Department of
Education

AUSTRALIAN CURRICULUM

MATHEMATICS
YEAR 7

3D objects
(Representations and volume)

MATHEMATICS
YEAR 7

3D objects
(Representations and volume)

Student's name: _____

Teacher's name: _____

First published 2012

ISBN 9780730744443

SCIS 1564090

© Department of Education WA 2012 (Revised 2020) https://creativecommons.org/licenses/by-nc/4.0/

Requests and enquiries concerning copyright should be addressed to:

Manager Intellectual Property and Copyright
Department of Education
151 Royal Street
EAST PERTH WA 6004

Email: copyright@education.wa.edu.au

Department of Education

This resource contains extracts from The Australian Curriculum Version 3.0 © Australian Curriculum, Assessment and Reporting Authority 2012.
ACARA neither endorses nor verifies the accuracy of the information provided and accepts no responsibility for incomplete or inaccurate information. In particular, ACARA does not endorse or verify that:

- the content descriptions are solely for a particular year and subject
- all the content descriptions for that year and subject have been used
- the author's material aligns with the Australian Curriculum content descriptions for the relevant year and subject.

You can find the unaltered and most up to date version of this material at www.australiancurriculum.edu.au. This material is reproduced with the permission of ACARA.

creativecommons.org/licenses/by-nc-sa/3.0/au/

Graphics used in this resource are sourced from http://openclipart.org under the creative commons license http://creativecommons.org/publicdomain/zero/1.0

This product will be registered through the National Copyright Unit for use in all Australian schools without remuneration.

Contents

Signposts 2

Introduction 3

Curriculum details 4

1. Plan (drawing) 7
2. Oblique projection 13
3. Isometric projection 19
4. Volume 31
5. Volume of rectangular prisms 37
6. Rectangular prism formula 43
7. Units and the volume formula 51
8. Summary 57
9. Review tasks 59

Solutions 69

© Department of Education WA 2012 – MATHSAC024

Signposts

Each symbol is a sign to help you.

Here is what each one means:

The recommended time you should take to complete this section.

An explanation of key terms, concepts or processes.

A written response.
Write your answer or response in your journal.

Correct this task using the answers at the end of the resource.

Calculators may not be used here.

Make notes describing how you attempted to solve the problem. Keep these notes to refer to when completing the Self-evaluation task. Your teacher may wish you to forward these notes.

© Department of Education WA 2012 – MATHSAC024

Introduction

This resource should take you approximately two weeks to complete. It comprises seven learning sections, a summary section and a review task section.

The learning sections have the following headings:

- **Key words**
  These are the main words that you need to understand and use fluently to explain your thinking.

- **Warm-up**
  Warm-up tasks should take you no longer than 10 minutes to complete. These are skills from previous work you are expected to recall from memory, or mental calculations that you are expected to perform quickly and accurately. If you have any difficulties in answering these questions, please discuss them with your teacher.

- **Review**
  Some sections have reviews immediately after the warm-up. The skills in these reviews are from previous work and are essential for that section. You will use these to develop new skills in mathematics. Please speak to your teacher immediately if you are having any trouble in completing these activities.

- **Focus problem**
  Focus problems are designed to introduce new concepts. They provide examples of the types of problems you will be able to solve by learning the new concepts in this resource. Do not spend too long on these but do check and read the solutions thoroughly.

- **Skills development**
  These help you consolidate new work and concepts. Most sections include skills development activities which provide opportunities for you to become skilled at using new procedures, apply your learning to solve problems and justify your ideas. Please mark your work after completing each part.

Correcting your work

Please mark and correct your work as you go. Worked solutions are provided to show how you should set out your work. If you are having any difficulty in understanding them, or are getting the majority of the questions wrong, please speak to your teacher immediately.

Journal

Please keep an exercise book to record your notes and to summarise your learning. At the end of each section, write definitions for the key words that were introduced for that section.

© Department of Education WA 2012 – MATHSAC024

Curriculum details

Content Descriptions

This resource provides learning and teaching to deliver the Australian Curriculum: Mathematics for the following Year 7 Content Descriptions.

