



Department of  
**Education**

# AUSTRALIAN CURRICULUM

## MATHEMATICS YEAR 7

### Comparisons (Ratios)



# **MATHEMATICS**

## **YEAR 7**

### **Comparisons (Ratios)**

**Student's name:** \_\_\_\_\_

**Teacher's name:** \_\_\_\_\_

First published 2012

ISBN 9780730744399

SCIS 1564080

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Department of  
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# Signposts

Each symbol is a sign to help you.

Here is what each one means:



The recommended time you should take to complete this section.



An explanation of key terms, concepts or processes.



A written response.  
Write your answer or response in your journal.



Correct this task using the answers at the end of the resource.



Calculators may not be used here.



Make notes describing how you attempted to solve the problem. Keep these notes to refer to when completing the Self-evaluation task. Your teacher may wish you to forward these notes.

# Introduction

This resource should take you approximately two weeks to complete. It comprises seven learning sections, a summary section and a review task section.

The learning sections have the following headings:

- **Key words**  
These are the main words that you need to understand and use fluently to explain your thinking.
- **Warm-up**  
Warm-up tasks should take you no longer than 10 minutes to complete. These are skills from previous work you are expected to recall from memory, or mental calculations that you are expected to perform quickly and accurately. If you have any difficulties in answering these questions, please discuss them with your teacher.
- **Review**  
Some sections have reviews immediately after the warm-up. The skills in these reviews are from previous work and are essential for that section. You will use these to develop new skills in mathematics. Please speak to your teacher immediately if you are having any trouble in completing these activities.
- **Focus problem**  
Focus problems are designed to introduce new concepts. They provide examples of the types of problems you will be able to solve by learning the new concepts in this resource. Do not spend too long on these but do check and read the solutions thoroughly.
- **Skills development**  
These help you consolidate new work and concepts. Most sections include skills development activities which provide opportunities for you to become skilled at using new procedures, apply your learning to solve problems and justify your ideas. Please mark your work after completing each part.

## Correcting your work

Please mark and correct your work as you go. Worked solutions are provided to show how you should set out your work. If you are having any difficulty in understanding them, or are getting the majority of the questions wrong, please speak to your teacher immediately.

## Journal

Please keep an exercise book to record your notes and to summarise your learning. At the end of each section, write definitions for the key words that were introduced for that section.

# Curriculum details

## Content Descriptions

This resource provides learning and teaching to deliver the Australian Curriculum: Mathematics for the following Year 7 Content Descriptions.

Recognise and solve problems involving simple ratios (ACMNA173)

Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)

Content descriptions	1	2	3	4	5	6	7	R
ACMNA173								
ACMNA174								
ACMNA154								

 Indicates the content description is explicitly covered in that section of the resource.

## Previous relevant Content Descriptions

The following Content Descriptions should be considered as prior learning for students using this resource.

### At Year 6 level

Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)

Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies (ACMNA129)

Multiply and divide decimals by powers of 10 (ACMNA130)

Make connections between equivalent fractions, decimals and percentages (ACMNA131)

## Proficiency strand statements at Year 7 level

*At this year level:*

*Understanding* includes describing patterns in uses of indices with whole numbers, recognising equivalences between fractions, decimals, percentages and ratios, plotting points on the Cartesian plane, identifying angles formed by a transversal crossing a pair of lines, and connecting the laws and properties of numbers to algebraic terms and expressions

*Fluency* includes calculating accurately with integers, representing fractions and decimals in various ways, investigating best buys, finding measures of central tendency and calculating areas of shapes and volumes of prisms

*Problem Solving* includes formulating and solving authentic problems using numbers and measurements, working with transformations and identifying symmetry, calculating angles and interpreting sets of data collected through chance experiments

*Reasoning* includes applying the number laws to calculations, applying known geometric facts to draw conclusions about shapes, applying an understanding of ratio and interpreting data displays

## General capabilities

General capabilities	1	2	3	4	5	6	7	R
Literacy								
Numeracy								
Information and communication technology (ICT) capability								
Critical and creative thinking								
Personal and social capability								
Ethical behaviour								
Intercultural understanding								



Indicates General capabilities are explicitly covered in that section of the resource.

## Cross-curriculum priorities

Cross-curriculum priorities	1	2	3	4	5	6	7	R
Aboriginal and Torres Strait Islander histories and cultures								
Asia and Australia's engagement with Asia								
Sustainability								



Indicates Cross-curriculum priorities are explicitly covered in that section of the resource.

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# 1. Comparisons (ratios)

When you complete this section you should be able to:

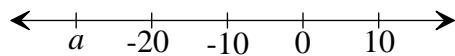
- recognise and write ratios.

## Key words

- ratio

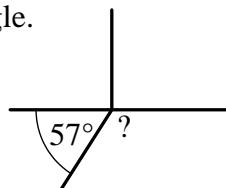
## Warm-up 1

1. Is 7 a factor of 21?
2.  $7 + 8 =$  \_\_\_\_\_
3. What is the missing number?



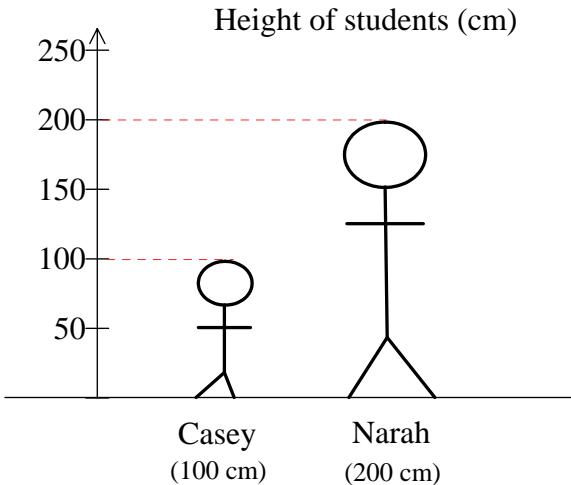
$$a = \underline{\hspace{2cm}}$$

4. Circle the greater fraction.  $\frac{3}{4}$  or  $\frac{1}{2}$
5. What is a half of 20?
6.  $5.3 + 5.3 =$  \_\_\_\_\_
7.  $4.5 \times 4 =$  \_\_\_\_\_
8. Write 0.125 as a fraction.
9. Complete: 7, 14, 21, \_\_\_\_\_
10. Determine the size of the missing angle.



## Focus problem 1

Narah and Casey were comparing their heights.



Casey suggested that she was 100 centimetres smaller than Narah.

Narah suggested that he was twice the size of Casey.

In what other ways can they compare their heights?



Check your work before continuing.

### Ratios everywhere...

Ratios are everywhere. In fact, you have probably used them many times.

Have you ever ordered some chips, been driven in a car or played a game of footy? If so, then you have used them!

The following is a short list of uses in the workforce.

- farmers – determining the amount of fertilizer to use on a crop
- nurses – medication dosages for patients
- bricklayers – determining the correct mix for the mortar
- mechanic – establishing correct fluid mixes
- welder – determining correct filler metals, power and speed for different welds
- accountants – checking the health of business (cash-flow rates, etc).

## Skills development 1

A **ratio** is a comparison of two (or more) amounts.

### Example

Write the following as a ratio.

A 42-kilogram person needs 7 millilitres of medicine.

### Solution

42:7



Note that in ratios units are not used.  
Also note that the order of the numbers is important, with 42:7 being very different to 7:42.

Note that the symbol ‘:’ is read as ‘to’. That is, the ratio from the example is read as ‘42 to 7’.

1. Write each of the following situations as a ratio.

(a) A packet of lollies contains 24 red lollies and 32 blue lollies.

24: \_\_\_\_\_

(b) A breakfast cereal contains 42 grams of sugar per 100 grams.

\_\_\_\_\_ : \_\_\_\_\_

(c) A cordial mix uses 1 part cordial to 4 parts water.

\_\_\_\_\_

2. The following shows three trucks and two cars using a stretch of the road.



For each of the following, complete the ratio.

(a) trucks to cars \_\_\_\_\_ : \_\_\_\_\_

(b) cars to trucks \_\_\_\_\_ : \_\_\_\_\_

(c) cars to vehicles \_\_\_\_\_ : \_\_\_\_\_

(d) trucks to vehicles \_\_\_\_\_ : \_\_\_\_\_

3. A bicycle has two sets of gears connected by a chain.

The front gear has 30 teeth and the rear gear has 20 teeth.



- (a) Write the **ratio**, comparing the front gear to the rear gear.

- (b) What would be the ratio if the front gear was changed to 10 teeth?

4. Complete the following.

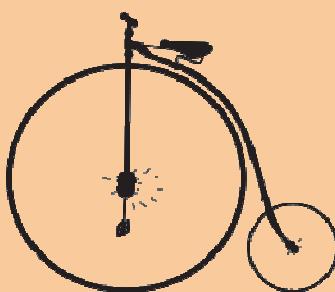
When working with ratios, only m \_\_\_\_\_ and d \_\_\_\_\_ are used.



Check your work before continuing.

### The Penny-farthing

What a weird bike!



The high wheel bicycle, or penny-farthing, replaced the ‘walking machine’ and the velocipede (or boneshaker) as the mode of transport for the young men of the times. (They were expensive, costing an average worker six-month’s pay!)

The pedals were attached to the large front wheel. One rotation of the pedal meant the large wheel would also rotate once. The larger the wheel, the further you could go using the one rotation.

Of course, they were dangerous and soon ‘safety bikes’ were invented that used gears and a chain, much like the bikes we have today.

## 2. More comparisons

When you complete this section you should be able to:

- identify the processes for division as the inverse of multiplication.

### Key words

- inverse operation

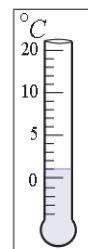
### Warm-up 2

1. Is 7 a common factor of 14 and 21? \_\_\_\_\_

2.  $18 - 9 =$  \_\_\_\_\_

3. The temperature was 1 degrees but it dropped 6 degrees.

What is the new temperature? \_\_\_\_\_



4. Insert <, > or = to make the following sentence true.

$$\frac{3}{5} \square \frac{5}{10}$$

5.  $\frac{1}{2} \times 48 =$  \_\_\_\_\_

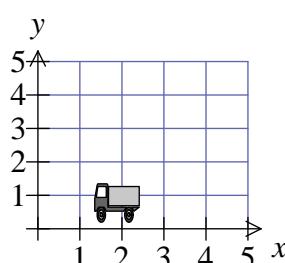
6. Round 5.7 to a whole number. \_\_\_\_\_

7.  $5 \overline{) 20.5}$

8. Write 25% as a decimal. \_\_\_\_\_

9. Complete: 80.6, 81.1, 81.6, \_\_\_\_\_

10.



The truck is at (2, 1).

If the truck moves 2 units up,  
where will it then be?

\_\_\_\_\_

## Focus problem 2

To calculate the potential speed of his go-cart, Aaron had to divide the gear number by 0.2. Unfortunately, he did not have a calculator.

