4, 9, 16, 25, 36, 49, 64, 81, 100

2. Represent these square numbers in diagrams.

   Students may be unsure how to represent in a diagram, so encourage them to draw a square with side values of the number and area of the square number. For example:




Side length 2 cm
Area 4 $cm^2$

Activity 2

We know from our work with integers that if we multiply two negative numbers the result is a positive number. This suggests that that squaring a negative number will give the same solution as its positive counterpart, for example, -2 x -2 = 4. In this case we are squaring -2 and 4 is the square number.

1. Find the first 10 square numbers using negatives.

   1, 4, 9, 16, 25, 36, 49, 64, 81, 100

2. Can we represent these square numbers as diagrams like those in Activity1? Why?

   We cannot represent these using squares as above as we cannot have a negative side length.

Activity 3

The reverse of this process is ‘finding the square root’. For example, the square root of 4 is 2. This is because the number we multiplied by itself to get an answer of 4, is 2. Thus squaring and square root are inverse operations in the same way that multiplication is the inverse of division. One operation undoes the other.

We usually calculate the square root by using a special button on our calculator. The button is √**,** which is meant to represent the letter ‘r’ for ‘root’.

Considering what we have seen in Activities 1 and 2, find all the possible square roots of the first 10 square numbers.

$\sqrt{1} = \pm 1$
$\sqrt{4} = \pm 2$
$\sqrt{9} = \pm 3$
$\sqrt{16} = \pm 4$
$\sqrt{25} = \pm 5$
$\sqrt{36} = \pm 6$
$\sqrt{49} = \pm 7$
$\sqrt{64} = \pm 8$
$\sqrt{81} = \pm 9$
$\sqrt{100} = \pm 10$

Activity 4

You need to find the square root of 841 and 1369 but don't have access to a calculator with the square root button. How might you find the solutions?

- Devise a method to find the solutions.
- State the solutions.
- Explain if your method was efficient or not.

$\sqrt{841} = \pm 29$
$\sqrt{1369} = \pm 37$

Answers will vary.

Activity 1

When we multiply a number by itself, we say that we are squaring the number. The result is then called a square number. For example, 2 x 2 = 4, here we are squaring the number 2, the solution 4, is a square number.

1. Write the first 10 square numbers.

2. Represent these square numbers in diagrams.

Activity 2

We know from our work with integers that if we multiply two negative numbers the result is a positive number. This suggests that that squaring a negative number will give the same solution as its positive counterpart, for example, -2 x -2 = 4. In this case we are squaring -2 and 4 is the square number.

1. Find the first 10 square numbers using negatives.

2. Can we represent these square numbers as diagrams like those in Activity1? Why?

Activity 3

The reverse of this process is ‘finding the square root’. For example, the square root of 4 is 2. This is because the number we multiplied by itself to get an answer of 4, is 2. Thus squaring and square root are inverse operations in the same way that multiplication is the inverse of division. One operation undoes the other.

We usually calculate the square root by using a special button on our calculator. The button is √**,** which is meant to represent the letter ‘r’ for ‘root’.

Considering what we have seen in Activities 1 and 2, find all the possible square roots of the first 10 square numbers.

Activity 4

You need to find the square root of 841 and 1369 but don't have access to a calculator with the square root button. How might you find the solutions?

- Devise a method to find the solutions.
- State the solutions.
- Explain if your method was efficient or not.

Department of Education

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Which is More?

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 133: WHICH IS MORE?

## Overview

In this task, students will use their knowledge and understanding of percentages and rounding to identify which pocket money plan is most beneficial for the student and which pocket money plan is most beneficial for the parent.

## Students will need

- calculator

## Relevant content descriptions from the Western Australian Curriculum

- Round decimals to a specified number of decimal places (ACMNA156)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. (ACMNA158)
- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

## Students can demonstrate

- *fluency* when they
  - calculate 50% and 25% of amounts
  - calculate simple multiplication
  - round up and down to the nearest dollar
- *understanding* when
  - adding the original amount with the percentage of that amount
  - showing mathematically which plan is better for Mark and which plan is better for his parents
  - identifying Mark's possible loss
- *reasoning* when they
  - outline which option Mark should choose and which option Mark's parents should choose
- *problem solving* when they
  - identify which option will give Mark the most savings when he finishes Year 12

Mark is in negotiations with his parents regarding pocket money. To date, he has not received any regular pocket money but his parents have promised that once he starts Year 7 in February, they will give him a regular amount. This amount will increase by 50% each year that he remains in education. His parents are considering the following options:

1. $10 each week for the first year, then increasing by 50% each year he remains in school, and rounding up to the nearest dollar.
2. $20 each week for the first year, then increasing by 25% each year he remains in school, and rounding down to the nearest dollar.
3. The school year he is in, multiplied by three, and rounding down to the nearest dollar.

Activity 1

1. Assuming Mark stays in school until Year 12, which option would be best for him to earn the most money?

| Year | Option 1 | Option 2 | Option 3 |
|---|---|---|---|
| Year 7 | $10 | $20 | $21 |
| Year 8 | $15 | $25 | $24 |
| Year 9 | $23 | $31 | $27 |
| Year 10 | $35 | $38 | $30 |
| Year 11 | $53 | $47 | $33 |
| Year 12 | $80 | $58 | $36 |

| Option 1 | Weekly Amount | Yearly Amount |
|---|---|---|
| Year 7 | $10 | $520 |
| Year 8 | $15 | $780 |
| Year 9 | $23 | $1196 |
| Year 10 | $35 | $1820 |
| Year 11 | $53 | $2756 |
| Year 12 | $80 | $4160 |
| Total | | $11 232 |

| Option 2 | Weekly Amount | Yearly Amount |
|---|---|---|
| Year 7 | $20 | $1040 |
| Year 8 | $25 | $1300 |
| Year 9 | $31 | $1612 |
| Year 10 | $38 | $1976 |
| Year 11 | $47 | $2444 |
| Year 12 | $58 | $3016 |
| Total | | $11 388 |

| Option 3 | Weekly Amount | Yearly Amount |
|---|---|---|
| Year 7 | $21 | $1092 |
| Year 8 | $24 | $1248 |
| Year 9 | $27 | $1404 |
| Year 10 | $30 | $1560 |
| Year 11 | $33 | $1716 |
| Year 12 | $36 | $1872 |
| Total | | $8892 |

Option 2 is best for Mark.

2. Assuming Mark stays in school until Year 12, which option would be best for his parents to give Mark the least possible money?

   Option 3

3. If Mark chooses to finish school in Year 10, how much money will he potentially lose?

   Option 1: $6 916
   Option 2: $5 460
   Option 3: $3 588

4. If Mark does finish school in Year 10, is the previous option still the best for him? Explain.

   Yes, Option 2 is still the best for him. Below is outlined what he would earn between Year 7 and Year 10.
   Option 1: $4 316
   Option 2: $5 928
   Option 3: $5 304

5. If Mark does finish school in Year 10, is the previous option still the best for his parents? Explain.

   No, they should choose Option 1 if he will finish school in Year 10. Below is outlined what they would give him between Year 7 and Year 10.
   Option 1: $4 316
   Option 2: $5 928
   Option 3: $5 304

<u>Activity 2</u>

Mark plans to stay in school until the end of Year 12. He decides that he will save according to the following:

- If his parents give him Option 1 he will save 5% of his pocket money in the first year and then increase his saving by 5% every year after that.
- If his parents give him Option 2 he will save 4% of his pocket money in the first year and then increase his saving by 4% every year after that.
- If his parents give him Option 3 he will save 8% of his pocket money in the first year and then increase his saving by 8% every year after that.

Which option will give him the most savings when he finishes Year 12? Do not round your answers.

Encourage students to change each percentage into a decimal and multiply to find the percentage of the amount. Emphasise that per cent = hundredths. Thus, for example; 5% of $520 = 0.05 x $520 = $26.

| Option 1 | Weekly Amount | Yearly Amount | Percentage | Savings |
|---|---|---|---|---|
| Year 7 | $10 | $520 | 5% | $26 |
| Year 8 | $15 | $780 | 10% | $78 |
| Year 9 | $23 | $1196 | 15% | $179.40 |
| Year 10 | $35 | $1820 | 20% | $364 |
| Year 11 | $53 | $2756 | 25% | $689 |
| Year 12 | $80 | $4160 | 30% | $1248 |
| Total | | $11 232 | | $2584.40 |
| | | | | |
| Option 2 | Weekly Amount | Yearly Amount | Percentage | Savings |
| Year 7 | $20 | $1040 | 4% | $41.6 |
| Year 8 | $25 | $1300 | 8% | $104 |
| Year 9 | $31 | $1612 | 12% | $193.44 |
| Year 10 | $38 | $1976 | 16% | $316.16 |
| Year 11 | $47 | $2444 | 20% | $488.80 |
| Year 12 | $58 | $3016 | 24% | $723.84 |
| Total | | $ 11 388 | | $1911.40 |
| | | | | |
| Option 3 | Weekly Amount | Yearly Amount | Percentage | Savings |
| Year 7 | $21 | $1092 | 8% | $87.36 |
| Year 8 | $24 | $1248 | 16% | $199. 68 |
| Year 9 | $27 | $1404 | 24% | $249.6 |
| Year 10 | $30 | $1560 | 32% | $499.20 |
| Year 11 | $33 | $1716 | 40% | $686.40 |
| Year 12 | $36 | $1872 | 48% | $898.56 |
| Total | | $8892 | | $2620.80 |

Option 3 is best.

Activity 3

Write two short paragraphs, one outlining which option Mark should choose and why; the other outlining which option Mark's parents should choose and why.

Answers will vary.

Mark is in negotiations with his parents regarding pocket money. To date, he has not received any regular pocket money but his parents have promised that once he starts year 7 in February, they will give him a regular amount. This amount will increase by 50% each year that he remains in education. His parents are considering the following options:

1. $10 each week for the first year, then increasing by 50% each year he remains in school, and rounding up to the nearest dollar.

2. $20 each week for the first year, then increasing by 25% each year he remains in school, and rounding down to the nearest dollar.

3. The school year he is in, multiplied by three, and rounding down to the nearest dollar.

<u>Activity 1</u>

1. Assuming Mark stays in school until Year 12, which option would be best for him to earn the most money?

2. Assuming Mark stays in school until Year 12, which option would be best for his parents to give Mark the least possible money?

3. If Mark chooses to finish school in Year 10, how much money will he potentially lose?

4. If Mark does finish school in Year 10, is the previous option still the best for him? Explain.

5. If Mark does finish school in Year 10, is the previous option still the best for his parents? Explain.

Activity 2

Mark plans to stay in school until the end of Year 12. He decides that he will save according to the following:

- If his parents give him Option 1 he will save 5% of his pocket money in the first year and then increase his saving by 5% every year after that.
- If his parents give him Option 2 he will save 4% of his pocket money in the first year and then increase his saving by 4% every year after that.
- If his parents give him Option 3 he will save 8% of his pocket money in the first year and then increase his saving by 8% every year after that.