Calculate volumes of rectangular prisms (ACMMG160)

Draw different views of prisms and solids formed from combinations of prisms (ACMMG161)

Content Descriptions	1	2	3	4	5	6	7	R
ACMMG160								
ACMMG161								

Indicates the content description is explicitly covered in that section of the resource.

Previous relevant Content Descriptions

The following Content Descriptions should be considered as prior learning for students using this resource.

At Year 4 level

Compare objects using familiar metric units of area and volume (ACMMG290)

At Year 5 level

Connect three-dimensional objects with their nets and other two-dimensional representations (ACMMG111)
Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108)

At Year 6 level

Connect volume and capacity and their units of measurement (ACMMG138)

Proficiency strand statements at Year 7 level

At this year level:

Understanding includes describing patterns in uses of indices with whole numbers, recognising equivalences between fractions, decimals, percentages and ratios, plotting points on the Cartesian plane, identifying angles formed by a transversal crossing a pair of lines, and connecting the laws and properties of numbers to algebraic terms and expressions

Fluency includes calculating accurately with integers, representing fractions and decimals in various ways, investigating best buys, finding measures of central tendency and calculating areas of shapes and volumes of prisms

© Department of Education WA 2012 – MATHSAC024

Problem Solving includes formulating and solving authentic problems using numbers and measurements, working with transformations and identifying symmetry, calculating angles and interpreting sets of data collected through chance experiments

Reasoning includes applying the number laws to calculations, applying known geometric facts to draw conclusions about shapes, applying an understanding of ratio and interpreting data displays

General capabilities

General capabilities	1	2	3	4	5	6	7	R
Literacy	■	■	■	■	■	■	■	■
Numeracy	■	■	■	■	■	■	■	■
Information and communication technology (ICT) capability		■						
Critical and creative thinking					■	■	■	■
Personal and social capability				■				
Ethical behaviour								
Intercultural understanding								

■ Indicates general capabilities are explicitly covered in that section of the resource.

Cross-curriculum priorities

Cross-curriculum priorities	1	2	3	4	5	6	7	R
Aboriginal and Torres Strait Islander histories and cultures							■	
Asia and Australia's engagement with Asia	■				■			
Sustainability								

■ Indicates cross-curriculum priorities are explicitly covered in that section of the resource.

This resource contains extracts from The Australian Curriculum Version 3.0 © Australian Curriculum, Assessment and Reporting Authority 2012.
ACARA neither endorses nor verifies the accuracy of the information provided and accepts no responsibility for incomplete or inaccurate information. In particular, ACARA does not endorse or verify that:

- the content descriptions are solely for a particular year and subject
- all the content descriptions for that year and subject have been used
- the author's material aligns with the Australian Curriculum content descriptions for the relevant year and subject.

You can find the unaltered and most up to date version of this material at www.australiancurriculum.edu.au. This material is reproduced with the permission of ACARA.

creativecommons.org/licenses/by-nc-sa/3.0/au/

© Department of Education WA 2012 – MATHSAC024

1. Plan (drawing)

When you complete this section you should be able to:

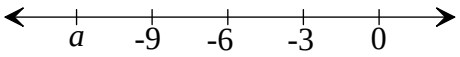
- represent three-dimensional objects using plans.

Key words

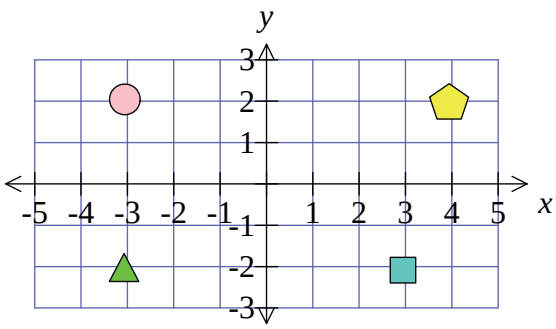
- two-dimensional
- three-dimensional
- plan
- elevation