As he was struggling with the calculation, his friend said that he should just multiply the gear number by 5.

But is Aaron's friend correct?



Check your work before continuing.

### Gears

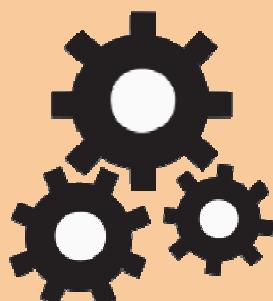
A gear is a toothed wheel. When combined with another gear it can transfer power and/or speed.

Gears have been around for over 4500 years, with evidence suggesting that both China and India used them for power generation.

A water-wheel mill (which uses the concept of gears), dating back to the first half of the 3<sup>rd</sup> century, is also known to exist in Turkey.

In the 18<sup>th</sup> Century, the Industrial Revolution saw the rise of metal gears, which in turn saw the development of accurate clocks, modern-day bicycles, automobiles, power generators and many other mechanical devices we now take for granted.

Three cheers for the gears!



## Skills development 2.1

### Example

A ratio is given as follows.

4:20

Show how you can start with 4 and obtain the result of 20.

### Solution

$$4 \times 5 = 20$$



But how did you know it was 5?



You can use the inverse operation to find it.

For example,  $20 \div 4 = 5$ .

So  $4 \times 5 = 20$ .

Note that you can also divide 4 by 20, which gives you 0.2.

That is,  $4 \div 0.2 = 20$ .

1. Show how you can obtain the second number given the first.

(a)  $5 \Rightarrow 20$

(i) multiplication: \_\_\_\_\_ (ii) division: \_\_\_\_\_

(b)  $20 \Rightarrow 5$

(i) multiplication: \_\_\_\_\_ (ii) division: \_\_\_\_\_

2. Match the equivalent operations (that is, the operations that give the same result).

$\times 2$

$\div 3$

$\times 6$

$\times 0.1$

$\div 4$

$\div 0.5$

$\times \frac{1}{3}$

$\div \frac{1}{6}$

$\div 10$

$\times 0.25$

## Skills development 2.2

Another way to understand the use of multiplication and division with ratios, is by looking at them through tables.

### Example

Complete the table to show the correct cost of tickets for the number of people.

<i>Number of people</i>	1	4	10	2	5		6
<i>Cost of tickets (\$)</i>	25	100	250			200	

### Solution

<i>Number of people</i>	1	4	10	2	5	8	6
<i>Cost of tickets (\$)</i>	25	100	250	50	125	200	150

$\times 2$

Of course, you will need to find all the other patterns to complete the table.

1. A worming medication for puppies suggests 2 millilitres per kilogram.

Complete the table to show the correct amount of medication for the weight.

<i>Puppy's weight (kg)</i>	1	2	5	12		
<i>Amount of medication (mL)</i>	2	4			6	20

2. A bag of mortar mix will be needed to lay approximately 30 bricks.

Complete the table to show the correct number of bags for the number of bricks.

<i>Number of bags of mortar mix</i>	1	2	5		8	
<i>Number of bricks</i>	30			90		300

3. Complete the following tables.

(a)

<i>Time</i> (hours)	1	2	3	4	5	10
<i>Pay</i> (\$)					80	

(b)

<i>Speed</i> (km/h)	1	2	3	4	5	10
<i>Distance</i> (km)					32	

You may have noticed that km/h is used in the table above. Note, the forward slash reads as ‘per’.



Check your work before continuing.



### 3. Simplifying ratios

When you complete this section you should be able to:

- simplify ratios.

#### Key words

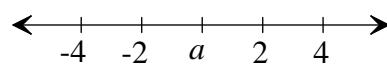
- common factor

#### Warm-up 3

1. Circle the prime number. 1, 3, 6, 10

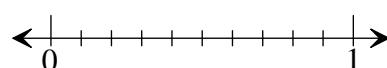
2.  $8 \times 5 =$  \_\_\_\_\_

3. What is the missing number?



$a =$  \_\_\_\_\_

4. Locate  $\frac{6}{10}$  on the number line.



5. What is a third of 24? \_\_\_\_\_

6. Estimate the sum by first rounding to whole numbers.

$9.9 + 8.6 \approx$  \_\_\_\_\_

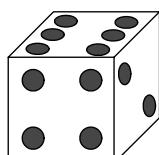
7. 
$$\begin{array}{r} 2.5 \\ \times \quad 7 \\ \hline \end{array}$$

8. Write  $\frac{2}{10}$  as a percentage. \_\_\_\_\_

9. Complete:  $1\frac{1}{5}, 2\frac{2}{5}, 3\frac{3}{5},$  \_\_\_\_\_

10.

A six-sided die is rolled.



Express, as a decimal, the probability that it lands on a number greater than 3.

\_\_\_\_\_

## Review 3.1

A factor (or divisor) is a number that divides exactly into another number.

### Example

List the factors of 12.



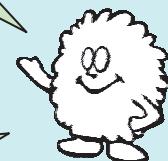
Hmm, what multiplies to give 12?

### Solution

1, 2, 3, 4, 6, and 12 are all factors of 12.

That's easy.  
It's just the  
times tables!

$$\begin{aligned}1 \times 12 &= 12 \\2 \times 6 &= 12 \\3 \times 4 &= 12\end{aligned}$$



- Determine the factors for each of the following.

(a) 20

$$1 \times \underline{\hspace{2cm}} = 20$$

$$2 \times \underline{\hspace{2cm}} = 20$$

$$4 \times \underline{\hspace{2cm}} = 20$$

∴ The factors are 1, 2, 4,       ,       ,       .

(b) 8

$$\underline{\hspace{2cm}} = 8$$

$$\underline{\hspace{2cm}} = 8$$

∴ The factors are       ,       ,       ,       .

(c) 24

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

(d) 13

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}}$$

## Review 3.2

A **common factor** is a number that divides exactly into two or more numbers.

### Example

Determine the common factor(s) for 12 and 16.

### Solution

The factors for 12 are 1, 2, 3, 4, 6 and 12.

The factors for 16 are 1, 2, 4, 8 and 16.

∴ Common factors are 1, 2 and 4.

1. Determine the common factors for each of the following pairs.

- (a) 4 and 10

The factors of 4 are: 1, 2, 4.

The factors of 10 are: 1, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

∴ The common factors are: 1 and \_\_\_\_\_.

- (b) 8 and 12

The factors of 8 are: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

The factors of 12 are: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

∴ The common factors are: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

- (c) 9 and 15

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- (d) 13 and 16

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Check your work before continuing.

## Focus problem 3

Yadra wanted to buy some material for a dress.

She saw the material advertised in two separate shops for:

- (i) \$15 per metre.
- (ii) \$60 for 4 metres.

Which one of the two shops offers the cheaper material?



Check your work before continuing.

### Silky silk

Silk is a soft, smooth fabric. It is believed to have originated in China and India more than 4500 years ago. Silk is considered to be one of the finest fabrics in the world.

In the northeast of Thailand, weavers raise their silk worms on a steady diet of mulberry leaves. The silk worms form cocoons from which the silk is then gathered.



## Skills development 3.1

### Example

The following ratios are equivalent  $1 : 4$  and  $3 : 12$ .

Complete the following showing the relationship between the amount of material and the cost.

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline 1 : 4 \\ \times 3 \quad \rightarrow \quad 3 : \quad 12 \quad \leftarrow \times \quad \end{array}$$

### Solution

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline 1 : 4 \\ \times 3 \quad \rightarrow \quad 3 : \quad 12 \quad \leftarrow \times 3 \end{array}$$

1. Complete each of the following showing the appropriate relationship.

(a)

$$\begin{array}{c} \text{Weight} : \text{Cooking time} \\ (\text{kg}) : (\text{min}) \\ \hline 1 : 60 \\ \times 5 \quad \rightarrow \quad 5 : \quad 12 \quad \leftarrow \times \quad \end{array}$$

(b)

$$\begin{array}{c} \text{Goals} : \text{Points} \\ \hline 1 : 6 \\ \times 6 \quad \downarrow \\ 1 : \quad 6 \\ 5 : \quad 30 \\ \uparrow \quad \times \quad \end{array}$$

(c)

$$\begin{array}{c} \text{Speed} : \text{Distance} \\ (\text{km/h}) : (\text{km}) \\ \hline 80 : 24 \\ \div 4 \quad \rightarrow \quad 20 : \quad \leftarrow \div \quad \end{array}$$

## Skills development 3.2

### Example

The following two ratios are equivalent.

Determine the missing value.

$$\begin{array}{c} \text{Cordial : Water} \\[1ex] 2 : 5 \\[1ex] 7 : \underline{\hspace{2cm}} \end{array}$$

### Solution

There are various ways of solving the problem. The following is one such method.

$$\begin{array}{c} \text{Cordial : Water} \\[1ex] \times ? \rightarrow 2 : 5 \quad \leftarrow \times ? \\[1ex] 7 : \underline{\hspace{2cm}} \end{array}$$

To find the value on which you need to multiply by, work in the opposite direction and use a division.

$$7 \div 2 = 3.5$$

So you need to multiply by 3.5.

$$\begin{array}{c} \text{Cordial : Water} \\[1ex] \times 3.5 \rightarrow 2 : 5 \quad \leftarrow \times 3.5 \\[1ex] 7 : \underline{\hspace{2cm}} \end{array}$$

$$5 \times 3.5 = 17.5$$

Therefore, the missing value is 17.5.

1. Complete each of the following showing the appropriate relationship.

(a)  $\begin{array}{c} \text{Cereal : Sodium} \\[1ex] (\text{per serving}) : (\text{mg}) \\[1ex] 1 : 30 \\[1ex] 4 : \underline{\hspace{2cm}} \end{array}$

(b)  $\begin{array}{c} \text{Hot chocolate : Sugars (g)} \\[1ex] (\text{per teaspoon}) \\[1ex] 2 : \underline{\hspace{2cm}} \\[1ex] 5 : 30 \end{array}$

## Skills development 3.3

To simplify a ratio, look for the greatest **common factor** and divide through by it.

### Example

Simplify the ratio 18 : 6.

**Solution**

(a) 18 : 6

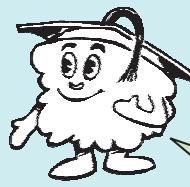
Factors of 18 are: 1, 2, 3, 6, 9, 18

Factors of 6 are: 1, 2, 3, 6.

So '6' is the greatest common factor.

$$\begin{array}{r} 18 \quad : \quad 6 \\ \div 6 \quad \rightarrow \quad 3 \quad : \quad 1 \quad \leftarrow \div 6 \end{array}$$

∴ The ratio becomes: 3 : 1.



Note that you can also get to the same result by using the other common factors. It just takes longer.

For example, you could divide by 3 and then by 2 to get the same result.

1. Simplify the following ratios.

(a) 4 : 10    The factors of 4 are: 1, 2, and 4.