Which option will give him the most savings when he finishes Year 12? Do not round your answers.

Activity 3

Write two short paragraphs, one outlining which option Mark should choose and why; and the other outlining which option Mark's parents should choose and why.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Nutty Nine

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 134:  NUTTY NINE

## Overview

In this task, students will investigate the magic of the number 9. They will need to carry out procedures flexibly and efficiently to find solutions. They will make connections between related concepts and progressively apply their familiar notions to develop a new idea. Students will be required to apply their existing strategies to seek solutions and reason mathematically to explain their thinking.

## Students will need

- calculator

## Relevant content descriptions from the Western Australian Curriculum

- Apply the associative, commutative and distributive laws to aid mental and written computation (ACMNA151)

## Students can demonstrate

- *fluency* when they
  - can perform the operations as outlined
- *understanding* when they
  - can show sufficient examples to support their belief that the statement is true for all cases
- *reasoning when they*
  - attempt to explain why the statement is true
  - comment on the number 9
- *problem solving* when they
  - investigate whether the statement works for larger numbers

Activity 1

If I multiply a number by 9, then add the digits of the results, I will always get a result of 9. For example, 9 x 18 = 162 and 1 + 6 + 2 = 9

1. Explain why this might happen.

   Answers will vary.

2. Show that this is true for all 2-digit numbers multiplied by 9?

   Answers will vary.
   It is true for all 2-digit numbers.
   Students should provide at least five examples.

3. Investigate whether it will work with 3-digit numbers?

   E.g., 9 x 238 = 2142 = 2 + 1 + 4 + 2 = 9
   Yes it will work for 3-digit numbers.
   Answers will vary.
   Students should provide at least five examples.

4. What will happen if the result is not 9 when the digits are added together?

   E.g., 9 x 254 = 2286 = 2 + 2 + 8 + 6 = 18
   Keep adding the digits of the result, 1 + 8 = 9
   Result will be 9.

Activity 2

Take any 2-digit number such a 53, then reverse the digit to get 35. Find the difference between the two numbers. Look what happens:

53 – 35 = 18
1 + 8 = 9

1. Does this work for all 2-digit numbers?

   Yes it will work for 2-digit numbers.
   Answers will vary.
   Students should provide at least five examples.

2. Investigate whether this will work for 3-digit numbers.
   E.g., 312 – 213 = 99 then 9 + 9 = 18 and 1 + 8 = 9
   Yes it will work for 3-digit numbers.
   Answers will vary.
   Students should provide at least five examples.

Activity 3

Take the number 9 and add any number to it, observe what happens:

9 + 53 = 62 → 6 + 2 = 8

↓

5 + 3 = 8

1. Can you explain what is observed in the above example?

   The sum of the digits of the number that is added to 9 is the same as the sum of the digits of the result of the addition.

2. Check that this works for all 2-digit numbers.

   Yes it will work for 2-digit numbers.
   Answers will vary.
   Students should provide at least five examples.

3. Investigate whether this works for 3-digit numbers.

   Yes it will work for 3-digit numbers.
   Answers will vary.
   Students should provide at least five examples.

Activity 4

Looking at the work that you have done through the above activities, comment on your understanding of the number 9 and its patterns in operations.

Answers will vary.

Activity 1

If I multiply a number by 9, then add the digits of the results, I will always get a result of 9. For example, 9 x 18 = 162 and 1 + 6 + 2 = 9

1. Explain why this might happen.

2. Show that this is true for all 2-digit numbers multiplied by 9?

3. Investigate whether it will work with 3-digit numbers?

4. What will happen if the result is not 9 when the digits are added together?

Activity 2

Take any 2-digit number such a 53, then reverse the digit to get 35. Find the difference between the two numbers. Look what happens:

$$53 - 35 = 18$$
$$1 + 8 = 9$$

1. Does this work for all 2-digit numbers?

2. Investigate whether this will work for 3-digit numbers.

Activity 3

Take the number 9 and add any number to it, observe what happens:

$9 + 53 = 62 \rightarrow 6 + 2 = 8$

$\downarrow$

$5 + 3 = 8$

1. Can you explain what is observed in the above example?

2. Check that this works for all 2-digit numbers.

3. Investigate whether this works for 3-digit numbers.

Activity 4

Looking at the work that you have done through the above activities, comment on your understanding of the number 9 and its patterns in operations.

Department of Education

The Mathematical Association of Western Australia Inc.

# YEAR 7 MATHEMATICS

## Number & Algebra Activity

# Match It

PRODUCED BY A DEPARTMENT OF EDUCATION - MAWA PARTNERSHIP PROJECT

WRITTEN FOR THE YEAR 7 AUSTRALIAN CURRICULUM

# TASK 136: MATCH IT

### Overview

In this task, students are required to match cards of fractions, decimals, percentages and diagrams. In each set of cards there are some missing, students are required to complete these blank cards in order to complete the four-card match.

### Students will need

- calculator
- scissors
- glue
- coloured paper (optional)

### Relevant content descriptions from the Western Australian Curriculum

- Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)

### Students can demonstrate

- *fluency* when they
  - convert fluently between fractions, decimals and percentages
- *understanding* when they
  - identify missing fractions, decimals and percentages
- *problem solving* when they
  - correctly draw missing diagrams and correctly match a fraction, decimal and percentage

In this task, you are required to match cards of fractions, decimals, percentages and diagrams. In each set of cards there are some blanks. You are required to complete these blank cards, then match each four-card set. Direct students as necessary.

| | |
|---|---|
| | — |
| — | – |
| – | – |
| — | — |
| — | |
| – | – |
| – | — |

| | |
|---|---|
| 0.25 | 0.6 |
| 0.7 | 0.25 |
| $0.\dot{6}$ | 0.8 |
| | $0.08\dot{3}$ |
| 0.5 | 0.75 |
| $0.\dot{3}$ | |
| 0.125 | 0.8 |

| | |
|---|---|
| 75% | 10% |
| | 50% |
| $33.\dot{3}\%$ | |
| 80% | 70% |
| 75% | $8.\dot{3}\%$ |
| 25% | 12.5% |
| 80% | $66.\dot{6}\%$ |

| | |
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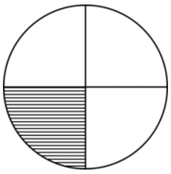
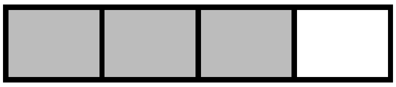
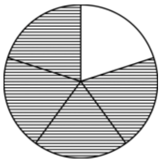
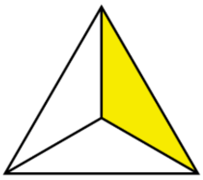
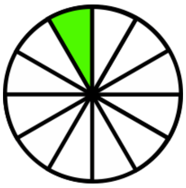
| |  |
|  |  |
|  |  |
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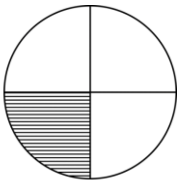
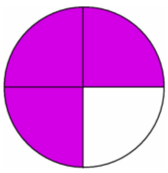
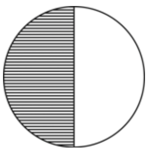
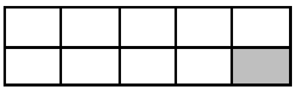
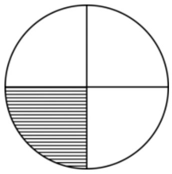
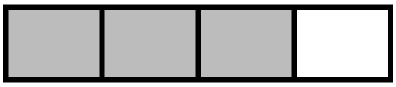
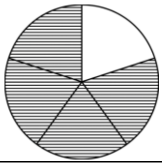
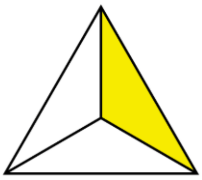
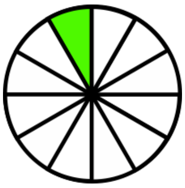
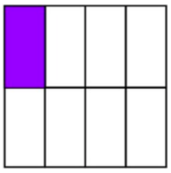
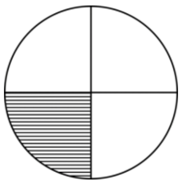
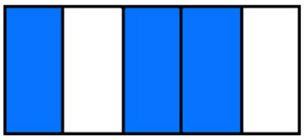
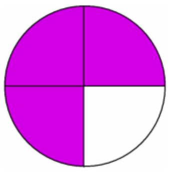
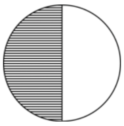
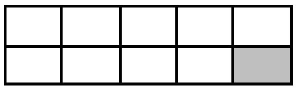
|  |  |
| |  |
|  |  |

Answer page:

| | |
|---|---|
| $\frac{7}{10}$, 0.7, 70%  | $\frac{1}{4}$, 0.25, 25%  |
| $\frac{3}{4}$, 0.75, 75%  | $\frac{8}{10}$, 0.8, 80%  |
| $\frac{1}{3}$, $0.\dot{3}$, $33.\dot{3}\%$  | $\frac{1}{12}$, $0.08\dot{3}$, $8.\dot{3}\%$  |
| $\frac{4}{5}$, 0.8, 80%  | $\frac{1}{8}$, 0.125, 12.5%  |
| $\frac{2}{3}$. $0.\dot{6}$, $66.\dot{6}\%$  | $\frac{3}{12}$, 0.25, 25%  |
| $\frac{3}{5}$, 0.6, 60%  | $\frac{9}{12}$, 0.75, 75%  |
| $\frac{1}{2}$, 0.5, 50%  | $\frac{1}{10}$, 0.1, 10%  |

In this task, you are required to match cards of fractions, decimals, percentages and diagrams. In each set of cards there are some blanks. You are required to complete these blank cards, then match each four-card set.

| | |
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|---|---|
| 0.25 | 0.6 |
| 0.7 | 0.25 |
| $0.\dot{6}$ | 0.8 |
| | $0.08\dot{3}$ |
| 0.5 | 0.75 |
| $0.\dot{3}$ | |
| 0.125 | 0.8 |

| | |
|---|---|
| 75% | 10% |
| | 50% |
| $33.\dot{3}\%$ | |
| 80% | 70% |
| 75% | $8.\dot{3}\%$ |
| 25% | 12.5% |
| 80% | $66.\dot{6}\%$ |

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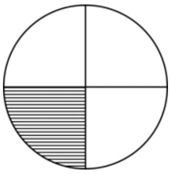
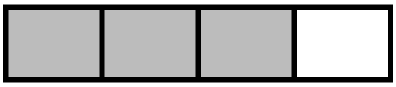
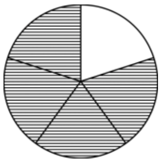
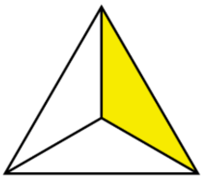
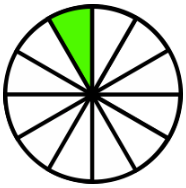
| |  |
|  |  |
|  |  |
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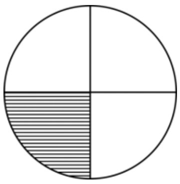
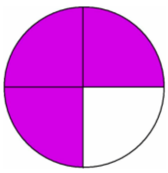
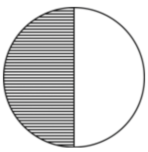
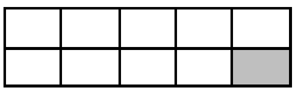
|  |  |
| |  |
|  |  |

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Activity

# Algebraic Thinking

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 301: ALGEBRAIC THINKING

**Overview**

This task focuses on the manipulation of mathematics materials, collecting data, making tables, and interpreting results and making generalisations. These skills are needed before students can understand formal algebra.