Warm-up 1

1. Circle the factors of 12. 1, 2, 3, 4, 5, 6
2. $6 + 8 =$ _____
3. What is the missing number?



$a =$ _____

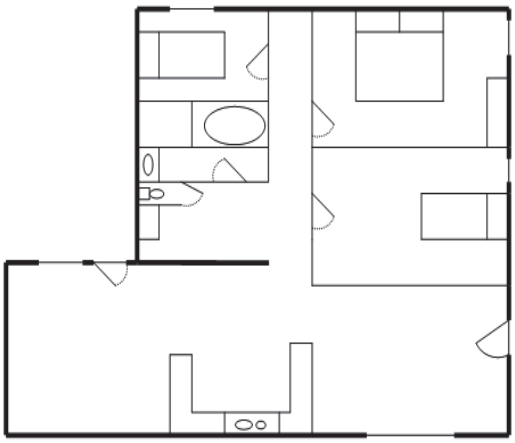
4. Circle the greater fraction. $\frac{1}{4}$ or $\frac{2}{6}$
5. What is a half of 20? _____
6. $9.1 + 1.2 =$ _____
7. $6.3 \times 4 =$ _____
8. Write 0.01 as a fraction. _____
9. Complete: 26, 32, 38, _____
10. 

What shape is at (4, 2)? _____

© Department of Education WA 2012 – MATHSAC024

Focus problem 1

This shows Veronica's house **plan**. It is a three bedroom, one bathroom house.



In the space below, draw a simple house plan of your home, or another familiar house or building.

Check your work before continuing.

Glass windows wonders

There is evidence that people from the Stone Age used naturally occurring pieces of glass for cutting. Some 5000 years ago, the Egyptians used glass to create simple items of jewellery. However, when did glass windows come about?

In ancient China and Japan, paper windows were cheap and widely used. In other places, windows were just holes in the wall, sometimes covered by fur or other animal skins. Wooden shutters came into vogue much later. It is believed that the first use of glass windows occurred 2000 years ago in Alexandria, Egypt.

It was only in the late 17th century that ordinary homes started to have glass windows, when production standards improved.

© Department of Education WA 2012 – MATHSAC024

Skills development 1.1

Plans are a set of **two-dimensional** drawings used to describe **three-dimensional** objects. These plans may consist of different views, such as the top (roof) plan, front and side **elevations**.

Example

Draw the following views of the car:

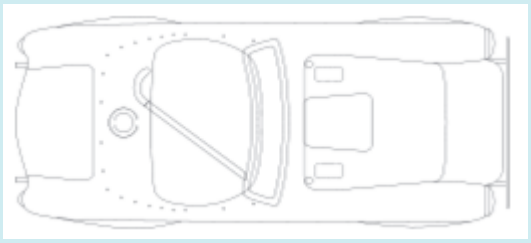
(a) top plan

(b) front elevation

(c) side elevation.

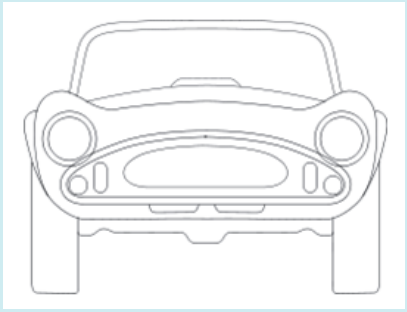
Solution

(a) top plan



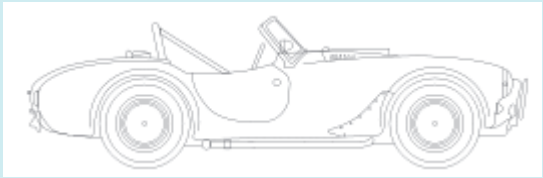
But where has the colour gone?

(b) front elevation



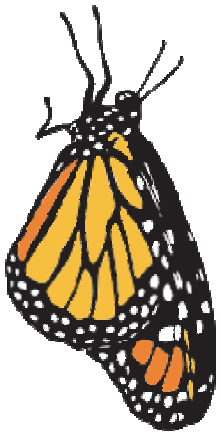
Plans are sketches so you don't need any colour. However, it is important to use rulers for the straight edges.