The factors of 10 are: 1, \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_.

$$\begin{array}{r} 4 \quad : \quad 10 \\ \div 2 \quad \rightarrow \quad 2 \quad : \quad \text{_____} \quad \leftarrow \div \text{_____} \end{array}$$

∴ The ratio becomes 2 : \_\_\_\_\_.

(b) 6 : 9    The factors of 6 are: \_\_\_\_\_

The factors of 9 are: \_\_\_\_\_

$$\begin{array}{r} 6 \quad : \quad 9 \\ \div \text{_____} \quad \rightarrow \quad 1 \quad : \quad \text{_____} \quad \leftarrow \div \text{_____} \end{array}$$

∴ The ratio becomes: \_\_\_\_\_ : \_\_\_\_\_.

2. Simplify the following ratios (if possible).

(a)  $15 : 25$

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(b)  $7 : 20$

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3. Who wrote the latest breakfast cookbook?

Match the equivalent ratios to find the answer to the problem.

(Write the letter representing the equivalent ratio underneath its Roman numeral).

(I)  $1 : 2$

(o)  $15 : 9$

(II)  $1 : 3$

(c)  $2 : 6$

(III)  $1 : 4$

(c)  $4 : 10$

(IV)  $5 : 3$

(h)  $6 : 9$

(V)  $2 : 1$

(n)  $8 : 2$

(VI)  $2 : 3$

(i)  $6 : 2$

(VII)  $2 : 5$

(r)  $5 : 10$

(VIII)  $3 : 1$

(b)  $6 : 8$

(IX)  $3 : 2$

(s)  $10 : 5$

(X)  $3 : 4$

(a)  $9 : 6$

(XI)  $4 : 1$

(p)  $2 : 8$

VII	VI	I	VIII	V

III

X	IX	II	IV	XI



Check your work before continuing.

## 4. Converting

When you complete this section you should be able to:

- convert ratios to fractions, decimals and percentages.

### Warm-up 4

1. Complete the pattern. 49, 36, 25, 16, \_\_\_\_\_

2.  $63 \div 7 =$  \_\_\_\_\_

3. The temperature was minus 3 degrees but it went up 6 degrees.

What is the new temperature? \_\_\_\_\_

4. Express the value of  $w$  as a fraction.

5.  $\frac{1}{4} \times 36 =$  \_\_\_\_\_

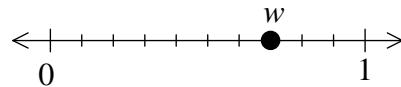
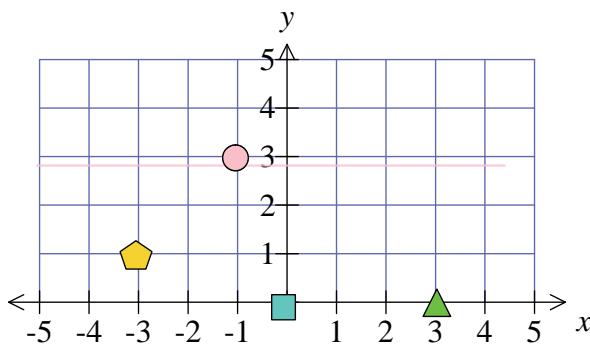
6.  $8.06 \times 100 =$  \_\_\_\_\_

7.  $12.06 \div 6 =$  \_\_\_\_\_

8. Write 27% as fraction.

9. Complete: 61, 67, 73, \_\_\_\_\_

10.



Which shape is at (-3, 1)?

\_\_\_\_\_

## Focus problem 4

A competition on the side of a chocolate bar wrapper suggests that 1 out of 5 are ‘instant winners’.

As a ratio, this can be written as 1 to 5.

But what percentage of wrappers are winners?



Check your work before continuing.

### Chocolate craziness!

The word ‘chocolate’ is derived from the Aztec word, ‘cacahuatl’, which means ‘bitter water’. Chocolate is extracted from the beans of the cocoa tree, scientifically known as the ‘Theobroma Cacao’. Yes, the spelling is different!

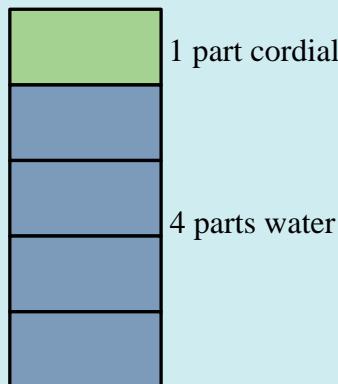
The tree grows a huge cacao pod approximately 15-30 cm long by 8-12 cm wide, which contains around 30 beans. It takes around 25 of these pods to get 1 kilogram of cocoa.

The beans are roasted and the centre is removed where it is then ground up into a paste called chocolate liquor. Milk and sugar are added to this paste to make chocolate.

## Skills development 4.1

Fractions can be derived from ratios.

For example, the following shows a cordial mix in the ratio of 1 part cordial to 4 parts water, or  $1 : 4$ .



From this situation, the following statements can be made.

- There is one-quarter ( $\frac{1}{4}$ ) the amount of cordial than there is water.
- The cordial makes up one-fifth ( $\frac{1}{5}$ ) of the cordial mix.
- There is four ( $\frac{4}{1}$ ) times as much water as there is cordial in the cordial mix.
- The water makes up four-fifths ( $\frac{4}{5}$ ) of the cordial mix.

Where did the 5 come from? I thought that the ratio was  $1 : 4$ ?



That's because  $1 + 4 = 5$ . That is, there are 5 parts altogether.



**Example**

The water to cement ratio, calculated by weight, for a particular pre-mix concrete is 2 : 5.

Determine each of the following.

- The weight of the water as a fraction of the weight of cement.
- The weight of the cement as a fraction of the weight of water.
- The weight of the water as a fraction of the total weight of the mix.
- The weight of the cement as a fraction of the total weight of the mix.

**Solution**

(a)  $\frac{2}{5}$

(b)  $\frac{5}{2}$  or  $2\frac{1}{2}$

(c)  $\frac{2}{7}$

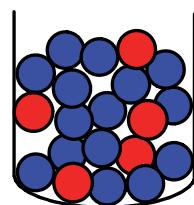
(d)  $\frac{5}{7}$



I guess it is important to understand both what the numbers represent and what is required.

- A container of candies has red and blue candies in the ratio of 1 : 3. Complete the following statements by adding the appropriate fraction. (You do not need to simplify the fraction.)

- There are \_\_\_\_\_ as many red candies as there are blue.
- There are \_\_\_\_\_ times as many blue candies as there are red.
- The red candies make up \_\_\_\_\_ of the total candies.
- The blue candies make up \_\_\_\_\_ of the total candies.



- The aspect ratio for an old television is 4 : 3. This ratio compares the width of the television screen to its height. Complete the following statements by adding the appropriate fraction. (You do not need to simplify the fraction.)

- The height of the television screen is \_\_\_\_\_ its width.
- The width of the television screen is \_\_\_\_\_ its height.

## Skills development 4.2

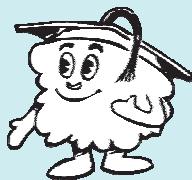
Fractions, decimals and percentages can be converted from one form to another. These conversions can be useful in calculations.

### Fractions to decimals

To convert a fraction into a decimal, divide the fraction out.

#### Example

Convert  $\frac{4}{10}$  into a decimal.



$\frac{4}{10}$  means the same as  $4 \div 10$ .

#### Solution

$$\frac{4}{10} = 0.4$$

You can use your calculator to find the answer.

$\therefore$  The fraction  $\frac{4}{10}$  is equivalent to 0.4.

1. Complete each of the following and convert the fractions into decimals.

(a)  $\frac{4}{5}$

$4 \div 5 =$  \_\_\_\_\_

$\therefore$  The fraction is equivalent to \_\_\_\_\_.

(b)  $\frac{4}{8}$

\_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_

$\therefore$  The fraction is equivalent to \_\_\_\_\_.

(c)  $\frac{8}{9}$

---



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(d)  $\frac{12}{3}$

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## Skills development 4.3

### Decimals to fractions

Decimals are often referred to as decimal fractions. This is because they are just a special way of writing fractions that have denominators that are 10, 100, 1000 etc.

#### Example

Convert the following decimals into fractions. (You do not need to simplify the fraction.)

(a) 0.7

(b) 0.102

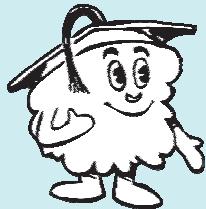
(c) 32.9

#### Solution

(a)  $0.7 = \frac{7}{10}$

(b)  $0.102 = \frac{102}{1000}$

(c)  $32.9 = \frac{329}{10}$



The number of zeros in the denominator is the same as the number of decimal places in the decimal.

1. Convert the following decimals into fractions.

(a)  $0.6 =$  \_\_\_\_\_

(b)  $0.35 =$  \_\_\_\_\_

(c)  $2.005 =$  \_\_\_\_\_

## Skills development 4.4

### Percentages

A percentage is a special type of ratio. It is a comparison with the number 100. For example, if a student correctly answered 15 out of 20 questions, the percentage can be found as follows.

$$\begin{array}{rcl} \times 5 & 15 & \text{out of } 20 \\ \Rightarrow & ? & \text{out of } 100 \\ & & \leftarrow \times 5 \end{array}$$

$$15 \times 5 = 75$$

$\therefore$  The student correctly answered 75% of the questions.

Can you see how the percentage sign (%) looks like 100, with the '1' moved between the zeros and tilted?



Note how the problem was set up and solved like a ratio question. There are other ways to solve it but the emphasis of this book is on ratios.



Percentages are not the focus for this module, but if you are interested in them or you need help in understanding them, you should speak to your teacher.

### Decimals to percentages

To convert a decimal into a percentage, multiply the decimal by 100.

#### Example

Convert the following decimals into percentages.

(a) 0.7

(b) 0.835

(c) 2.05

#### Solution

(a)  $0.7 \times 100 = 70$

(b)  $0.835 \times 100 = 83.5$

(c)  $2.05 \times 100 = 205$

$\therefore 0.7 = 70\%$

$\therefore 0.835 = 83.5\%$

$\therefore 2.05 = 205\%$

- Convert the following decimals to percentages.

(a) 0.67

(b) 0.05

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(c) 0.394

(d) 3.1

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## **Skills development 4.5**

## Percentages to decimals

To convert a percentage into a decimal, divide by 100.

## Example

Convert the following decimals into percentages.



## Solution

$$(a) \quad 27 \div 100 = 0.27$$

$$(b) \quad 309 \div 100 = 3.09$$

$$(c) \quad 0.5 \div 100 = 0.005$$

$$\therefore 27\% = 0.27$$

$$\therefore 309\% = 3.09$$

$$\therefore 0.5\% = 0.005$$

1. Convert the following percentages into decimals.

- (a) 4%

- (b) 21%

- (c) 1.08%

- (d) 251%

## Skills development 4.6

### Fractions into percentages

There are different ways to convert fractions into percentages. For example, you could use equivalent fractions or set up a ratio-like problem. However, the quickest and easiest method is to convert the fraction into a decimal and then convert that into a percentage.