**Students will need**

- counters
- grid paper
- matchsticks
- 2-cm blocks

**Relevant content descriptions from the Western Australian Curriculum**

- Introduce the concept of variables as a way of representing numbers using letters (ACMNA175)

**Students can demonstrate**

- *fluency* when they
  - calculate accurately with integers
  - recognise common unit fractions and decimals
- *understanding* when they
  - recognise different ways of determining the answer
- *reasoning* when they
  - state the rule in their own terms and link this to generalisations
- *problem solving* when they
  - solve a pattern using integers ,fractions and decimals

## Activity 1

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 15 | 13 | 11 | 9 | 7 | 5 | 3 |

1. (a) What is happening in the pattern? The pattern is going down by two.

   (b) State the rule in your own terms? For each new position subtract two from the previous position.

   (c) Predict the $10^{th}$ position. 3

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 104 | 100 | 96 | 92 | 88 | 84 | 80 |

2. (a) What is happening in the pattern? The pattern is going down by 4.

   (b) State the rule in your own terms. For each new position subtract 4 from the previous position.

   (c) Predict the $10^{th}$ position. 68

## Activity 2

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 2 | 5 | 11 | 23 | 47 | 95 | |

1. (a) What is happening in this pattern? It is a growing pattern. It is going up by 3, 6, 12, 24, 48, . .

   (b) State the rule in your own words. The difference between positions is doubling each time.

   (c) Predict the $7^{th}$ position. 191

Now look at these.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| PATTERN | 3 | 4 | 6 | 10 | 18 | 34 |

2. (a) What is happening in this pattern? A growing pattern, going up by 1, 2, 4, 8, 16 etc.

(b) State the rule in your own words. The difference between positions is doubling each time.

(c) Predict the 10th position. 514

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 5 | 10 | 15 | 20 | 25 | 30 | 35 |

3. (a) What is happening in this pattern? Pattern is going up by 5.

(b) State the rule in your own words? Multiply the position by 5; e.g., $4 \times 5 = 20$.

(c) Can you see a link between position and pattern? Predict the 12th term. $12 \times 5 = 60$

(d) Write a generalization in your own words. $p \times 5$ for any term, where $p$ = the position number.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 7 | 12 | 17 | 22 | 27 | 32 | 37 |

4. (a) What is happening in the pattern? Pattern is going up by 5.

(b) State the rule in your own words. Each term is going up by +5.

(c) Can you see a link between the position and pattern? The starting position is 5 + 2.

(d) Write a generalization in your own words. Multiply position by 5 and add 2.

(e) Predict the 10th term. $10 \times 5 + 2 = 52$. Predict the 20th position. $20 \times 5 + 2 = 102$

<u>Activity 3</u>

1. Decimals Patterns.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 |

(a) What is happening in the pattern? Each position is going up by 0.2.

(b) Continue the pattern to the 7th position. Above

(c) Can you see a link between the position and pattern? Position × 0.2

(d) Write a generalization in your own words. $p$ x 0.2 for any term, where $p$ = the position.

(e) Predict 10th position. $10 \times 0.2 = 2$ Predict 20th position. $20 \times 0.2 = 4$

2. Fraction Patterns.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | ¼ | ½ | ¾ | 1 | 1¼ | 1½ | 1¾ |

(a) What is happening in the pattern? Pattern going up by 1/4

(b) Continue the pattern to the 7th position. 1¾

(c) State the rule in your own terms. Add ¼ to each new term.

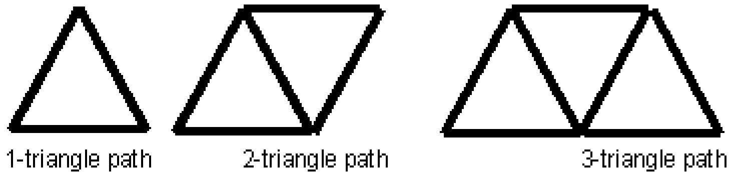
(d) Can you see a link between the position and pattern? You could multiply each position by ¼ to get the next term.

(e) Write a generalization in your own words. $p \times \frac{1}{4}$

(f) Predict the 10th position. 10 × ¼ = 2.5 Predict the 20th position. 20 × ¼ = 5

Activity 4

1. Make a triangular pattern with matchsticks like the one shown.




(a) Record the information in the table below.

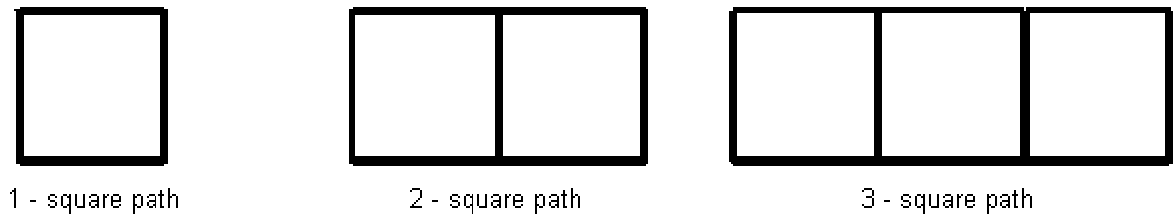
| Position (element) number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of triangles | 1 | 2 | | | | |
| Number of matchsticks | 3 | 5 | | | | |

(b) How many matchsticks are used to make 6 linear triangles? 13

(c) How many matchsticks are used to make 10 linear triangles? 21

(d) What is the rule for the number of matchsticks used, if $p$ = position number, $t$ = number of triangles, and $m$ = matchsticks? $p + t + 1 = m$

2. Now do the same for a square pattern.



(a) Record information in the table below.

| Position(element) Number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of squares | 1 | 2 | 3 | 4 | 5 | |
| Number of matchsticks | 4 | 7 | 12 | 19 | 28 | |

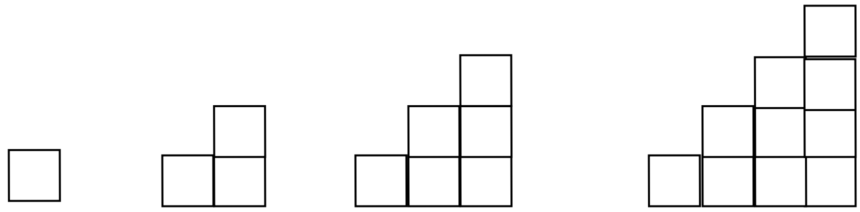
(b) How many matchsticks are used to make 6 linear squares? 39

(c) How many matchsticks are used to make 10 linear squares? 103

(d) What is the rule for the number of matchsticks used, if $p$ = position number, $s$ = number of squares, and $m$ = number of matchsticks? $p \times s + 3 = m$

## Activity 5

These shapes can be been made using toothpicks or matchsticks.



Shape 1 Shape 2 Shape 3 Shape 4

| Position (Element) | Shape 1 | Shape 2 | Shape 3 | Shape 4 |
|---|---|---|---|---|
| Number of matchsticks | 4 | 10 | 18 | 28 |
| Perimeter | 4 | 8 | 12 | 16 |

(a) What is the perimeter of the 5th shape? 20

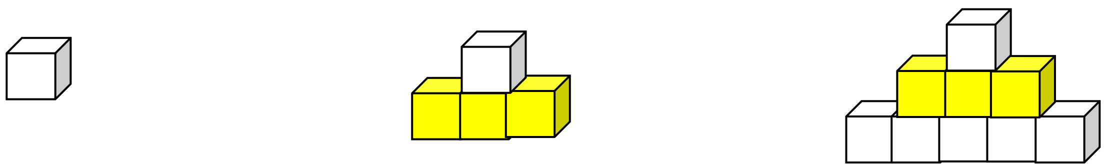
(b) What is the perimeter of the 6th shape? 24

(c) Can you see a pattern in the number of matches being used? Number of matchsticks increasing by 6, 8, 10, . . .

(d) Can you see a pattern in calculating the perimeter?
If you multiply the element by 4 you get the perimeter.

## Activity 6

Make the pattern with 2-cm blocks. Design a table; make some predictions. What is the rule? Make a generalisation in your own words. Various answers.



Activity 1

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 15 | 13 | 11 | 9 | 7 | 5 | 3 |

1. (a) What is happening in the pattern?

   (b) State the rule in your own terms.

   (c) Predict the $10^{th}$ position.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 104 | 100 | 96 | 92 | 88 | 84 | 80 |

2. (a) What is happening in the pattern?

   (b) State the rule in your own terms.

   (c) Predict the $10^{th}$ position.

Activity 2

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 2 | 5 | 11 | 23 | 47 | 95 | |

1. (a) What is happening in this pattern?

   (b) State the rule in your own words.

   (c) Predict the $7^{th}$ position.

Now look at these.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| PATTERN | 3 | 4 | 6 | 10 | 18 | 34 |

2. (a) What is happening in this pattern?

   (b) State the rule in your own words.

   (c) Predict the $10^{th}$ position.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 5 | 10 | 15 | 20 | 25 | 30 | 35 |

3. (a) What is happening in this pattern?

   (b) State the rule in your own words.

   (c) Can you see a link between the position and pattern? Predict the 12$^{th}$ term.

   (d) Write a generalization in your own words.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 7 | 12 | 17 | 22 | 27 | 32 | 37 |

4. (a) What is happening in the pattern?

   (b) State the rule in your own words?

   (c) Can you see a link between the position and pattern?

   (d) Write a generalization in your own words.

   (e) Predict the 10$^{th}$ term.

<u>Activity 3</u>

1. Decimals Patterns.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | 0.2 | 0.4 | 0.6 | 0.8 | | | |

   (a) What is happening in the pattern?

   (b) Continue the pattern to the 7$^{th}$ position.

   (c) Can you see a link between the position and pattern?

   (d) Write a generalization in your own words.

   (e) Predict the 10th position. Predict the 20th position.

2. Fraction Patterns.

| POSITION | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| PATTERN | ¼ | ½ | ¾ | 1 | 1¼ | 1½ | |

(a) What is happening in the pattern?

(b) Continue the pattern to the $7^{th}$ position

(c) State the rule in your own terms?

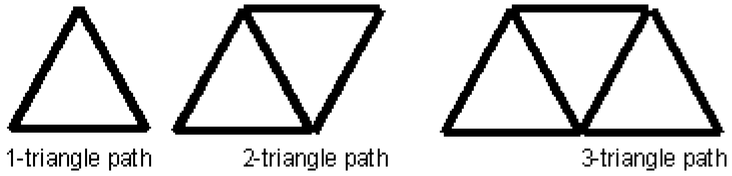
(d) Can you see a link between the position and pattern?