(c) side elevation

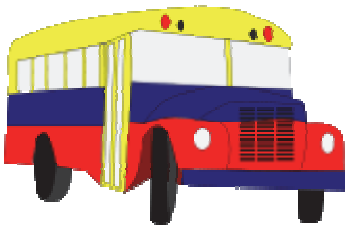


© Department of Education WA 2012 – MATHSAC024

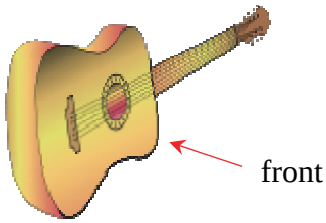
1. Draw the top plan of the Monarch butterfly with its wings fully opened.



2. Draw the side **elevation** of the bus.



3. Draw the front elevation of the guitar.



Check your work before continuing.

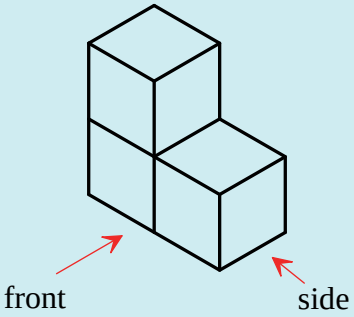
© Department of Education WA 2012 – MATHSAC024

Skills development 1.2

Technical plans, such as those used in engineering and architecture, will often consist of multiple views. This helps in describing correctly the intended **three-dimensional** shape.

Example

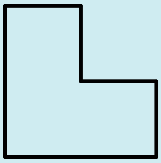
Draw the top **plan**, front and side **elevations** of the following shape.



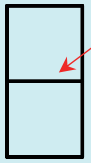
Solution

top plan

front elevation



side elevation

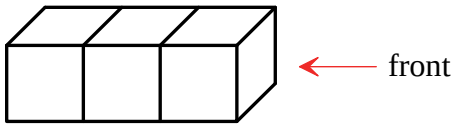


Note that the line in the middle shows a change in depth. (The top view also shows a change in depth.)

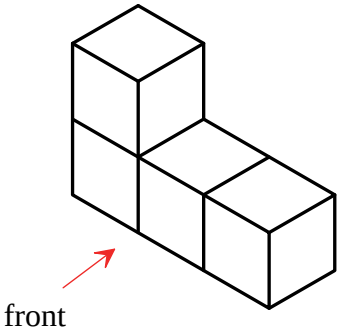
Also note how the front has no change in depth so only the outline of the face is used.

1. Draw the top plan, front and side elevations for each of the following.

(a)

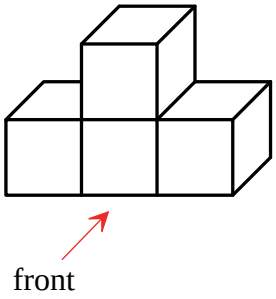


(b)

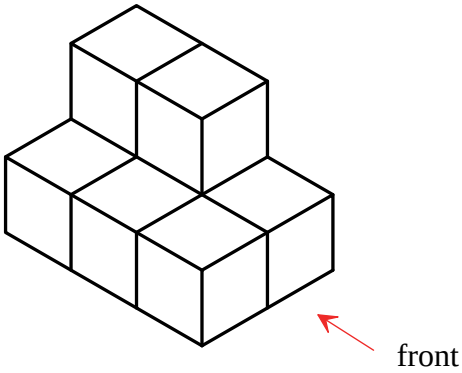


© Department of Education WA 2012 – MATHSAC024

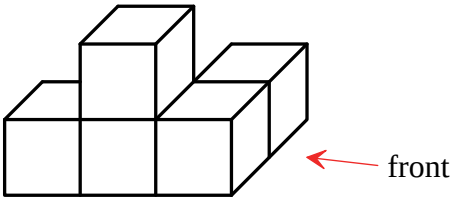
(c)



(d)



(e)



2. Draw the top **plan**, front and side **elevations** of a familiar object.

Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

2. Oblique projection

When you complete this section you should be able to:

- draw oblique projections.