#### Example

Convert the following fractions into percentages.

(a)  $\frac{7}{10}$

(b)  $\frac{3}{5}$

(c)  $\frac{7}{4}$

#### Solution

(a)  $7 \div 10 = 0.7$

(b)  $3 \div 5 = 0.6$

(c)  $7 \div 4 = 1.75$

$0.7 \times 100 = 70$

$0.6 \times 100 = 60$

$1.75 \times 100 = 175$

$\therefore \frac{7}{10} = 70\%$

$\therefore \frac{3}{5} = 60\%$

$\therefore 1.75 = 175\%$

- Convert the following fractions into percentages.

(a)  $\frac{6}{10}$

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(b)  $\frac{3}{4}$

---

---

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(c)  $\frac{25}{8}$

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## **Skills development 4.7**

## Percentages into fractions

To convert a percentage into a fraction, you could use equivalent fractions, set up a ratio-like problem or convert it into a decimal and then convert that into a fraction. If the percentage is a whole number, the quickest and easiest method is to remove the percentage sign and replace it with a denominator of 100. (If it is not, you will need to use one of the other methods.)

## Example

Convert the following fractions into percentages.



## Solution

- (a)  $\frac{70}{100}$       (b)  $\frac{4}{100}$       (c)  $\frac{205}{100}$

1. Convert these fractions into percentages into fractions.

- (a)  $63\% =$  \_\_\_\_\_      (b)  $5\% =$  \_\_\_\_\_      (c)  $300\% =$  \_\_\_\_\_

## **Skills development 4.8**

1. Use the following fractions, percentages and decimals to complete the table below.

$$150\% \quad 0.4 \quad \frac{1}{2} \quad 0.333... \quad 50\% \quad \frac{4}{1} \quad 4 \quad 0.6 \quad \frac{3}{2}$$

Fraction	Decimal	Percentage
$\frac{2}{5}$		40%
$\frac{1}{3}$		33.333... %
	1.5	
		400%
$\frac{3}{5}$		60%
	0.5	

2. What has four wheels and flies?

Match the equivalent fractions, decimals and percentages.

Then place the letter that matches into the appropriate box below.

I.      0.5

(a)       $\frac{1}{4}$

II.       $\frac{3}{5}$

(r)      100%

III.      25%

(b)      0.333...

IV.      5

(a)      2

V.       $\frac{1}{3}$

(k)      66.666... %

VI.      200%

(e)       $\frac{3}{2}$

VII.       $\frac{5}{2}$

(a)       $\frac{1}{2}$

VIII.      1.5

(u)      75%

IX.      0.8

(g)      2.5

X.      1

(c)      0.4

XI.      0.75

(g)      60%

XII.      40%

(t)       $\frac{4}{5}$

XIII.       $\frac{2}{3}$

(r)      500%

I

II	III	IV	V	VI	VII	VIII

IX	X	XI	XII	XIII



Check your work before continuing.



# 5. Solving rate problems

When you complete this section you should be able to:

- solve rate problems.

## Keywords

- rate

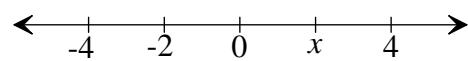
## Warm-up 5

1. What is the greatest common factor of 6 and 15?

2.  $18 + 14 =$  \_\_\_\_\_

3. What is the missing number?

$x =$  \_\_\_\_\_



4.  $\frac{1}{4} + \frac{3}{4} =$  \_\_\_\_\_

5. What is a tenth of 60? \_\_\_\_\_

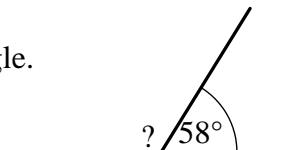
6.  $6400 \text{ m} =$  \_\_\_\_\_ km

7.  $8 + 8 \times 5 =$  \_\_\_\_\_

8. Write  $\frac{3}{4}$  as decimal. \_\_\_\_\_

9. Complete: 6, 5.5, 5, \_\_\_\_\_

10. Determine the size of the missing angle.



## Focus problem 5

Jenny is driving in her car on the Forest Highway and is travelling at a speed of 110 km/h.

How far will she travel in 3 hours?



Check your work before continuing.

### To speed or not to speed?

The first maximum speed limit was introduced in the 1860s, in the United Kingdom. It was 10 miles per hour (or 16 km/h). In the United Arab Emirates, they once posted a maximum speed limit of 160 km/h, however, this was later reduced to 140 km/h.



On certain parts of Germany's autobahns there is still no speed limit.

But what speed is safe and sustainable?

In the 1970s, when oil became too expensive, governments around the globe reduced the speed limit to save on fuel.

The leading cause of death amongst children between 10 – 19 years of age is road accidents, with speed being the main contributor.

## Skills development 5.1

A **rate** is a particular kind of ratio which involves two quantities with different units.

**Example**

Simon is paid at a rate of \$14 per hour. How much will he earn in 8 hours?

**Solution**

$$14 \times 8 = 112$$

$\therefore$  Simon will earn \$112.



Because you are trying to find the first part of the rate, multiplication is used.

- Determine the amount earned for each of the following rates and hours worked.

- \$20 per hour for 5 hours.

$$20 \times 5 = \underline{\hspace{2cm}}$$

$\therefore$  The amount earned is                 .

- \$15.95/h for 8 hours.

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- 7 hours work at \$22 per hour.

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- A car is travelling at 80 km/h. How far will it travel in 1.5 hours?

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- Jaiden's heart beats 72 beats per minute. How many beats will it do in one hour?

Hint: 1 hour = 60 minutes.

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Check your work before continuing.

## Skills development 5.2

### Example

Sharon is paid \$14 per hour.

How long will she need to work to earn 84 dollars?

### Solution

$$84 \div 14 = 6$$

$\therefore$  Sharon will need to work 6 hours.



Because you are trying to find the second part of the rate, a division is used.

- Determine the time it will take to earn each of the amounts given the following **rates**.

- \$150 given a pay rate of \$10 per hour

$$150 \div 10 = \underline{\hspace{2cm}}$$

$\therefore$  It will take  $\underline{\hspace{2cm}}$  hours.

- \$260 given a pay rate of \$18.95 per hour

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- \$2500 given a pay rate of \$500 per week

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- How long will it take a car to travel 200 kilometres if its speed is 80 km/h?

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- How many kilograms of meat can you buy with \$20 if it costs \$32.50 per kilo?

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Check your work before continuing.

## Skills development 5.3

Making a **rate** is simple. Just divide one of the parts by the other.

### Example

Roxy worked a 37.5 hour week and was paid \$543.75.

What is Roxy's rate of pay?



Note that Roxy's pay rate could have been written as 'hours per dollar'.

However, this changes the meaning and generally time is used as the last term of the rate.

### Solution

$$543.75 \div 37.5 = 14.5$$

$\therefore$  Roxy's rate of pay is \$14.50 per hour.

- Determine the appropriate rates.

- \$150 for 10 hours of work

$$150 \div 10 = \underline{\hspace{2cm}}$$

$\therefore$  The rate is  $\underline{\hspace{2cm}}$  per hour.

- 6 hours of work for \$520.

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- Find the daily rate given \$2500 for two weeks' worth of work. (A week is 7 days.)

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- Determine the speed (rate) of a car that completes 230 km in 1.5 hours.

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- Determine the rate if 6 metres of timber is selling for \$12.

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Check your work before continuing.



# 6. Solving ratio problems

When you complete this section you should be able to:

- solve ratio problems.

## Warm-up 6

1.  $0.62 \times 10 =$  \_\_\_\_\_

2.  $321 - 19 =$  \_\_\_\_\_

3. The temperature is minus 2 degrees.

How much will it need to increase to get to 4 degrees? \_\_\_\_\_

4.  $\frac{3}{6} - \frac{2}{6} =$  \_\_\_\_\_

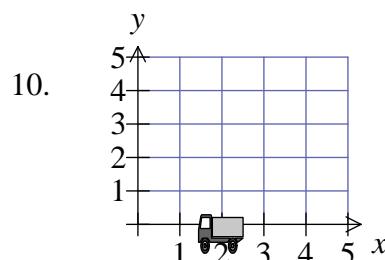
5.  $\frac{1}{7} \times 21 =$  \_\_\_\_\_

6.  $7500 \text{ mg} =$  \_\_\_\_\_ g

7.  $8 \times 7 - 6 =$  \_\_\_\_\_

8. Write 0.75 as a percentage. \_\_\_\_\_

9.  $2\frac{7}{8}, 2\frac{5}{8}, 2\frac{3}{8},$  \_\_\_\_\_



At what point is the truck? \_\_\_\_\_

## Focus problem 6

While camping, Aaron was asked to make the damper mix for his entire family, including his aunties, uncles and cousins.

He knew the following basic recipe would be enough for four people.



2 cups of self-raising flour  
1 pinch of salt  
1 tablespoon of butter  
 $\frac{1}{2}$  cup of milk

However, Aaron needed to make the mix for 20 people.

How much of each ingredient will he need?



Check your work before continuing.

### Bush tucker

Gubinges, better known as Kakadu plums, have the highest amount of vitamin C of any fruit. Vitamin C, or L-ascorbic acid, is an important nutrient that maintains people's health, protecting the body against oxidation. It contains antioxidants.

Aboriginal people have known the benefits of this fruit for a long time. The bark from the tree was also used to treat wounds and other sores.

## Skills development 6.1

To solve problems involving ratios, you should follow the following steps.

Step 1: Write down what you know.

Step 2: Solve for the unknown.

Step 3: Answer the question.

### Example

Gina wanted to mix up some cordial. The label read, ‘Mix 1 part cordial to 4 parts water’.

If Gina was to mix 200 millilitres of cordial, how much water should she use?

### Solution

Step 1: Write out what you know.

$$\begin{array}{rcl} \text{Cordial} & : & \text{Water} \\ \hline 1 & : & 4 \\ 200 & : & ? \end{array}$$



Although not needed, the headings help organise your information.

Step 2: Solve for the unknown.

$$\begin{array}{rcl} \text{Cordial} & : & \text{Water} \\ \hline 1 & : & 4 \\ \times 200 \quad \rightarrow & 200 & : \quad ? \quad \leftarrow \times 200 \\ 4 \times 200 = 800 & & \end{array}$$

Remember, there are other ways to solve the problem. Choose the best method for you!



Step 3: Answer the question.

$\therefore$  Gina should use 800 millilitres of water.

1. A medicine container suggests taking 5 millilitres per 10 kilograms. How many millilitres should a 70 kg person take?

$$\begin{array}{rcl} \text{Medicine} & : & \text{Weight} \\ (\text{mL}) & & (\text{kg}) \\ \hline 5 & : & 10 \\ ? & : & 70 \end{array}$$

$\therefore$  A 70 kg person should take \_\_\_\_\_ mL of the medicine.