(e) Write a generalization in your own terms?

(f) Predict the $10^{th}$ position?    Predict the $20^{th}$ position?

Activity 4

1. Make a triangular pattern with matchsticks like the one shown




(a) Record the information in the table below.

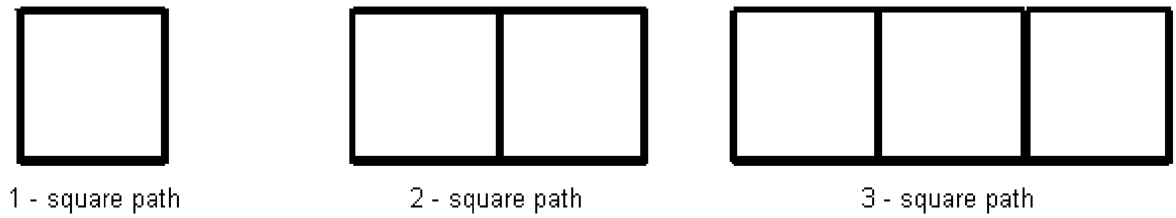
| Position(element) number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of triangles | 1 | 2 | | | | |
| Number of matchsticks used | 3 | 5 | | | | |

(b) How many matchsticks are used to make 6 linear triangles?

(c) How many matchsticks are used to make 10 linear triangles?

(d) What is the rule for the number of matchsticks used, if $p$ = position number, $t$ = number of triangles, and $m$ = matchsticks?

2. Now do the same for a square pattern




(a) Record the information in the table below.

| Position(element) Number | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of squares | 1 | 2 | | | | |
| Number of matchsticks | 4 | 7 | | | | |

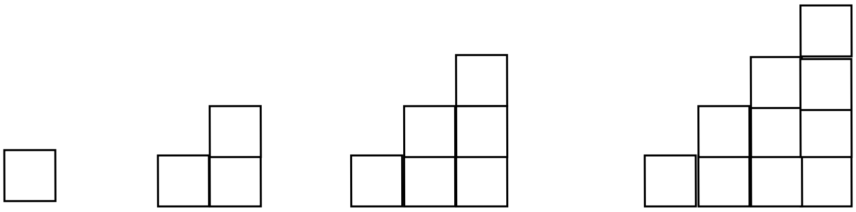
(b) How many matchsticks are used to make 6 linear squares?

(c) How many matchsticks are used to make 10 linear squares?

(d) What is the rule for the number of matchsticks used?

Activity 5

These shapes have been made using toothpicks



Shape 1 Shape 2 Shape 3 Shape 4

| Position (Element) | Shape 1 | Shape 2 | Shape 3 | Shape 4 |
|---|---|---|---|---|
| Number of matchsticks | 4 | 10 | 18 | 28 |
| Perimeter | 4 | 8 | 12 | 16 |

(a) What is the perimeter of the 5th shape?

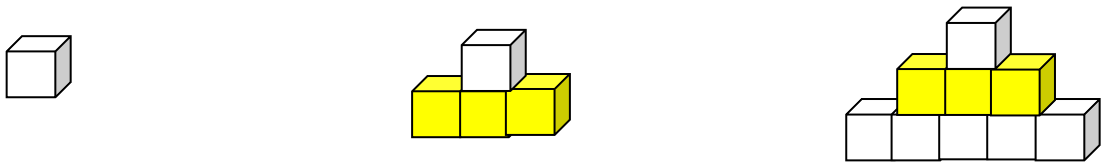
(b) What is the perimeter of the 6th shape?

(c) Can you see a pattern in the number of matches being used?

(d) Can you see a pattern in calculating the perimeter?

Activity 6

Make the pattern with 2cm blocks. Design a table; make some predictions. What is the rule? Make a generalisation in your own words.



Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Random Ratios

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 302: RANDOM RATIOS

## Overview

The problems in this activity ask students to solve realistic ratio problems.
Using a table is a good strategy to teach students to help them solve these types of problems.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Recognise and solve problems using simple ratios (ACMNA173)

## Students can demonstrate

- *fluency* when they
  - calculate accurately
- *understanding* when they
  - solve rate and ratio problems using fractions or percentages
- *problem solving* when they
  - choose the most efficient method to solve a particular problem

Activity

1. The average human heart beats at 72 beats per minute.

   (a) How many times does it beat in 15 seconds? 18

   (b) How many in an hour? 4320

   (c) How many in a day? 103 680

   (d) How many in a year? Approx. 37 843 200

2. Take your pulse and record the number of beats in 30 seconds.

   (a) How many times does it beat in an hour?

   (b) How many times does it beat in a day?

   (c) How many times does it beat in a month?

3. Use the table below to record your data. This table will vary for each student

Personal Heart Rate Table.

| Time | Number of Heartbeats |
|---|---|
| 30 sec | |
| 1 min | |
| 2 min | |
| 5 min | |
| 10 min | |
| 1 hour | |
| 1 day | |

4. Now calculate how many beats your heart has made in your life-time.

   This answer will vary depending on each student's data.

5 Bobby has a bag full of marbles that he keeps in his room. He has 35 red marbles and 25 green marbles. Find the ratio of red marbles to green marbles, and put it in its simplest form. 7 : 5

Marbles Table

| Red Marbles | Green Marbles |
|---|---|
| 35 | 25 |
| 7 | 5 |

6. To make 20 biscuits, Jacinta uses 5 cups of flour to 1 cup of milk. If she uses 3 cups of milk, how many cups of flour will she use and how many biscuits will be made?

Biscuits Table

| Number of biscuits | Number of cups of flour | Number of cups of milk |
|---|---|---|
| 20 | 5 | 1 |
| 40 | 10 | 2 |
| 60 | 15 | 3 |

7. Construct your own tables to solve the following problems.

   (a) If we know that 5 cups of flour make 20 biscuits, can we figure out how many biscuits are made with 20 cups of flour? 80

   (b) When 2000 kilograms of paper are recycled or reused, 18 trees are saved. How many trees are saved if 5000 kilograms of paper is recycled? 45

   (c) It costs $90 to feed a family of three for one week.

      (i) How much will it cost to feed a family of five for one week? $150

      (ii) How much will it cost to feed a family of six for a week? $180

      (iii) How much will it cost to feed a family of eight for a week? $240

8. In a school there are four boy scouts to every three girl scouts.

   (a) If there are forty-two girl scouts, how many boy scouts are there? 56

   (b) If there are 81 girl scouts, how many boy scouts are there? 108

9. To make green paint, a painter mixes yellow paint and blue paint in the ratio of three to two. If he used twelve litres of yellow paint, how much blue paint did he use? 8

10. When a honeyeater flies, it beats its wings an average of 23 times in ten seconds.

   (a) How many times will it beat its wings in one minute? 138.

   (b) How many times will it beat its wings in 90 seconds? 207

<u>Activity</u>

1. The average human heart beats at 72 beats per minute.

   (a) How many times does it beat in 15 seconds?

   (b) How many in an hour?

   (c) How many in a day?

   (d) How many in a year?

2. Take your pulse and record the number of beats in 30 seconds.

   (a) How many times does it beat in an hour?

   (b) How many times does it beat in a day?

   (c) How many times does it beat in a month?

3. Use the table below to record your data.

Personal Heart Rate Table.

| Time | Number of Heartbeats |
|---|---|
| 30 sec | |
| 1 min | |
| 2 min | |
| 5 min | |
| 10 min | |
| 1 hour | |
| 1 day | |

4. Now calculate how many beats your heart has made in your life-time.

5. Bobby has a bag full of marbles that he keeps in his room. He has 35 red marbles and 25 green marbles. Find the ratio of red marbles to green marbles, and put it in its simplest form.

Marbles table

| Red Marbles | Green Marbles |
|---|---|
| 35 | 25 |
| | |

6. To make 20 biscuits, Jacinta uses 5 cups of flour to 1 cup of milk. If she uses 3 cups of milk, how many cups of flour will she use and how many biscuits will be made?

Biscuits Table

| Number of biscuits | Number of cups of flour | Number of cups of milk |
|---|---|---|
| 20 | 5 | 1 |
| 40 | 10 | 2 |
| ? | ? | 3 |

7. Construct your own tables to solve the following problems.

(d) If we know that 5 cups of flour make 20 biscuits, can we figure out how many biscuits are made with 20 cups of flour?

(e) When 2000 kilograms of paper are recycled or reused, 18 trees are saved. How many trees are saved if 5000 kilograms of paper is recycled?

(f) It costs $90 to feed a family of three for one week

(ii) How much will it cost to feed a family of five for one week?

(ii) How much will it cost to feed a family of six for a week?

(iii) How much will it cost to feed a family of eight for a week?

8. In a school there are four boy scouts to every three girl scouts.

(a) If there are forty-two girl scouts, how many boy scouts are there?

(b) If there are 81 girl scouts, how many boy scouts are there?

9. To make green paint, a painter mixes yellow paint and blue paint in the ratio of three to two. If he used twelve litres of yellow paint, how much blue paint did he use?

10. When a honeyeater flies, it beats its wings an average of 23 times in ten seconds.

(a) How many times will it beat its wings in one minute?

(b) How many times will it beat its wings in 90 seconds?

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Part of a Whole

Produced by a Department of Education - MAWA partnership project

written for the Year 7 Australian Curriculum

# TASK 301: PART OF A WHOLE

## Overview

In this task students are asked to solve authentic examples for the quantities to be expressed and understanding the reasons for the calculations. The aspect that is focussed on is called the **unitary method** of solving problems, where calculations are based on one part of the whole. The most important step is to identify which is the 'part' and which is the 'whole'.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Express one quantity as a fraction of another, with and without the use of digital technologies (ACMNA155)

## Students can demonstrate

- *understanding* when they
  - know which aspect of the problem is the whole and which aspect is the part of the whole
- *reasoning* when they
  - understand the appropriate calculation method to use
- *problem solving*
  - successfully solve word problems

Activity

1. A block of cheese weighs ½ kilogram. Greg cuts the block of cheese into twenty equal cube-like pieces. What fraction of one kilogram did each piece of cheese weigh?