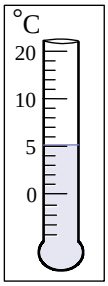
Key words

- oblique projection

Warm-up 2

1. Circle the common factor of 9 and 21. 2, 3, 4, 5, 6

2. $16 - 9 =$ _____



3. The temperature was 5 degrees but it dropped 8 degrees.

   What is the new temperature? _____

4. Insert <, > or = to make the following sentence true.

   $\frac{2}{5} \square \frac{1}{4}$

5. $\frac{2}{2} \times 100 =$ _____

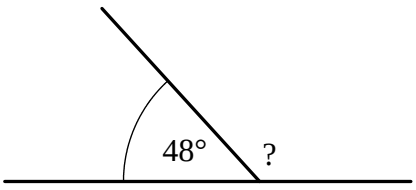
6. Round 7.5 to a whole number. _____

7. $4\overline{)32.8}$

8. Write $66\frac{2}{3}\%$ as a decimal. _____

9. Complete: 3.6, 3.9, 4.2, _____

10. Determine the size of the missing angle.



_____

© Department of Education WA 2012 – MATHSAC024

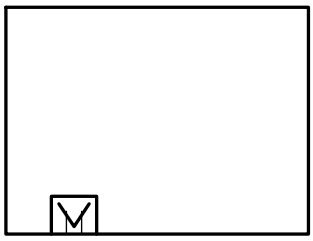
Focus problem 2

Giles had a plan drawn up for an integrated rail unloader facility, where the storing of the ore takes place before conveyance onto the ships. The plan is shown below.

top plan

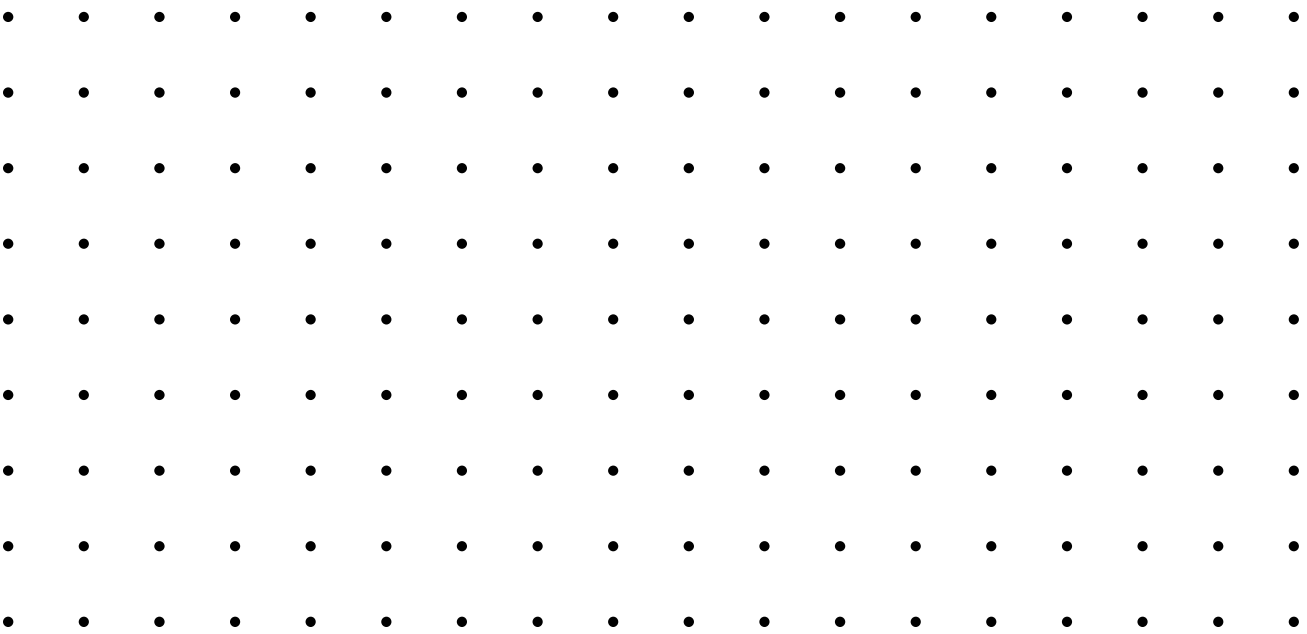
front elevation

side elevation



The problem was that Giles could not visualise what the facility looked like, so he requested someone to draw him a face-on drawing (**oblique projection**).