2. A bicycle gear is in the ratio of  $2 : 5$ . That is, for every 2 rotations (turns) of the pedal, the wheel rotates 5 times.
- (a) Given the pedal is rotated 20 times, how many times does the wheel rotate?

Pedal rotation : Wheel rotation

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:

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- (b) How many times does the pedal need to turn to have the wheel turn 30 times?

Pedal rotation : Wheel rotation

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3. A car is travelling at 65 km/h. How far will it travel in 1.5 hours?

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4. A shop needs to add 10% onto its prices.
- (a) Given that an item costs \$400, determine the new cost.

Percent : Cost

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100 : 400

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110 : ?

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- (b) Determine the new cost of an item that originally costs \$95.
- 
- 
5. Four out of five students are planning to go to the school's disco. If the school has 350 students, how many students are planning to go to the disco?
- 
-

6. Jaylene works for 8 hours and earned \$324.40.

- (a) How much does Jaylene get paid per hour?

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- (b) How much will Jaylene earn working 10 hours?

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- (c) How many hours will Jaylene need to work to earn \$600?

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7. In 2010, Japan had approximately 338 000 people per 1000 square kilometres.

- (a) How many people do they have per square km?

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- (b) If Japan has approximately 377 000 square kilometres, what is its population?

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Check your work before continuing.

## 7. ‘Best buys’

When you complete this section you should be able to:

- determine ‘best buys’.

### Warm-up 7

1.  $762 \div 10 =$  \_\_\_\_\_

2.  $52 \div 4 =$  \_\_\_\_\_

3. The temperature is 7 degrees.

How much will it need to decrease to get to minus 6 degrees? \_\_\_\_\_

4.  $\frac{3}{8} + \frac{3}{4} =$  \_\_\_\_\_

5.  $\frac{1}{8} \times 32 =$  \_\_\_\_\_

6.  $3600 \text{ mL} =$  \_\_\_\_\_ L

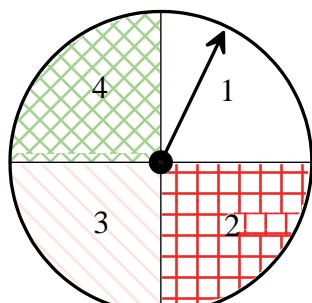
7.  $10 \div (2 + 3) =$  \_\_\_\_\_

8. Find 50% of \$60. \_\_\_\_\_

9. Describe the rule for the following pattern.

12, 17, 22, 27, 32, 37, ...

10.



Determine the probability the spinner will land on 1 or a 2.  
\_\_\_\_\_

Express your answer as a percentage.  
\_\_\_\_\_

## Review 7.1

### Example

Determine the cost per item for each of the following.

- (a) \$12.50 for 10 bottles
- (b) 12 cans for \$8.40

### Solution

(a)  $12.50 \div 10 = 1.25$

$\therefore \$1.25$  per bottle

(b)  $\$8.40 \div 12 = 0.7$

$\therefore \$0.70$  or 70 cents per can



The question tells you what to do!

It says ‘cost per item’ so divide the cost by the number of items.

1. Determine the cost per item.

- (a) 6 cans for \$12.90

$$\underline{\quad} \div 6 = \underline{\quad}$$

$\therefore \underline{\quad}$  per can

- (b) \$24 for 30 bottles of water

$$24 \div \underline{\quad} = \underline{\quad}$$

$\therefore \underline{\quad}$  per bottle

- (c) \$22.80 for 5 reams of paper

$$\underline{\quad}$$

$$\underline{\quad}$$

- (d) A dozen eggs costs \$5.34. (Hint: A dozen means 12.)

$$\underline{\quad}$$

$$\underline{\quad}$$



Check your work before continuing.

## Review 7.2

### Example

Determine the cost per given quantity (as shown in the brackets).

- \$6.75 for 450 millilitres (cost per 100 mL)
- 50 grams for \$1.55 (cost per 100 g)

### Solution

$$(a) 6.75 \div 450 \times 100 = 1.5$$

$\therefore \$1.50$  per 100 mL

$$(b) 1.55 \div 50 \times 100 = 3.1$$

$\therefore \$3.10$  per 100 g

You can also use multiplication to get your answer.

For example, if you multiply 50 grams by 2 you will also get 100 grams.

$$\text{So } 2 \times 1.55 = 3.1$$

which is \$3.10 per 100 g.



- Determine the cost per 100 grams of the following.

- \$3.60 for 500 grams of peanuts

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- 25 grams for \$1.33

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- Determine the cost per 100 millilitres of the following.

- \$1.30 for 375 mL can of soft drink

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- 2 litres of soft drink for \$1.99. (Hint: 1 L = 1000 mL)

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Check your work before continuing.

## Focus problem 7

Santiago's mum gave him \$5 dollars to buy some milk at the local deli. When Santi arrived at the deli, he found that he had a choice between buying a 2-litre container of milk for \$3.50, a 3-litre container for \$4.90, or a 600 mL container for \$1.30.

Which choice of containers of milk offers the 'best buy'?

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What are some of the reasons that Santi may use to purchase one of the other choices?

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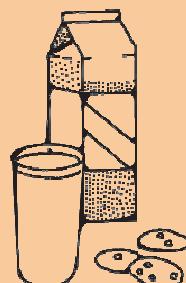
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Check your work before continuing.

### Milky madness

Humans first learned to regularly drink milk from their domesticated animals some 10 000 years ago in South-East Asia. It has since become a significant dietary food, containing many important elements including protein, calcium and vitamin C.



Early last century, milk was stored in glass bottles called pints (568 mL). However, due to the cost of cleaning, storing and transporting these bottles, cardboard and plastic containers are now used. The plastic used is a high-density polyethylene, which is easily recyclable, if given the chance.

## Skills development 7.1

### Example

Determine the ‘best buy’ out of the following.

- A. \$8.45 for 20 batteries
- B. 8 batteries for \$3.99
- C. 2 batteries for 99 cents

### Solution

A.  $8.45 \div 20 = 0.4225$

$\therefore \$0.4225$  or 42.25 cents per battery



Note, that when you are making comparisons, ensure that all units are the same before doing any calculations.

B.  $3.99 \div 8 = 0.49875$

$\therefore \$0.49875$  or 49.875 cents per battery

C.  $0.99 \div 2 = 0.495$

$\therefore \$0.495$  or 49.5 cents per battery

For example, the two batteries were selling for 99 cents, but I changed this into dollars before I did my calculations. This then matched the other two items so a fair comparison can be made.

Therefore, the ‘best buy’ is the 20 pack of batteries (A).

1. Complete the following and determine the ‘best buy’.



3 movies for \$8.95

or

2 movies for \$5.95



Remember that ‘best buy’ in this resource refers to the cheapest price.

$8.95 \div \underline{\quad} = \underline{\quad}$

$\therefore \underline{\quad}$  per movie

$5.95 \div \underline{\quad} = \underline{\quad}$

$\therefore \underline{\quad}$  per movie

$\therefore$  The        pack of movies is the better buy.

2. Determine the ‘best buys’ out of the following groups of items.
- (a) 12 pack of dog food for \$26.80 or 5 pack of dog food for \$11.15

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- (b) \$5.30 for 50 vitamin tablets or 250 vitamin tablets for \$25.99

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- (c) \$15.80 for 400 grams of steak or 1.2 kg of steak for \$47.50

(Hint: Make sure all units are the same before doing any comparisons.)

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- (d) 35 lollies for 82 cents or \$5.85 for 250 lollies

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Check your work before continuing.

## 8. Summary

- A ratio is comparison of two (or more) amounts.
- A ratio can be written in the from  $a : b$ .
- The symbol ‘:’ is read as ‘to’.
- The inverse operation to multiplication is division (and vice versa).
- A ratio can be represented by a fraction, decimal or percentage.
- A rate is a particular kind of ratio. It involves two quantities measured in different units.
- Ratios can help you in determining ‘best buys’.



## 9. Review tasks

The following tasks will assist you to consolidate your learning and understanding of the concepts introduced in this resource, and assist you to prepare for assessments.

### Task A

Name: \_\_\_\_\_

Suggested time: 55 minutes

Actual time taken: \_\_\_\_\_

#### Instructions

Complete this work on your own.

You may use a calculator but show how you got your answer.

Attempt every question. Take as long as you need and record the time in the space provided above after you have finished.

1. Write each of the following situations as a ratio.

- (a) A muesli bar contains 23 grams of carbohydrates to 10 grams of dietary fibre.

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- (b) A cordial mix uses 1 part cordial to 5 parts water.

2. The following is a pictorial representation of the modes of transport used to get to an island during a period of one week.



Determine the following ratios.

- (a) planes to boats \_\_\_\_\_

- (b) boats to planes \_\_\_\_\_

- (c) planes to modes of transport \_\_\_\_\_

- (d) boats to modes of transport \_\_\_\_\_

3. A nurse wants to determine the amount of salt in an intravenous saline solution.  
Complete the table to show the correct amount of salt per 10 mL of solution.

<i>Amount of saline solution (10 mL)</i>	1	2	5	12		
<i>Amount of salt (Osm)</i>	3	6			15	30

4. Simplify the following ratios.

(a)  $6 : 30$

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(b)  $24 : 16$

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5. Complete the table below.

Fraction	Decimal	Percentage
	0.5	
$\frac{1}{4}$	0.25	
	5	
$\frac{2}{3}$		66. $\bar{6}$ %

6. Ada is paid by the hour.

(a) Complete the table to show her pay.

<i>Time</i> (hours)	1	2	3	4	5	10
<i>Pay</i> (\$)				60		

(b) What is Ada's hourly pay rate? \_\_\_\_\_

7. (a) A car is travelling at 60 km/h.

(i) How far will it travel in 1.5 hours?

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(ii) How long will it take it to travel 210 kilometres?

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(b) Another car completes 280 km in 4 hours.

What is its speed?

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8. A shop needs to add 10% onto its prices for GST.

(a) Given that an item costs \$200, determine the new cost.

Percent : Cost

100 : 200

110 : ?

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(b) Determine the new cost of an item that originally costs \$148.

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- (c) An item has 10% of its price added for GST. Explain how you could determine the original price.

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9. All cars built after 1998 are able to use a mix of ethanol and petrol to at least a rating of E10, which means 10% of the fuel is ethanol.

(a) What percentage of the fuel in a mixture of E10 is petrol? \_\_\_\_\_

(b) Determine the ratio of ethanol to petrol in a mixture of E10. \_\_\_\_\_

Ella fills her tank with 40 litres of the E10 fuel. She determined that the amount of ethanol she was putting into her car was 10% of 40 litres.

(c) How much of each of the following is Ella adding to the tank?

(i) ethanol

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(ii) petrol

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(d) Use the amounts of ethanol and petrol found in part (c) to write the comparison as a ratio.

(e) Compare the ratio found in part (b) with part (d) and comment on your findings.

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10. Nathan works for 6 hours and earns \$214.50

(a) What is Nathan's hourly pay rate?

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- (b) How much will Nathan earn working 10 hours?