- $^{1}/_{10}$
- $^{1}/_{20}$
- $^{1}/_{25}$
- $^{1}/_{40}$

2. 24 000 spectators attended the sporting event. If $^{3}/_{5}$ were children, how many adults attended the event? 9600

3. James started with a number. He halved it, took away 4, halved the answer and then subtracted 2. The final answer was 4. What was James' starting number? 32

4. A car dealer claims that by buying a new car, Fred will pay $^{1}/_{5}$ less for petrol than he pays for the car he currently drives. If the car Fred currently drives costs $^{1}/_{6}$ less to fill up with petrol than his neighbour Frank's car, and Frank pays $3966 per year for petrol, what will it cost Fred to put petrol in a new car for one year (assume both cars will be travelling the same distance)? $2644

5. Angela and four brothers and sisters spent the weekend helping each other to paint the exterior of their family home. Each of them painted a fraction of the family house. Who painted the largest area?
(a) Angela: $^{2}/_{7}$
(b) Tony: $^{2}/_{12}$
(c) Frank: $^{2}/_{9}$
(d) Gina: $^{1}/_{5}$
(e) Joe: $^{1}/_{8}$

6. This recipe for chocolate cookies makes enough cookies for 15 students. To make the cookies you will need ½ cup margarine, 1 cup of sugar, 1 egg, 1 cup self-raising flour and 2½ tablespoons of cocoa. Your classroom has 30 students. How many tablespoons of cocoa will you need so that everyone in the class gets a chocolate cookie? 5

Activity

1. A block of cheese weighs ½ kilogram. Greg cuts the block of cheese into twenty equal cube-like pieces. What fraction of one kilogram did each piece of cheese weigh?

- $^1/_{10}$
- $^1/_{20}$
- $^1/_{25}$
- $^1/_{40}$

2. 24 000 spectators attended the sporting event. If $^3/_5$ were children, how many adults attended the event?

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5. Angela and four brothers and sisters spent the weekend helping each other to paint the exterior of their family home. Each of them painted a fraction of the family house. Who painted the largest area?
(a) Angela: $^2/_7$
(b) Tony: $^2/_{12}$
(c) Frank: $^2/_9$
(d) Gina: $^1/_5$
(e) Joe: $^1/_8$

6. This recipe for chocolate cookies makes enough cookies for 15 students. To make the cookies you will need ½ cup margarine, 1 cup of sugar, 1 egg, 1 cup self-rising flour and 2½ tablespoons of cocoa. Your classroom has 30 students. How many tablespoons of cocoa will you need so that everyone in the class gets a chocolate cookie?

1. A block of cheese weighs ½ kilogram. Greg cuts the block of cheese into twenty equal cube-like pieces. What fraction of one kilogram did each piece of cheese weigh?

    - 1/10 b) 1/20 c)1/25 d) 1/40

2. 24 000 spectators attended the sporting event. If 3/5 were children, how many adults attended the event?

3. James started with a number. He halved it, took away 4, halved the answer and then subtracted 2. The answer was 4. What was James's starting number?

4. A car dealer claims that by buying a new car Fred will pay 1/5 less for petrol then he pays for the car he currently drives. If the car Fred currently drives costs 1/6 less to fill up with petrol than his neighbour Frank's car, and Frank pays $ 3966 per year for petrol, what will it cost Fred to put petrol in a new car for one year (assume both cars will be travelling the same distance)?

5. Angela and four brothers and sisters spent the weekend helping each other to paint the outside of their family home. Each of them painted a fraction of the family house. Who painted the largest area?
   (a) Angela: 2/7
   (b) Tony: 2/12
   (c) Frank: 2/9
   (d) Gina: 1/5
   (e) Joe: 1/8

6. This recipe for chocolate cookies makes enough cookies for 15 students. To make the cookies you will need ½ cup margarine, 1 cup of sugar, 1 egg, 1 cup self-rising flour and 2 ½ tablespoons of cocoa. Your classroom has 30 students. How many tablespoons of cocoa will you need so that everyone in the class gets a chocolate cookie?

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# Positive and Negative Numbers

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 304: POSITIVE AND NEGATIVE NUMBERS

**Overview**
This task focus on the knowledge needed to add positive and negative integers and how this is the foundation knowledge for further development of algebra understanding.

**Students will need**
- calculator

**Relevant content descriptions from the Western Australian Curriculum**
- Compare, order, add and subtract integers (ACMNA280)

**Students can demonstrate**
- *fluency* when they
  - calculate accurately with positive and negative integers
- *understanding* when they
  - connect the relationship between positive and negative integers
  - recognise different ways of determining the answer
- *problem solving* when they
  - solve word problems involving positive and negative numbers

Activity 1

Calculate these expressions and show (place a Y) whether the result is positive or negative.

| Expression | Positive | Negative |
|---|---|---|
| 8 + -6 = | Y | |
| -7 + -22 = | | Y |
| -8 - 5 = | | Y |
| 12 -18 = | | Y |
| -25 + 7 = | | Y |
| 36 + 23 = | Y | |
| -3 - -4 | Y | |
| 35 - -29 | Y | |
| 18 + 27 = | Y | |
| -6 + -5 = | | Y |

Activity 2

From the table below, record all the expressions that match each of the following numbers.

(a) Which combinations make 12?

(+8) - (-4); (+15) + (-3); (+4) - (-8); (+9) + (+3); (+6) - (-6); (+7) - (-5)

(b) Which combinations make 15?

(+12) + (+3); (+19) + (-4); (+18) - (+3); (+12) - (-3), (+11) - (-4); (- 3) - (-18); (+9) - (-6)

(b) Which combinations make 20?

(+15) + (+5); (+7) - (-13); (+32) + (-12); (+15) - (-5); (+12) - (-8),

| | | | |
|---|---|---|---|
| **(+12)+ (+3)** | **(+8) - (-4)** | **(+15)+ (+5)** | **(+7) - (-13)** |
| **(+15) + (-3)** | **(+19) + (-4)** | **(+4) - (-8)** | **(+18) - (+3)** |
| **(+32) + (-12)** | **(+12) - (-3)** | **(+15) - (-5)** | **(+9) - (-3)** |
| **(+11) - (-4)** | **(+9) + (+3)** | **(- 3) - (-18)** | **(+6) - (- 6)** |
| **(+12) - (-8)** | **(+7) - (-5)** | **(- 3) - (-18)** | **(+9) - (-6)** |

Activity 3

1. Solve these word problems by selecting the correct answer form the ones below each problem.

(a) Annie was on the $15^{th}$ floor of the building and ended up in the underground parking lot on level two below ground. How many floors had the lift travelled?

A. 15 B. 13 C. 2 D. 17

(b) Peter was rock climbing and he was making good progress and reached a height of 100 metres. All of a sudden one of his ropes slipped and he plummeted 34 metres. How far up the rock face is Peter now?

A. 134 metres B. 76 metres C. 66 metres D. 34 metres

(c) The leader board at the Vines Golf Championship showed Evelyn's score after 3 holes was 14. However, she scored -2 on the next hole. What was Evelyn's score after 4 holes of golf?

A. 4 B. 10 C. 12 d. 16

2. After 4 rounds of 18 holes of a Golf Tournament the following players had these scores. Which player is winning the match? Explain your answer

| Player | To Par | Round 1 | Round 2 | Round 3 | Round 4 | Total |
|---|---|---|---|---|---|---|
| Jordan | -10 | 67 | 73 | 67 | 71 | 278 |
| Jason | -13 | 66 | 72 | 67 | 70 | 275 |
| Sergio | -11 | 70 | 69 | 68 | 70 | 277 |
| Zac | -15 | 66 | 71 | 70 | 66 | 273 |
| Louis | -14 | 67 | 71 | 67 | 69 | 274 |

The player winning the match is ZAC because he has the lowest score on -15 below par

3. According to the bank statement, Jack ended the month with a $5000 balance.
If Jack started the month with a balance of $8000, which integer represents the change in his bank account balance?

A. $4000 B. $3000 C. -$3000 D. -$4000

Activity 1

Calculate these expressions and show (place a Y) whether the result is positive or negative.

| Expression | Positive | Negative |
|---|---|---|
| 8 + -6 = | | |
| -7 + -22 = | | |
| -8 - 5 = | | |
| 12 -18 = | | |
| -25 + 7 = | | |
| 36 + 23 = | | |
| -3 - -4 | | |
| 35 - -29 | | |
| 18 + 27 = | | |
| -6 + -5 = | | |

Activity 2

From the table below, record all the expressions that match each of the following numbers.

(a) Which combinations make 12?

(b) Which combinations make 15?

(c) Which combinations make 20?

| | | | |
|---|---|---|---|
| **(+12) + (+3)** | **(+8) - (-4)** | **(+15) + (+5)** | **(+7) - (-13)** |
| **(+15) + (-3)** | **(+19) + (-4)** | **(+4) - (-8)** | **(+18) - (+3)** |
| **(+32) + (-12)** | **(+12) - (-3)** | **(+15) - (-5)** | **(+9) - (-3)** |
| **(+11) - (-4)** | **(+9) + (+3)** | **(- 3) - (-18)** | **(+6) - (- 6)** |
| **(+12) - (-8)** | **(+7) - (-5)** | **(- 3) - (-18)** | **(+9) - (-6)** |

Activity 3

1. Solve these word problems by selecting the correct answer from the ones below each problem.

(a) Annie was on the $15^{th}$ floor of the building and ended up in the underground parking lot on level two below ground. How many floors had the lift travelled?

B. 15 B. 13 C. 2 D. 17

(b) Peter was rock climbing and he was making good progress and had reached a height of 100 metres. All of a sudden one of his ropes slipped and he plummeted 34 metres. How far up the rock face is Peter now?

B. 134 metres B. 76 metres C. 66 metres D. 34 metres

(c) The leader board at the Vines Golf Championship showed Evelyn's score after 3 holes was 14. However, she scored -2 on the next hole. What was Evelyn's score after 4 holes of golf?

B. 4 B. 10 C. 12 D. 16

2. After 4 rounds of 18 holes of a Golf Tournament the following players had these scores. Which player is winning the match? Explain your answer.

| Player | To Par | Round 1 | Round 2 | Round 3 | Round 4 | Total |
|---|---|---|---|---|---|---|
| Jordan | -10 | 67 | 73 | 67 | 71 | 278 |
| Jason | -13 | 66 | 72 | 67 | 70 | 275 |
| Sergio | -11 | 70 | 69 | 68 | 70 | 277 |
| Zac | -15 | 66 | 71 | 70 | 66 | 273 |
| Louis | -14 | 67 | 71 | 67 | 69 | 274 |

3. According to the bank statement, Jack ended the month with a $5000 balance.
If Jack started the month with a balance of $8000, which integer represents the change in his bank account balance?

B. $4000 B. $3000 C. -$3000 D. -$4000

Department of Education

# Year 7 Mathematics

## Number & Algebra Activity

# Distributive Understanding

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 305: DISTRIBUTIVE UNDERSTANDING

## Overview

This task links the distributive property to whole number and decimal number multiplication and shows how this thinking is needed when students progress to the use of pronumerals in multiplication.

## Students will need

- No requirements

## Relevant content descriptions from the Western Australian Curriculum

- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

## Students can demonstrate

- *fluency* when they
  - calculate accurately using whole number and decimal number multiplication
- *understanding* when they
  - understand the distributive process in multiplying with whole numbers and decimal numbers
- *reasoning* when they
  - explain how the distributive property for whole numbers and decimal numbers can be used with pronumerals.

Activity 1

The Distributive Law

26
X 54

| | |
|---|---|
| 4x6 | 24 |
| 4x20 | 80 |
| 50x6 | 300 |
| 50x20 | 1000 |
| Ans: | 1404 |

Notice that the 20+6 is multiplied by 4, and the 20+6 is multiplied by 50. What we do with whole numbers is the same strategy with pro-numerals.