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- (c) Nathan plans to go on holidays and wants to save \$500 for his trip. How many hours will he need to work to earn the \$500?

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11. In 2012, India's approximate population was 1220 million people. India covers approximately 3.2 million square kilometres.

- (a) Determine India's approximate 2012 population density (number of people per square kilometre).

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China covers 9.6 million square metres. Its 2012 estimated population is approximately 1350 million.

- (b) What is China's approximate 2012 population density?

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- (c) Which of the two countries has the highest 2012 population density?

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FYI: Australia's 2012 population density is around 3 people per square kilometre!

12. Determine the 'best buys' out of the following items.

- (a) 6 pack of juice for \$3.18 or 3 pack of juice for \$1.62

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- (b) \$7.30 for 15 party pies or 2 party pies for 97 cents

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## Task B

Name: \_\_\_\_\_

Suggested time: 55 minutes

Actual time taken: \_\_\_\_\_

### Instructions

Complete this work on your own.

You may use a calculator but show how you got your answer.

Attempt every question. Take as long as you need and record the time in the space provided above after you have finished.

After receiving their latest power bill, Ruby's family decided that it was time to install solar power. Their average daily consumption was 14 kW per day.

A company is offering the following solar power specials.

- System A:  $8 \times 190$  W panels with a 1.5 kW inverter for \$1450.
- System B:  $12 \times 190$  W panels with a 2 kW inverter for \$3450.
- System C:  $16 \times 190$  W panels with a 3 kW inverter for \$4950.



1. Which offer is the 'best buy' for Ruby's household?

(Show all your working and give your reasoning for your choice.)

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On average, a 1 kW solar panel system based in Perth produces 4.4 kWh per day.

The State Government will buy back power at 7 cents per kW.

2. Does this change your decision? (Give your reasoning and support all your claims by showing your working.)

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The efficiency of an inverter is measured by how well it converts the DC electricity into AC electricity. This usually ranges from 95% to 97.5% for most models.

3. Will this change your decision? (Give your reasoning and support all your claims by showing your working.)

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4. Investigate your own living situation.

How much power do you use per day?

What is the cost per kW that you are charged?

If you are using solar power, what are the savings (or costs) to your household over a one-year period? What are the savings (or costs) over a ten-year period?

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## Self-evaluation task

Please complete the following.

### How well did you manage your own learning using this resource?

	Always	Usually	Rarely	Not sure
Each section took approximately 45 minutes to complete.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I needed extra help.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I marked and corrected my work at the end of each section.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I made the journal entries and summaries when asked.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I have kept to my work schedule.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

### How much mathematics have you learnt using this resource?

	Always	Usually	Rarely	Not sure
<i>Understanding</i>				
I recognise equivalences between fractions, decimals, percentages and ratios.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I understand that rate and ratio problems can be solved using fractions or percentages.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<i>Fluency</i>				
I calculated accurately.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I can recognise and write ratios.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I can simplify ratios.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I understand that rate and ratio problems can be solved using fractions or percentages.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
I can determine best buys.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<i>Problem Solving</i>				
I choose between fractions or percentages to solve simple ratio problems efficiently.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
<i>Reasoning</i>				
I applied an understanding of ratio to solve problems.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Write a list of topics for which you need additional assistance.

# Solutions

## 1. Comparisons

### Solutions to Warm-up 1

1. Yes it is as  $21 \div 7 = 3$ .
2. 15
3.  $a = (-30)$
4.  $\frac{3}{4}$  should be circled.
5. 10
6. 10.6
7. 18
8.  $\frac{125}{100} = \frac{5}{4}$
9. 28
10.  $123^\circ$

### Solution to Focus problem 1

What you were asked to do was to compare the heights of Casey and Narah.

To do this, you can use the following methods.

Method 1 – using differences (addition/subtraction)	Method 2 – using ratios (multiplication/division)
Casey is 100 cm smaller than Narah	Narah is twice Casey's height
Narah is 100 cm taller than Narah	Casey is half Narah's height

In this module, we will be learning about ratios. That is, we will be using multiplication and/or division for our ‘comparisons’.

A ratio is comparison of two (or more) amounts.

As an example, the ratio of the above heights is written as 100 : 200.

The symbol ‘:’ is read as ‘to’.



## Solutions to Skills development 1

1. (a) 24 : 32  
 (b) 42 : 100  
 (c) 1 : 4
2. (a) Trucks to cars 3 : 2  
 (b) Cars to trucks 2 : 3  
 (c) Cars to vehicles 2 : 5  
 (d) Trucks to vehicles 3 : 5

3. (a) 30 : 20  
 (b) 10 : 20



Remember, if you write the ratio the other way round it changes the meaning.

4. When working with ratios, only multiplication and division are used.

## 2. More comparisons

### Solutions to Warm-up 2

1. Yes, it is as  $14 \div 7 = 2$  and  $21 \div 7 = 3$ .
2. 9
3.  $(-5)$  degrees
4.  $\frac{3}{5} > \frac{5}{10}$
5. 24
6. 6
7. 4.1
8. 0.25
9. 82.1
10.  $(2, 3)$

### Solution to Focus problem 2

What you were asked to find was if dividing by 0.2 is the same as multiplying by 5.

Multiplication and division are opposite or inverse operations. That is, multiplication undoes division (and vice versa).

For example, if we multiply the gear number ‘4’ by 5, the result is 20. To undo this, we can divide by 5 to get the gear number again.

$$4 \times 5 = 20$$

$$20 \div 5 = 4$$

But how are 0.2 and 5 related?

As a fraction, 5 can be written as  $\frac{5}{1}$ . Likewise, 0.2 as a fraction is written as  $\frac{1}{5}$ .

So  $\frac{5}{1}$  is the inverse of  $\frac{1}{5}$  (and vice versa).

Multiplying by  $\frac{5}{1}$  gives the same result as dividing by  $\frac{1}{5}$  (0.2), and dividing by 5 gives the same result as multiplying by  $\frac{1}{5}$  (0.2).

So why do I need to know this?



Using inverse operations will help you when you work with ratios.



## Solutions to Skills development 2.1

1. (a)  $5 \Rightarrow 20$

(i) multiplication:  $\times 4$

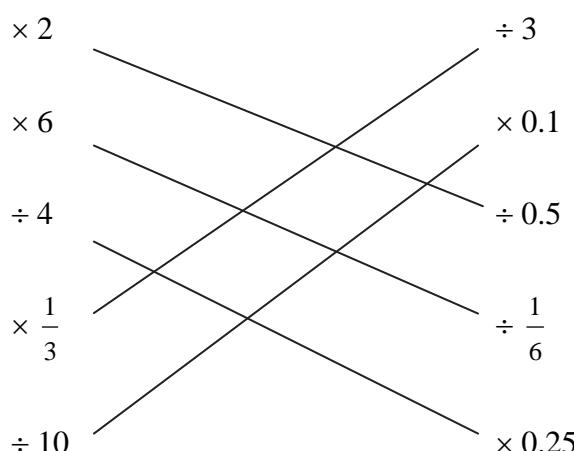
(ii) division:  $\div \frac{1}{4}$  (or 0.25)

(b)  $20 \Rightarrow 5$

(i) multiplication:  $\times \frac{1}{4}$  (or 0.25)

(ii) division:  $\div 4$

2.



## Solutions to Skills development 2.2

1.

<i>Puppy's weight (kg)</i>	1	2	5	12	3	10
<i>Amount of medication (mL)</i>	2	4	10	24	6	20

2.

<i>Number of bags of mortar mix</i>	1	2	5	3	8	10
<i>Number of bricks</i>	30	60	150	90	240	300

3.

(a)

<i>Time (hours)</i>	1	2	3	4	5	10
<i>Pay (\$)</i>	16	32	48	64	80	160

(b)

<i>Speed (km/h)</i>	1	2	3	4	5	10
<i>Distance (km)</i>	6.4	12.8	19.2	25.6	32	64

### 3. Simplifying ratios

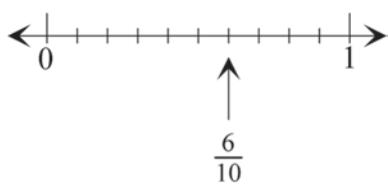
#### Solutions to Warm-up 3

1. 3 should be circled.

2. 40

3. 0

4.



5. 8

6.  $10 + 9 = 19$ 

7. 17.5

8. 20%

9.  $4\frac{4}{5}$ 

10. 0.5

## Solutions to Skills review 3.1

1. (a)  $20 = 1 \times 20$   
 $20 = 2 \times 10$   
 $20 = 4 \times 5$   
 $\therefore$  The factors are 1, 2, 4, 5, 10 and 20.
- (b)  $8 = 1 \times 8$   
 $8 = 2 \times 4$   
 $\therefore$  The factors are 1, 2, 4 and 8.
- (c)  $24 = 1 \times 24$   
 $24 = 2 \times 12$   
 $24 = 3 \times 8$   
 $24 = 4 \times 6$   
 $\therefore$  The factors are 1, 2, 3, 4, 6, 8, 12 and 24.
- (d)  $13 = 1 \times 13$   
 $\therefore$  The factors are 1 and 13.

Two factors? 13 must be a prime!



## Solutions to Skills review 3.2

1. Determine the common factors for each of the following pairs.
- (a) 4 and 10  
The factors of 4 are: 1, 2, 4.  
The factors of 10 are: 1, 2, 5, 10.  
 $\therefore$  The common factors are: 1 and 2.
- (b) 8 and 12  
The factors of 8 are: 1, 2, 4 and 8.  
The factors of 12 are: 1, 2, 3, 4, 6 and 12.  
 $\therefore$  The common factors are: 1, 2 and 4.
- (c) 9 and 15  
The factors of 9 are: 1, 3 and 9.  
The factors of 15 are: 1, 3, 5, 15  
 $\therefore$  The common factors are : 1 and 3.
- (d) 13 and 16  
The factors of 13 are: 1 and 13.  
The factors of 16 are: 1, 2, 4, 8 and 16.  
 $\therefore$  The common factor is 1.

## Solution to Focus problem 3

What you were asked to find was the cheaper material. However, it is a trick question as both are the same price. (You do get more for \$60 but you pay more too!) Ratios may look different, but in fact they may be the same (or equivalent). For example, the following ratios are all equivalent.

$$15 : 1 \quad 60 : 4 \quad 30 : 2 \quad 75 : 5$$

Consider the equivalent ratios  $1 : 15$  and  $4 : 60$ . Multiplying both sides of the first ratio by 4 gives the second ratio.

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline \end{array}$$

$$\begin{array}{rcl} & 1 & : & 15 \\ \times 4 \rightarrow & 4 & : & 60 & \leftarrow \times 4 \end{array}$$

The relationship between the ratios holds true in all directions, just the multiplier (or divisor) changes.

For example, the following shows how the relationship continues across the ratios.