Calculate the following by drawing a table like the one above. Then explain what you have done.

*(a)* 24 x 36 = 864 (b) 45x 23 = 1 035 (c) 17 x 56 = 952

| | |
|---|---|
| 6 × 4 | 24 |
| 6× 20 | 120 |
| 30 × 4 | 120 |
| 30 ×20 | 600 |
| Ans: | 864 |

Examples (b) and (c) should each also be completed similarly in a table.
Various explanations will be given.

Activity 2

Look at the same approach when using decimal numbers.

5.28 x 34 = ?

| | |
|---|---|
| 4 x 0.08 | 0.32 |
| 4 x 0.2 | 0.8 |
| 4 x 5 | 20 |
| 30 x 0.08 | 2.4 |
| 30 x 0.2 | 6 |
| 30 x 5 | 150 |
| Product | 179.52 |

Calculate each of the following by drawing a table like the one in the decimal multiplication example. Then explain what you have done.

(a) 7.85 x 64 = 502.4 (b) 3.76 x 18 = 67.68 (c) 1.23 x 5.6 = 6.888

| | |
|---|---|
| 4 × 0.05 | 0.2 |
| 4 × 0.8 | 3.2 |
| 4 × 7 | 28 |
| 60 × 0.05 | 3 |
| 60 × 0.8 | 48 |
| 60 × 7 | 420 |
| | 502.4 |

Examples (b) and (c) should each also be completed similarly in a table.
Various explanations will be given.

Activity 3

When using the distributive property in algebra, the process that was used in whole number multiplication and in decimal number multiplication also applies to pronumerals.

Look at the following examples.

Example 1:

$4(a + b) = 4a + 4b$

Notice that the 4 has to be multiplied by the $a$ and then the $b$.

Example 2:

$7(2c – 3d + 5) = 14c – 21d + 35$.

Notice that the 7 must be multiplied by the $2c$, then the $-3d$, and then the 5.

Your challenge is to complete $(a – 3)(b + 4)$, then explain your method.

The $a$ needs to be multiplied by $b$ and the 4, which is equal to $ab + 4a$.

The -3 now needs to be multiplied by $b$ and the 4, which is equal to $-3b – 12$.

Put both parts of the multiplication together and you get $ab + 4a - 3b - 12$.

Activity 1

The Distributive Law

26
X 54

| | |
|---|---|
| 4x6 | 24 |
| 4x20 | 80 |
| 50x6 | 300 |
| 50x20 | 1000 |
| Ans: | 1404 |

Notice that the 20+6 is multiplied by 4, and the 20+6 is multiplied by 50. What we do with whole numbers is the same strategy with pro-numerals.

Calculate the following by drawing a table like the one above. Then explain what you have done.

(a) 24 x 36 = ? (b) 45x 23 = ? (c) 17 x 56 = ?

Activity 2

Look at the same approach when using decimal numbers.

5.28 x 34 =?

| | |
|---|---|
| 4 x 0.08 | 0.32 |
| 4 x 0.2 | 0.8 |
| 4 x 5 | 20 |
| 30 x 0.08 | 2.4 |
| 30 x 0.2 | |
| 30 x 5 | 50 |
| Product | 179.52 |

Calculate each of the following by drawing a table like the one in the decimal multiplication example. Then explain what you have done.

(a) $7.85 \times 64 = ?$ (b) $3.76 \times 18 = ?$ (c) $1.23 \times 5.6 = ?$

Activity 3

When using the distributive property in algebra, the process that was used in whole number multiplication and in decimal number multiplication also applies to pronumerals.

Look at the following examples.

Example 1:

$4(a + b) = 4a + 4b$

Notice that the 4 has to be multiplied by the *a* and then the *b*.

Example 2:

$7(2c – 3d + 5) = 14c – 21d + 35$.

Notice that the 7 must be multiplied by the 2*c*, then the -3*d*, and then the 5.

Your challenge is to complete $(a – 3)(b + 4)$, then explain your method.

Department of Education

The Mathematical Association of Western Australia Inc. MAΣWA

# Year 7 Mathematics

## Number & Algebra Activity

# Order of Operations

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 306: ORDER OF OPERATIONS

### Overview

Students need to identify order of operations in contextualised problems, preserving the order by inserting brackets in numerical expressions, then recognising how order is preserved by convention.

### Students will need

- calculators

### Relevant content descriptions from the Western Australian Curriculum

- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

### Students can demonstrate

- *fluency* when they
  - calculate accurately in number sentences with the four operations and indices
- *understanding* when they
  - explain the order of operations
  - justify their thinking when solving expression from word problems
- *reasoning* when they
  - solve written expressions and write written expression to solve word problems

Activity 1

There are rules for the order in which we carry out operations in a numerical expression. These are the rules for the order of operations.

1. Always do calculations in brackets first [B];
2. Next do any calculations with indices or powers [I];
3. Multiplication and division next, in order from left to right [MD]; and
4. Finally, addition and subtraction, in order from left to right [AS]

This pneumonic will help you remember the order. **BIMDAS.** But remember that multiplication and division are done together as they occur, and similarly for addition and subtraction.

Here is an example:

| | |
|---|---|
| $3^2 + 8 \times 2 - (9 - 5)^2 - 20 \times 0.5 + 12 \div 3 \times 2$ | B (operations to remove the brackets) |
| $= 3^2 + 8 \times 2 - 4^2 - 20 \times 0.5 + 12 \div 3 \times 2$ | I (operations to remove the indices) |
| $= 9 + 8 \times 2 - 16 - 20 \times 0.5 + 12 \div 3 \times 2$ | MD (multiplication & division operations) |
| $= 9 + 16 - 16 - 10 + 8$ | AS (addition & subtraction operations) |
| $= 7$ | |

Calculate the following, showing all the steps of working to justify both your method and your answers.

1. $5^3 - 30 \div 3 + 4^2 = 125 - 30 \div 10 + 16 = 125 - 3 + 16 = 138$

2. $12 \div 2 \times 6 + 2^2 - 4 = 12 \div 2 \times 6 + 4 - 4 = 36 + 4 - 4 = 36$

3. $2^2 \times 5 + (27 - 15) \div 4 = 2^2 \times 5 + 12 \div 4 = 4 \times 5 + 12 \div 4 = 20 + 3 = 23$

4. $9 \times 3^2 \div 9 - 2^2 = 9 \times 9 \div 9 - 4 = 9 - 4 = 5$

5. $44 + 11 - 4^2 \div 2^3 \times 3 = 44 + 11 - 16 \div 8 \times 3 = 44 + 11 - 6 = 49$

6. $6^3 - 52 + 9 = 216 - 52 + 9 = 173$

7. $(32 + 28) \div 2^2 \times 3^2 - 9 = 60 \div 2^2 \times 3^2 - 9 = 60 \div 4 \times 9 - 9 = 135 - 9 = 126$

8. $6 - 12 \div 2^2 + 9^2 \times 2 = 6 - 12 \div 4 + 81 \times 2 = 6 - 3 + 162 = 165$

9. Your turn: Create your own numerical expression and test it out on your partner

   Answers will vary.

Activity 2 Target Number

From the table on the below, record all the expressions that match each of the following numbers.

(a) Which combinations make 15?
$3 + 2 \times 6$, $8 \times 2 - 1$, $6 \times 3 - 3$, $4 \times 3 + 12 \div 4$, $2.5 \times 5 + 2.5$, $5 \times 4 - 10 \div 2$, $20 - 1 \times 5$

(b) Which combinations make 18?
$3 + 3 \times 5$, $2.4 \times 5 + 6$, $7 \times 3 - 3$, $12 \times 2 - 6$, $5 \times 3 + 9 \div 3$, $0.5 \times 20 + 8$,

(c) Which combinations make 24?
$9 \times 2 + 6$, $5 \times 6 - 6$, $4 \times 5 + 12 \div 3$, $4 \times 3 + 3 \times 4$, $7 \times 4 - 2^2$, $5.5 \times 6 - 3^2$, $3^2 \times 3 - 3$

| | | | |
|---|---|---|---|
| **$3+2\times6$** | **$3+3\times5$** | **$2.4\times5+6$** | **$8\times2-1$** |
| **$9\times2+6$** | **$6\times3-3$** | **$7\times3-3$** | **$5\times6-6$** |
| **$12\times2-6$** | **$4\times5+12\div3$** | **$5\times3+9\div3$** | **$4\times3+12\div4$** |
| **$2.5\times5+2.5$** | **$4\times3+3\times4$** | **$7\times4-2^2$** | **$0.5\times20+8$** |
| **$5.5\times6-3^2$** | **$5\times4-10\div2$** | **$20-1\times5$** | **$3^2\times3-3$** |

Activity 3

Find the missing number in each of the equations below.

1. $15 \div 5 - 7 = -4$

2. $2 + 5 \times 3 = 17$

3. $36 \div 3 - 10 = 2$

4. 6 ÷ 2 + 7 × 5 = 38

5. 12 - 6 + 9 × 3 = 33

6. 9 - 3 + 8 × 2 - 2 = 20

7. Your turn: Write a number sentence with a missing number and see if other members of the class can work it out.
   Answers will vary

Activity 4: Word Problems

There will be various justifications for these.

1. Evelyn works at the supermarket for 2 hours a day for five days of the week, and for 4 hours on both Saturdays and Sundays. The pay is $12.50 per hour during the week and $18.00 per hour on Saturdays and Sundays. Write an expression and justify it.

2 x 5 x $12.50 + 4 x 2 x $18.00= $125 + $144 = $269

2. Paula scores 24 points fewer than Rom, who scores 56 points. Tracey scores half as many points as Paula. How many points does Tracey score? Write an expression and justify it.
(56 -24)÷ 2 = 32 ÷2 = 16

3. An AFL football team scored 12 goals and 18 behinds. If teams are awarded 6 points for a goal and 1 point for a behind how many points did the team achieve. Write an expression and justify your answer.

12 x 6 + 18 = 90

4. Use the order of operations and the digits 2, 4, 6, and 8 to create an expression with a value of 2. You may add brackets, exponents and negatives at your will.

$4^2$ - 6 – 8. Various answers for this question.

5. A certain small company has 56 employees. Of these, 12 receive a wage of $240 per day and the rest receive $180 per day. Each employee's week is equal to 6 working days. How much does the company pay out for each week if they need to pay the salary and 9% superannuation for each employee? Write an expression and justify your answer.

[12 x $240 + (56 -12) x $180] x 6 x 1.09 = $70 632

6. Greg had $76 and withdrew $210 from his bank account. He bought a pair of trousers for $85.99, 2 shirts for $32.50 each, and 1 pairs of shoes for $55.00 each. How much money did Greg have left at the end of the shopping day? Write an expression and justify your answer.

$76 + $210 - $85.99 – 2 x $ 32.50 – $55.00 = $80.00

Activity 1

There are rules for the order in which we carry out operations in a numerical expression. These are the rules for the order of operations.

3. Always do calculations in brackets first [B];
4. Next do any calculations with indices or powers [I];
3. Multiplication and division next, in order from left to right [MD]; and
5. Finally, addition and subtraction, in order from left to right [AS]

This pneumonic will help you remember the order. **BIMDAS.** But remember that multiplication and division are done together as they occur, and similarly for addition and subtraction.

Here is an example:

| | |
|---|---|
| $3^2 + 8 \times 2 - (9 - 5)^2 - 20 \times 0.5 + 12 \div 3 \times 2$ | B (operations to remove the brackets) |
| $= 3^2 + 8 \times 2 - 4^2 - 20 \times 0.5 + 12 \div 3 \times 2$ | I (operations to remove the indices) |
| $= 9 + 8 \times 2 - 16 - 20 \times 0.5 + 12 \div 3 \times 2$ | MD (multiplication & division operations) |
| $= 9 + 16 - 16 - 10 + 8$ | AS (addition & subtraction operations) |
| $= 7$ | |

Calculate the following, showing all the steps of working to justify both your method and your answers.

1. $5^3 - 30 \div 3 + 4^2$

2. $12 \div 2 \times 6 + 2^2 - 4$

3. $2^2 \times 5 + (3^3 - 15) \div 4$

4. $9 \times 3^2 \div 9 - 2^2$

5. $44 + 11 - 4^2 \div 2^3 \times 3$

6. $6^3 - 52 + 9 \times$

7. $32 + 28 \div 2^2 \times 3^2 - 95$

8. $6 - 12 \div 2^2 + 9^2 \times 2$

9. Your turn: Create your own numerical expression and test it out on your partner

## Activity 2 Target Number

From the table on the next page, record all the expressions that match each of the following numbers.

(a) Which combinations make 15?

(b) Which combinations make 18?

(c) Which combinations make 24?

| | | | |
|---|---|---|---|
| **3+2×6** | **3+3×5** | **2.4×5+6** | **8×2-1** |
| **9×2+6** | **6×3-3** | **7×3- 3** | **5×6-6** |
| **12×2-6** | **4×5+12÷3** | **5×3+9÷3** | **4×3+12÷ 4** |
| **2.5×5+2.5** | **4×3+3×4** | **7×4-2²** | **0.5×20+8** |
| **5.5×6-3²** | **5×4-10÷2** | **20-1×5** | **3²×3-3** |

Activity 3

Find the missing number in each of the equations below.

1. __ ÷ 5 – 7 = -4

2. 2 + 5 × __ = 17

3. __ ÷ 3 – 10 = 2

4. __ ÷ 2 + 7 × 5 = 38

5. 12 - 6 + __ × 3 = 33

6. 9 - 3 + 8 × 2 - __ = 20

7. Your turn: Write a number sentence with a missing number and see if other members of the class can work it out.

Activity 4: Word Problems

1. Evelyn works at the supermarket for 2 hours a day for five days of the week, and for 4 hours on both Saturdays and Sundays. The pay is $12.50 per hour during the week and $18.00 per hour on Saturdays and Sundays. Write an expression and justify it.

2. Paula scores 24 points fewer than Rom, who scores 56 points. Tracey scores half as many points as Paula. How many points does Tracey score? Write an expression and justify it.

3. An AFL football team scored 12 goals and 18 behinds. If teams are awarded 6 points for a goal and 1 point for a behind how many points did the team achieve. Write an expression and justify your answer.

4. Use the order of operations and the digits 2, 4, 6, and 8 to create an expression with a value of 2. You may add brackets, exponents and negatives at your will.

5. A certain small company has 56 employees. Of these, 12 receive a wage of $240 per day and the rest receive $180 per day. Each employee's week is equal to 6 working days. How much does the company pay out for each week if they need to pay the salary and 9% superannuation for each employee? Write an expression and justify your answer.

6. Greg had $76 and withdrew $210 from his bank account. He bought a pair of trousers for $85.99, 2 shirts for $32.50 each, and 1 pairs of shoes for $55.00 each. How much money did Greg have left at the end of the shopping day? Write an expression and justify your answer.

Department of Education

The Mathematical Association of Western Australia Inc.

# Year 7 Mathematics

## Number & Algebra Activity

# The Turn Around

Produced by a Department of Education - MAWA partnership project
written for the Year 7 Australian Curriculum

# TASK 307: THE TURN AROUND

## Overview

This task links the commutative property to addition and multiplication and shows how this thinking is needed before students progress to the use of pronumerals in addition expressions ($a + b = b + a$) and in multiplication expressions ($a \times b = b \times a$). This task also focusses on the addition expression of $a + b = x + y$ and the multiplication expression $a \times b = x \times y$.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

## Students can demonstrate

- *fluency* when they
    - calculate accurately
- *understanding* when they
    - connect the relationship between commutativity to algebra addition and multiplication expressions
- *reasoning* when they
    - generalise from addition and multiplication tables
- *problem solving* when they
    - Explore all possibilities to give different solutions

Activity 1

We can partition 12 into a number of different combination pairs.

| 12 | |
|---|---|
| 7 | 5 |
| 5 | 7 |
| 8 | 4 |
| 4 | 8 |
| 9 | 3 |
| 3 | 9 |
| 10 | 2 |
| 2 | 10 |
| 11 | 1 |
| 1 | 11 |

You need to see that the numbers can be added in any order:

**7 + 5 = 5 +7**

This thinking is the same when adding pronumerals in formal algebra:

$$\boldsymbol{a + b = b + a}$$

1. Make up all other $x + y = y + x$ examples from the table above

8 + 4 = 4 + 8, 2 +10 =10 + 2, 11 + 1 = 1 + 11, 3 + 9 = 9 + 3

2. Since all these combination are equal to 12, we can also say that 7 + 5 is equal to 8 + 4. This could be generalised as $a + b = x + y$.
Find some other $a + b = x + y$ examples in the table above.

11+ 1 =10 + 2, 9 +3 = 4 + 8 are just two examples.

Activity 2

1. Look at the number 8.25
Make a table of numbers that combine to make 8.25. Many possible answers. Examples:

| 8.25 | |
|---|---|
| 8 | 0.25 |
| 7 | 1.25 |
| 6 | 2.25 |
| 5 | 3.25 |
| 4 | 4.25 |
| 3 | 5.25 |
| 2 | 6.25 |

2. Make up your own $x + y = y + x$ expressions in your table

2 + 6.25 = 6.25 +2, 8 + 0.25 = 0.25 + 8, and others from the table created by students.

3. Write some $a + b = x + y$ expressions from your table.

2 + 6.25 = 8 + 0.25, 7 +1.25 = 3 +5.25. There are many more such combinations

Activity 3: Multiplication Number Sentences

| 24 | | | |
|---|---|---|---|
| 6 | multiplied by | 4 | = 24 |
| 4 | multiplied by | 6 | = 24 |
| 8 | multiplied by | 3 | = 24 |
| 3 | multiplied by | 8 | =24 |
| 12 | multiplied by | 2 | = 24 |
| 2 | multiplied by | 12 | = 24 |
| 24 | multiplied by | 1 | = 24 |
| 1 | multiplied by | 24 | =24 |

1. If 6 x 4 = 24 and 4 × 6 =24, we can say that $a \times b = b \times a$
Make up your own $a \times b = b \times a$ expressions from this table.

8 x 3 = 3 x 8, 2 x 12 = 12 x 2. There are many more combinations.

2. We can see that 6 x 4 = 24 and 12 x 2 = 24, so 6 x 4 = 12 x 2.
We could write this as $a \times b = c \times d$.
Make up your own $a \times b = c \times d$ expressions from this table.

6 x 4 = 8 x 3, 4 x 6 = 3 x 8. There are many more combinations.

3. Now use the same process with 36. Find all the factors of 36.

| 36 | | | |
|---|---|---|---|
| 1 | multiplied by | 36 | = 36 |
| 36 | multiplied by | 1 | = 36 |
| 18 | multiplied by | 2 | = 36 |
| 2 | multiplied by | 18 | = 36 |
| 12 | multiplied by | 3 | = 36 |
| 3 | multiplied by | 12 | = 36 |
| 9 | multiplied by | 4 | = 36 |
| 4 | multiplied by | 9 | = 36 |
| 6 | multiplied by | 6 | = 36 |

4. From the table above, make up your own expression of $a \times b = b \times a$.

   *4 x 9 = 9 x 4,  18 x 2 = 2 x 18.* There are more combinations.

5. Make up your own $a \times b = c \times d$ expressions from this table.

   1 x 36 = 18 x 2 , 12 x 3 = 6 x 6.  There are more combinations.

Note that numbers that are subtracted cannot have the order reversed, so subtraction is **NOT** commutative.

$$7 - 5 \neq 5 - 7$$
$$x - y \neq y - x$$

Similarly, numbers that are divided cannot have the order reversed, so division is **NOT** commutative.

$$10 \div 5 \neq 5 \div 10$$
$$x \div y \neq y \div x$$

Activity 1

We can partition 12 into a number of different combination pairs.

| 12 | |
|---|---|
| 7 | 5 |
| 5 | 7 |
| 8 | 4 |
| 4 | 8 |
| 9 | 3 |
| 3 | 9 |
| 10 | 2 |
| 2 | 10 |
| 11 | 1 |
| 1 | 11 |

You need to see that the numbers can be added in any order:

**7 + 5 = 5 +7**

This thinking is the same when adding pronumerals in formal algebra:

$$\boldsymbol{a + b = b + a}$$

1. Make up all other $x + y = y + x$ examples from the table above.

2. Since all these combination are equal to 12, we can also say that 7 + 5 is equal to 8 + 4. This could be generalised as $a + b = x + y$.
Find some other $a + b = x + y$ examples in the table above.

Activity 2

1. Look at the number 8.25
Make a table of numbers which combine to make a sum of 8.25.

| 8.25 | |
|---|---|
| 8 | 0.25 |
| | |
| | |
| | |
| | |
| | |
| | |

2. Make up your own $x + y = y + x$ expressions from your table.

3. Find some other a +b = x + y expressions in your table.

Activity 3: Multiplication Number Sentences

| 24 | | | |
|---|---|---|---|
| 6 | multiplied by | 4 | = 24 |
| 4 | multiplied by | 6 | = 24 |
| 8 | multiplied by | 3 | = 24 |
| 3 | multiplied by | 8 | =24 |
| 12 | multiplied by | 2 | = 24 |
| 2 | multiplied by | 12 | = 24 |
| 24 | multiplied by | 1 | = 24 |
| 1 | multiplied by | 24 | =24 |

1. If 6 x 4 = 24 and 4 × 6 = 24, we can say that $a \times b = b \times a$.
Make up your own $a \times b = b \times a$ expressions from this table.

2. We can see that 6 x 4 = 24 and 12 x 2 = 24, so 6 x 4 = 12 x 2.
We could write this as $a \times b = c \times d$.
Make up your own $a \times b = c \times d$ expressions from this table.