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline \end{array}$$

$$\begin{array}{rcl} & \times 15 & \\ & \downarrow & \\ 1 & : & 15 \\ \hline 4 & : & 60 \\ & \uparrow & \\ & \times 15 & \end{array}$$

And going the opposite way, the inverse operation is used.

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline \end{array}$$

$$\begin{array}{rcl} & \div 4 \rightarrow & \\ & & \leftarrow \div 4 \\ 1 & : & 15 & \\ 4 & : & 60 & \end{array}$$
  

$$\begin{array}{c} \text{Material} : \text{Cost} \\ \hline \end{array}$$

$$\begin{array}{rcl} & \div 15 & \\ & \downarrow & \\ 1 & : & 15 \\ \hline 4 & : & 60 \\ & \uparrow & \\ & \div 15 & \end{array}$$

Multiplication and division are inverse operations. That is, one undoes the other.



## Solution to Focus problem 3

A ratio can be shown in different ways. Consider the following example.

A packet of lollies has 1 red lolly for every 3 blue lollies.

In ratio form, this is shown as 1 : 3 which is read as ‘1 to 3’.

See how ‘:’ is read  
as ‘to’.



This ratio can be converted into fraction, decimal and percentage forms.

Form	Example	Description
ratio	1 : 3	1 red lolly to 3 blue lollies
fraction	$\frac{1}{4}$ of the lollies are red	$\frac{3}{4}$ of the lollies are blue
decimal	0.25 of the lollies are red	0.75 of the lollies are blue
percentage	25% are red lollies	75% are blue lollies

## Solutions to Skills development 3.1

1.

(a)

$$\begin{array}{rcl} \text{Weight} & : & \text{Cooking time} \\ (\text{kg}) & & (\text{min}) \end{array}$$

$$\begin{array}{rcl} \times 5 \quad \boxed{1} & : & 60 \\ & \boxed{5} & : \quad \boxed{12} & \times 5 \end{array}$$

(b)

$$\begin{array}{rcl} \text{Goals} & : & \text{Points} \end{array}$$

$$\begin{array}{rcl} \times 6 \quad \boxed{1} & : & 6 \\ & \boxed{5} & : \quad \boxed{30} & \times 6 \end{array}$$

(c)

$$\begin{array}{rcl} \text{Speed} & : & \text{Distance} \\ (\text{km/h}) & & (\text{km}) \end{array}$$

$$\begin{array}{rcl} \div 4 \quad \boxed{80} & : & 24 \\ & \boxed{20} & : \quad \boxed{6} & \div 4 \end{array}$$

## Solutions to Skills development 3.2

1. Solutions may vary. (You can use the inverse operation if desired.)

(a)

$$\begin{array}{rcl} \text{Cereal} & : & \text{Sodium} \\ (\text{per serving}) & : & (\text{mg}) \\ \hline & & \\ \times 4 & \begin{array}{c} 1 \\ \rightarrow \\ 4 \end{array} & : \quad \begin{array}{l} 30 \\ \leftarrow \\ \hline \end{array} \quad \times 4 \end{array}$$

$$30 \times 4 = 120$$

$\therefore$  The missing value is 120.

(b)

$$\begin{array}{rcl} \text{Hot chocolate} & : & \text{Sugars (g)} \\ (\text{per teaspoon}) & : & \\ \hline & & \times 6 \\ & \begin{array}{c} 2 \\ \rightarrow \\ 5 \end{array} & : \quad \begin{array}{l} \downarrow \\ \hline \end{array} \\ & & \\ & \begin{array}{c} 5 \\ \rightarrow \\ 30 \end{array} & : \quad \begin{array}{l} \uparrow \\ \times 6 \end{array} \end{array}$$

$$2 \times 6 = 12$$

$\therefore$  The missing value is 12.

## Solutions to Skills development 3.3

1. (a)  $4 : 10$     The factors of 4 are: 1, 2, and 4.  
                     The factors of 10 are: 1, 2, 5 and 10.

$$\begin{array}{r} 4 \quad : \quad 10 \\ \div 2 \quad \rightarrow \quad 2 \quad : \quad 5 \\ \hline \end{array} \quad \leftarrow \div 2$$

$\therefore$  The ratio becomes  $2 : 5$ .

- (b)  $6 : 9$     The factors of 6 are: 1, 2, 3 and 6.  
                     The factors of 9 are: 1, 3 and 9.

$$\begin{array}{r} 6 \quad : \quad 9 \\ \div 3 \quad \rightarrow \quad 1 \quad : \quad 3 \\ \hline \end{array} \quad \leftarrow \div 3$$

$\therefore$  The ratio becomes  $1 : 3$ .

2. (a)  $3 : 5$   
       (b) The ratio cannot be simplified.

3.

VII	VI	I	VIII	V	III	X	IX	II	IV	XI
c	h	r	i	s	p	b	a	c	o	n

## 4. Converting

### Solutions to Warm-up 4

- 9 (They are all squares numbers.)
- 9
- 3 degrees
- $\frac{7}{10}$
- 9
- 806
- 2.01
- $\frac{27}{100}$
- 79
- pentagon

## Solution to Focus problem 4

What you were asked to do was to find the percentage given the ratio of 1 to 5, or  $1 : 5$ .

A percentage is a special type of ratio. It is a comparison made to 100.

To find the percentage you could convert the ratio so that it compares it to 100.

$$\begin{array}{rcccl} & 1 & : & 5 & \\ \times 20 \rightarrow & 20 & : & 100 & \leftarrow \times 20 \end{array}$$

Since  $1 : 5$  is equivalent to  $20 : 100$ , 20% of the wrappers would be winners.

## Solutions to Skills development 4.1

1. (a) There are  $\frac{1}{3}$  as many red candies as there are blue.
  - (b) There are  $\frac{3}{1}$  times as many blue candies as there are red.
  - (c) The red candies make up  $\frac{1}{4}$  of the total candies.
  - (d) The blue candies make up  $\frac{3}{4}$  of the total candies.
2. (a) The height of the television screen is  $\frac{3}{4}$  its width.
  - (b) The width of the television screen is  $\frac{4}{3}$  its height.

## Solutions to Skills development 4.2

1. (a)  $4 \div 5 = 0.8 \quad \therefore$  The fraction is equivalent to 0.8.
- (b)  $4 \div 8 = 0.5 \quad \therefore$  The fraction is equivalent to 0.5.
- (c) 0.888...
- (d) 4

## Solutions to Skills development 4.3

1. (a)  $\frac{6}{10}$
- (b)  $\frac{35}{100}$
- (c)  $\frac{2005}{1000}$  or  $2\frac{5}{1000}$

## Solutions to Skills development 4.4

1. (a) 67%

- (b) 5%
- (c) 39.4%
- (d) 310%

### Solutions to Skills development 4.5

1. (a) 0.04
- (b) 0.21
- (c) 0.0108
- (d) 2.51

### Solutions to Skills development 4.6

1. (a) 60%
- (b) 75%
- (c) 312.5%

### Solutions to Skills development 4.7

1. (a)  $\frac{63}{100}$
- (b)  $\frac{5}{100}$
- (c)  $\frac{300}{100}$  or 3

### Solutions to Skills development 4.8

1.

Fraction	Decimal	Percentage
$\frac{2}{5}$	0.4	40%
$\frac{1}{3}$	0.333...	33.333... %
$\frac{3}{2}$	1.5	150%
$\frac{4}{1}$	4	400%
$\frac{3}{5}$	0.6	60%
$\frac{1}{2}$	0.5	50%

2.

I
a

II	III	IV	V	VI	VII	VIII
g	a	r	b	a	g	e

IX	X	XI	XII	XIII
t	r	u	c	k

## 5. Solving rate problems

### Solutions to Warm-up 5

1. The greatest common factor is 3.

2. 32

3.  $x = 2$

4.  $\frac{4}{4}$  or 1

5. 6

6. 6.4 km

7. 48

8. 0.75

9. 4.5

10.  $122^\circ$

### Solution to Focus problem 5

What you were asked to do was to solve a rate problem. A rate is a special type of ratio.

So 110 km/h is the same as 110 : 1, which means it can be solved just like a ratio.

$$\begin{array}{ccc} \text{km} & & \text{h} \\ 110 & : & 1 \\ \times 3 \rightarrow & & \leftarrow \times 3 \\ ? & : \underline{3} & \end{array}$$

$$110 \times 3 = 330$$

$\therefore$  Jenny would travel 330 kilometres.

Note that you can use the above method anytime to solve rates. However, because rates are given per unit, a straightforward multiplication (or division) can be made. This means only two lines of working are required.

For example, you could show the following.

$$110 \times 3 = 330$$

$\therefore$  Jenny would travel 330 kilometres.

## Solutions to Skills development 5.1

1. (a) \$20 per hour for 5 hours.

$$20 \times 5 = 100$$

$\therefore$  The amount earned is \$100.

- (b) \$127.60

- (c) \$154

2.  $80 \times 1.5 = 120$

$\therefore$  The car would travel 120 km in 1.5 hours.

3.  $72 \times 60 = 4320$

$\therefore$  Jaiden's heart will beat 4320 times in an hour.

## Solutions to Skills development 5.2

1. (a)  $150 \div 10 = 15$

$\therefore$  It will take 15 hours.

(b)  $260 \div 18.95 = 13.72\dots$

$\therefore$  It will take 14 hours (to the nearest hour).

(c)  $2500 \div 500 = 5$

$\therefore$  It will take 5 weeks.

2.  $200 \div 80 = 2.5$

$\therefore$  It will take 2.5 hours.

3.  $32.50 \div 20 = 1.625$

$\therefore$  1.625 kilogram of meat can be bought.

## Solutions to Skills development 5.3

1. (a) \$150 for 10 hours of work.

$$150 \div 10 = 15$$

$\therefore$  The rate is \$15 per hour.

(b)  $520 \div 6 = 86.66\dots$

$\therefore$  The rate is \$86.67 per hour (nearest cent).

(c)  $2500 \div 14 = 178.571\dots$

$\therefore$  The rate is \$178.57 per day (nearest cent).

2.  $230 \div 1.5 = 153.333\dots$

$\therefore$  The speed is 153 km/hour (nearest kilometre).

3.  $12 \div 6 = 2$

$\therefore$  \$2 per metre is the rate.

## 6. Solving ratio problems

### Solutions to Warm-up 6

1. 6.2
2. 302
3. 6 degrees
4.  $\frac{1}{6}$
5. 3
6. 7.5 g
7. 50
8. 75%
9.  $2\frac{1}{8}$
10. (2, 0)

### Solution to Focus problem 6

What you were asked to find was the amount for each of the ingredients.

There are several ways to do the problem. You could use:

- ratios only
- rates (the unit ratio)
- fractions
- percentages.

(There are other methods of course, but these are the only methods you are required to use in this book.)

An example of each method follows.

#### Ratios only

The amount of flour needed is shown below.

No of people	:      Ingredients	
4	:      2	
$\times 5$	$\rightarrow$	$\leftarrow \times 5$
20	: _____ ?	
$2 \times 5 = 10$		

$\therefore$  Aaron would need 10 cups of self-raising flour.

#### Rates

The amount of salt needed is shown below.

The rate is given as  $1 \div 4 = 0.25$  pinches of salt per person.

$$20 \times 0.25 = 10$$

$\therefore$  Aaron would need 5 pinches of salt.

## Fractions

$$\frac{\text{Amount of butter}}{\text{Amount of people}} = \frac{1}{4} = \frac{?}{20}$$

$$\frac{1}{4} = \frac{?}{20}$$

$\times 5$

$$1 \times 5 = 5$$

$\therefore$  Aaron would need 5 teaspoons of butter.

## Percentages

$$\frac{20}{4} \times 100 = 500\%$$

$$\text{So } 500\% \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{500}{100}$$

$$= 2.5 \text{ (or } 2\frac{1}{2}\text{)}$$

$\therefore$  Aaron would need 2.5 cups of milk.

But which method do I use?



That depends on the situation and the method you are most comfortable using.



## Solutions to Skills development 6.1

1.

$$\begin{array}{rcl} \text{Medicine} & : & \text{Weight} \\ (\text{mL}) & & (\text{kg}) \\ \hline & & \\ \times 7 & \left[ \begin{array}{c} 5 \\ ? \end{array} \right] & \left[ \begin{array}{c} 10 \\ 70 \end{array} \right] \times 7 \end{array}$$

$$5 \times 7 = 35$$

$\therefore$  A 70 kg person should take 35 mL of the medicine.

2.

$$\begin{array}{rcl} \text{Pedal rotation} & : & \text{Wheel rotation} \\ \hline & 2 & : & 5 \\ \times 10 \quad \rightarrow & 20 & : & ? \quad \leftarrow \times 10 \end{array}$$

$$5 \times 10 = 50$$

$\therefore$  If the pedals rotate 20 times, the wheel rotates 50 times.

(b)

$$\begin{array}{rcl} \text{Pedal} & : & \text{Wheel} \\ \text{rotation} & : & \text{rotation} \\ \hline & 2 & : & 5 \\ \times 6 \quad \rightarrow & ? & : & 30 \quad \leftarrow \times 6 \end{array}$$

$$2 \times 6 = 12$$

$\therefore$  If the wheel rotates 30 times, the pedals will need to rotate 12 times.

3.

$$\begin{array}{rcl} \text{Distance} & : & \text{Time} \\ \hline & 65 & : & 1 \\ \times 1.5 \quad \rightarrow & ? & : & 1.5 \quad \leftarrow \times 1.5 \end{array}$$

$$65 \times 1.5 = 97.5$$

$\therefore$  The car will travel 97.5 km in 1.5 hours.

4. (a)

$$\begin{array}{rcl} \text{Percent} & : & \text{Cost} \\ \hline & 100 & : & 400 \\ \times 1.1 \quad \rightarrow & 110 & : & ? \quad \leftarrow \times 1.1 \end{array}$$

$$400 \times 1.1 = 440$$

$\therefore$  The new cost is \$440.

(b)

$$\begin{array}{rcl} \text{Percent} & : & \text{Cost} \\ \hline 100 & : & 95 \\ \times 1.1 & \rightarrow & \left[ \quad \right] \times 1.1 \\ 110 & : & ? \leftarrow \end{array}$$

$$95 \times 1.1 = 104.5$$

$\therefore$  The new cost is \$104.50.

5. 280 students
6. (a) \$40.55  
(b) \$405.50  
(c)  $600 \div 40.55 = 14.796\dots$   
 $\therefore$  She will need to work 15 hours (to nearest hr).
7. (a) 338 people per square kilometre  
(b)  $377\,000 \times 338 = 127\,426\,000$

The population of Japan is 127 million people (nearest million).

## 7. ‘Best buys’

### Solutions to Warm-up 7

1. 76.2
2. 13
3. 13 degrees
4.  $\frac{9}{8}$  or  $1\frac{1}{8}$
5. 4
6. 3.6 L
7. 2
8. \$30.
9. The numbers are increasing by 5. (Add 5 each time.)
10. 50%.

### Solutions to Skills review 7.1

1. (a)  $12.9 \div 6 = 2.15$   
 $\therefore$  \$2.15 per can
- (b)  $24 \div 30 = 0.8$   
 $\therefore$  \$0.80 or 80 cents per bottle

- (c)  $22.8 \div 5 = 4.56$   
 $\therefore \$4.56$  per ream of paper
- (d)  $5.34 \div 12 = 0.445$   
 $\therefore \$0.445$  or 44.5 cents per egg

## Solutions to Skills review 7.2

- (a)  $3.6 \div 500 \times 100 = 0.72$   
 $\therefore \$0.72$  or 72 cents per 100 grams of peanuts
  - (b)  $1.33 \div 25 \times 100 = 5.32$   
 $\therefore \$5.32$  per 100 grams
- (a)  $1.30 \div 375 \times 100 = 0.34666$   
 $\therefore \$0.34666$  or 34.666 cents per 100 mL
  - (b)  $\$1.99 \div 2000 \times 100 = 0.0995$   
 $\therefore \$0.0995$  or 9.95 cents per 100 mL

Your method may be different to what is shown, but do not worry, as long as you arrive at the same result (and you are accurate) it is not an issue.



## Solution to Focus problem 7

What you were asked to find was the ‘best buy’ out of the three given choices. Generally, ‘best buy’ refers to the cheapest price for the same amount of an item. There are a number of ways to solve this problem. The following shows you one of the ways.

Firstly, write down what you know.

Container 1: 2-litre container of milk for \$3.50

Container 2: 3-litre container of milk for \$4.90

Container 3: 600 mL of milk for \$1.30

Secondly, decide how you are going to compare them (and then do the appropriate conversions). Will it be per litre, 100 mL or perhaps per mL?

Container 1:  $3.50 \div 2 = 1.75$

$\therefore \$1.75$  per litre

I decided to convert them into litre amounts as two of the items are already in litres.



Container 2:  $4.90 \div 3 = 1.6333\dots$

$\therefore \$1.65$  per litre (rounded to the nearest 5 cents)

Container 3:  $1.30 \div 600 \times 1000 = 2.1666\dots$

$\therefore \$2.15$  per litre (rounded to the nearest 5 cents)

You could also convert container 3 by first converting the 600 mL into litres and then dividing by that amount. For example,  
 $600 \text{ ml} = 0.6 \text{ L}$ .



$\therefore$  Since the 3-litre container has the cheapest price per litre it is the ‘best buy’.

Santi may choose one of the other containers because he knows 3 litres is too much for his family to drink.

## Solutions to Skills development 7.1

1.  $8.95 \div 3 = 2.98333\dots$

$\therefore \$2.98333\dots$  per movie

$$5.95 \div 2 = 2.975$$

$\therefore \$2.975$  per movie

Therefore, the 2 pack of movies is the better buy.

2. Solutions may vary slightly. If you have any concerns that your method might not be correct, discuss it with your teacher.

(a)  $26.8 \div 12 = 2.233\dots$

$\therefore \$2.233\dots$  per pack

$$11.15 \div 5 = 2.23$$

$\therefore \$2.23$  per pack



This was a close one!  
The only difference is that the price per pack for the 12 pack has a recurring or repeating decimal, which only just makes it more expensive.

Therefore the 5 pack of dog food is the better buy.

(b)  $5.3 \div 50 = 0.106$

$\therefore \$0.106$  or 10.6 cents per tablet

$$25.99 \div 250 = 0.10396$$

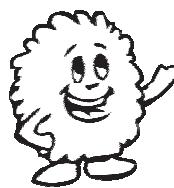
$\therefore \$0.10396$  or 10.396 cents per tablet



Remember, you can do these problems a number of ways.

Therefore the 250 vitamin tablets is the better buy.

I compared them both by using the price for 50 tablets.



It was easier because I only had to convert one of the values.

(c)  $15.8 \div 400 \times 100 = 3.95$

$\therefore \$3.95$  per 100 grams

$$47.5 \div 1200 \times 100 = 3.95833\dots$$

$\therefore \$3.95833\dots$  per 100 grams



Remember to make sure all units are the same before you do any calculations.

Therefore, the 400 grams of steak was the better buy.

(d)  $82 \div 35 = 2.342\ldots$   
 $\therefore 2.3428571\ldots$  cents per lolly

$$585 \div 250 = 2.34$$
 $\therefore 2.34$  cents per lolly

Therefore, the 250 lollies is the better buy.

## Solutions to Review tasks

### Solutions to Task A

1. (a)  $23 : 10$   
(b)  $1 : 5$

2. (a)  $3 : 1$   
(b)  $1 : 3$   
(c)  $3 : 4$   
(d)  $1 : 4$

3

<i>Amount of saline solution (10 mL)</i>	1	2	5	12	3	10
<i>Amount of salt (Osm)</i>	3	6	15	36	15	30

4. (a)  $1 : 5$   
(b)  $3 : 2$

5.

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{5}{1}$	5	500%
$\frac{2}{3}$	$0.\bar{6}$	$66.\bar{6}\%$

6. (a)

<i>Time</i> (hours)	1	2	3	4	5	10
<i>Pay</i> (\$)	15	30	45	60	75	150

- (b) \$15 per hour
7. (a) (i) 90 km  
      (ii) 3.5 hours  
 (b) 70 km/hr
8. (a) \$220  
 (b) \$162.80  
 (c) Solutions may be expressed differently but dividing by 1.1 would give you the original price.
9. (a) 90%  
 (b) 10 : 90 or 1 : 9  
 (c) (i) 4 L of ethanol      (ii) 36 L of petrol  
 (d) 4 : 36  
 (e) 4 : 36 is equivalent to 1 : 9
10. (a) \$35.75 per hour  
 (b) \$357.50  
 (c)  $500 \div 35.75 = 13.98\dots$   
 So he needed to work 14 hours.
11. (a)  $1220 \div 3.2 = 381.25$   
 381 people per square kilometre (nearest whole)  
 (b)  $1350 \div 9.6 = 140.625$   
 141 people per square kilometre (nearest whole)  
 (c) India has the highest population density.
12. (a) 6 pack of juice is the ‘best buy’.  
 (b) 2 party pies is the ‘best buy’.

## Solutions to Task B

1. Reasoning may vary. For example, the largest device may be chosen to support a growing family’s future needs. However, the ‘best buy’ in terms of electricity generated per dollar, is System A.

The rates are shown below.

$$\text{System A: } 1450 \div 1.5 = \$966.666 \text{ per kW}$$

$$\text{System B: } 3450 \div 2 = \$1725 \text{ per kW}$$

$$\text{System C: } 4950 \div 3 = \$1650 \text{ per kW}$$

2. Solutions may vary. Students should give a reason and support their claims by appropriate working.
3. Solutions may vary. Students should give a reason and support their claims by appropriate working.
4. Solutions will vary. Students should use their own figures to determine if solar power is viable for their household.





**Department of  
Education**

MATHSAC019  
COMPARISONS  
ISBN: 9780730744399