3. Now do the same process with 36. Find all the factors of 36.

| 36 | | | |
|---|---|---|---|
| 1 | multiplied by | | = 36 |
| 36 | multiplied by | | |
| 18 | multiplied by | | |
| 2 | multiplied by | | |
| 12 | multiplied by | | |
| 3 | multiplied by | | |
| 9 | multiplied by | | |
| 4 | multiplied by | | |
| 6 | multiplied by | | |

4. From the table above, make up your own expression of $a \times b = b \times a$.

5. Make up your own $a \times b = c \times d$ expressions from this table.

Note that numbers that are subtracted cannot have the order reversed, so subtraction is **NOT** commutative.

$$\mathbf{7 - 5 \neq 5 - 7}$$
$$\mathbf{x - y \neq y - x}$$

Similarly, numbers that are divided cannot have the order reversed, so division is **NOT** commutative.

$$\mathbf{10 \div 5 \neq 5 \div 10}$$
$$\mathbf{x \div y \neq y \div x}$$

Department of Education

# Year 7 Mathematics

## Number & Algebra Activity

# Maths by Association

Produced by a Department of Education - MAWA partnership project

Written for the Year 7 Australian Curriculum

# TASK 308: MATHS BY ASSOCIATION

## Overview

This task links the associative property to addition and multiplication and shows how this thinking is needed before students progress to the use of pronumerals in addition expressions such as ($a + b + c = a + c + b$) and in multiplication expressions such as ($a \text{ x } b \text{ x } c = a \text{ x } c \text{ x } b$). This task also focusses on the addition expression $a + b + c = x + y + z$ and the multiplication expression of $a \times b \times c = x \times y \times z$.

## Students will need

- calculators

## Relevant content descriptions from the Western Australian Curriculum

- Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

## Students can demonstrate

- *fluency* when they
  - calculate accurately
- *understanding* when they
  - connect the relationship in the associative property to algebra addition and multiplication expressions
  - recognise different ways of determining the answer
- *reasoning* when they
  - generalise from addition and multiplication tables
- *problem solving* when they
  - Explore all possibilities to give different solutions

Activity 1

An operation in mathematics is associative if a change in grouping does not change the result. This means the parenthesis (or brackets) can be moved, or do not have to be used.

| 20 | | |
|---|---|---|
| 17 | 2 | 1 |
| 16 | 3 | 1 |
| 15 | 3 | 2 |
| 14 | 4 | 2 |
| 12 | 5 | 3 |
| 10 | 6 | 4 |
| 8 | 7 | 5 |
| 8 | 6 | 6 |

Numbers that are added can be grouped in any order.
(8 + 5) + 7 = 8 + (5 + 7), so 13 + 7 = 8 + 12
This form could also be written in an algebraic equation as $(a + b) + c = a + (b + c)$.

Numbers can be written in any order, so it could be $(a + b) + c = c + (a + b)$.

1. From the table, create your own equations, using the associative property.

(10 + 6) + 4 = 10 + (6 +4); (15 + 3) + 2 = 15 + (3 + 2), and so on.
There are other combinations that could be used.

2. We could also create expressions such as 16 + 3 + 1 = 8 + 7 + 5.
From the table above, create your own *balanced number sentences.*

12 + 5 + 3 = 10 + 6 + 4; 17 + 2 + 1 = 8 + 6 + 6
There are many other combinations that could be used.

Activity 2

1. Try the same process with the number 36. You should be able to find at least 10 combinations to complete this table.

| 36 | | |
|---|---|---|
| 1 | 1 | 34 |
| 2 | 2 | 32 |
| 3 | 3 | 30 |
| 4 | 4 | 28 |
| 5 | 5 | 26 |
| 6 | 6 | 24 |
| 7 | 7 | 22 |
| 8 | 8 | 20 |
| 9 | 9 | 18 |

| | | |
|---|---|---|
| 10 | 10 | 16 |

There are many other combinations that could be made by extending the table and/or varying the addends; e.g., 11 + 12 + 13 = 36.

2. From the table, create your own equations of the *algebraic* form $(a + b) + c = a + (b + c)$ and of the form $(a + b) + c = c + (a + b)$.

(9 + 9) +18 = 9 + (9 + 18);   (7 + 7) + 22 = 22 + (7 + 7)

There are many other combinations that could be used.

3. We could also create equations such as 24 + 8 + 4 = 31 + 3 + 2.
From the table above, create your own *balanced number sentences.*

3 + 3 + 30 = 7 + 7 + 22. And using other combinations, 25 + 6 + 5 = 13 + 4 + 19.
There are many other combinations that could be used.

## Activity 3

Numbers that are multiplied can be grouped in any order.

| 60 | | | | | |
|---|---|---|---|---|---|
| 1 | multiplied by | 2 | multiplied by | 30 | = 60 |
| 2 | multiplied by | 2 | multiplied by | 15 | = 60 |
| 1 | multiplied by | 3 | multiplied by | 20 | = 60 |
| 2 | multiplied by | 2 | multiplied by | 15 | = 60 |
| 2 | multiplied by | 3 | multiplied by | 10 | = 60 |
| 1 | multiplied by | 5 | multiplied by | 12 | = 60 |
| 2 | multiplied by | 5 | multiplied by | 6 | = 60 |
| 4 | multiplied by | 5 | multiplied by | 3 | = 60 |

Note that (2 × 3) × 10 = 2 × (3 × 10), and this can be generalised algebraically as follows:
$(a \text{ x } b) \text{ x } c = a \text{ x } (b \text{ x } c)$

1. From the table, create your own number sentences of the *algebraic* form,
$(a \text{ x } b) \text{ x } c = a \text{ x } (b \text{ x } c)$.

(2 x 5) x 6 = 2 x (5 x 6),   (4 x 5) x 3 = 4 x (5 x 3)
There are many other combinations that could be used.

2. You could also create expressions such as 2 × 3 × 10 = 2 × 5 × 6.
From the table, create your own *balanced number sentences.*

4 x 5 x 3 = 2 x 5 x 6, 1x 3 x 20 = 2 x 3 x 10.
There are many other combinations that could be used.

Activity 4

1. Find different multiplication combinations for 100 and enter them in the table below.

| 100 | | | | | |
|---|---|---|---|---|---|
| 2 | multiplied by | 10 | multiplied by | 5 | = 100 |
| 4 | multiplied by | 5 | multiplied by | 5 | = 100 |
| 50 | multiplied by | 2 | multiplied by | 1 | = 100 |
| 25 | multiplied by | 2 | multiplied by | 2 | = 100 |
| 20 | multiplied by | 5 | multiplied by | 1 | = 100 |
| 40 | multiplied by | 2.5 | multiplied by | 1 | = 100 |
| 8 | multiplied by | 5 | multiplied by | 2.5 | = 100 |
| 100 | multiplied by | 1 | multiplied by | 1 | = 100 |

2. From the table, create your own number sentences of the *algebraic* form
($a$ x $b$) x $c$ = $a$ x ($b$ x $c$).

(2 x10) x 5 = 2 x (10 x 5), (20 x 1) x 5 = 20 x (1 x 5)

There are other combinations that could be used.

3. You could also create expressions such as $a$ x $b$ x $c$ = $f$ x $g$ x $h$.
From the table, make up your own *balanced number sentences.*

2 x 10 x 5 = 25 x 2 x 2; 8 x 5 x 2.5= 20 x 5 x 1
There are other combinations that could be used.

Note:

Numbers that are subtracted are NOT associative.
(24 – 8) – 3 ≠ 24 – (8 – 3)
$(x - y) - z \neq x - (y - z)$

Numbers that are divided are NOT associative.

(24 ÷ 8) ÷ 3 ≠ 24 ÷ (8 ÷ 3)
$(x \div y) \div z \neq x \div (y \div z)$

Activity 1

An operation in mathematics is associative if a change in grouping does not change the result. This means the parenthesis (or brackets) can be moved, or do not have to be used.

| 20 | | |
|---|---|---|
| 17 | 2 | 1 |
| 16 | 3 | 1 |
| 15 | 3 | 2 |
| 14 | 4 | 2 |
| 12 | 5 | 3 |
| 10 | 6 | 4 |
| 8 | 7 | 5 |
| 8 | 6 | 6 |

Numbers that are added can be grouped in any order.
(8 + 5) + 7 = 8 + (5 + 7), so 13 + 7 = 8 + 12
This form could also be written in an algebraic equation as $(a + b) + c = a + (b + c)$.

Numbers can be written in any order, so it could be $(a + b) + c = c + (a + b)$.

1. From the table, create your own equations, using the associative property.

2. We could also create expressions such as 16 + 3 + 1 = 8 + 7 + 5.
From the table above, create your own *balanced number sentences.*

Activity 2

1. Try the same process with the number 36. You should be able to find at least 10 combinations to complete this table.

| 36 | | |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

2. From the table, create your own equations of the *algebraic* form $(a + b) + c = a + (b + c)$ and of the form $(a + b) + c = c + (a + b)$.

3. We could also create equations such as 24 + 8 + 4 = 31 + 3 + 2.
From the table above, create your own *balanced number sentences.*

Activity 3

Numbers that are multiplied can be grouped in any order.

| 60 | | | | | |
|---|---|---|---|---|---|
| 1 | multiplied by | 2 | multiplied by | 30 | = 60 |
| 2 | multiplied by | 2 | multiplied by | 15 | = 60 |
| 1 | multiplied by | 3 | multiplied by | 20 | = 60 |
| 2 | multiplied by | 2 | multiplied by | 15 | = 60 |
| 2 | multiplied by | 3 | multiplied by | 10 | = 60 |
| 1 | multiplied by | 5 | multiplied by | 12 | = 60 |
| 2 | multiplied by | 5 | multiplied by | 6 | = 60 |
| 4 | multiplied by | 5 | multiplied by | 3 | = 60 |

Note that (2 × 3) × 10 = 2 × (3 × 10), and this can be generalised algebraically as follows:
($a$ x $b$) x $c$ = $a$ x ($b$ x $c$)

1. From the table, create your own number sentences of the *algebraic* form,
($a$ x $b$) x $c$ = $a$ x ($b$ x $c$).

2. You could also create expressions such as 2 × 3 × 10 = 2 × 5 × 6.
From the table, create your own *balanced number sentences.*

Activity 4

1. Find different multiplication combinations for 100 and enter them in the table below.

| 100 | | | | | |
|---|---|---|---|---|---|
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |
| | multiplied by | | multiplied by | | = 100 |

2. From the table, create your own number sentences of the *algebraic* form
($a$ x $b$) x $c$ = $a$ x ($b$ x $c$).

3. You could also create expressions such as $a$ x $b$ x $c$ = $f$ x $g$ x $h$.
From the table, make up your own *balanced number sentences.*

Note:

Numbers that are subtracted are NOT associative.
(24 – 8) – 3 ≠ 24 – (8 – 3)
$(x - y) - z \neq x - (y - z)$

Numbers that are divided are NOT associative.

(24 ÷ 8) ÷ 3 ≠ 24 ÷ (8 ÷ 3)
$(x \div y) \div z \neq x \div (y \div z)$