Draw an oblique projection of the facility below.

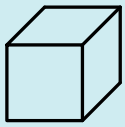


Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

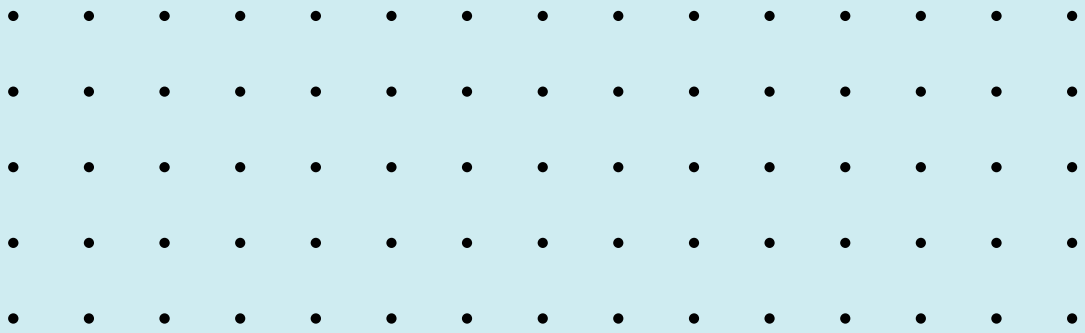
Skills development 2

Oblique projection (or face-on drawing)

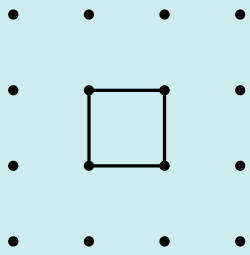


This is a two-dimensional representation of a cube.

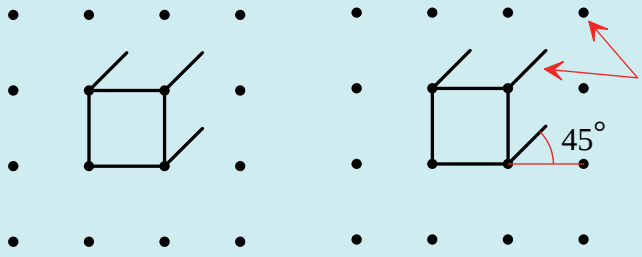
Redraw the cube using the dots and the steps below. You may like to draw a few.



Step 1 – Draw the front face.

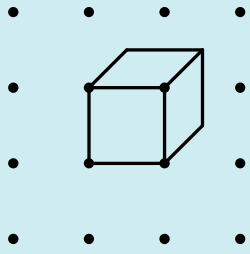


Step 2 – Draw the slanted or oblique lines at an angle and halfway to the next dot.



The slanted line is drawn at an angle, halfway to the next dot.

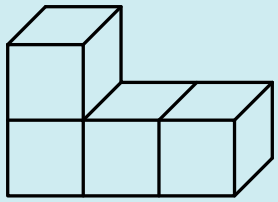
Step 3 – Complete the shape. (Add the other vertical and horizontal lines.)

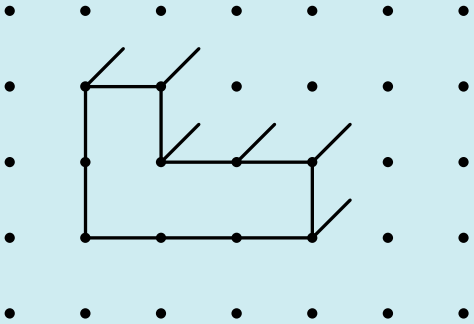


© Department of Education WA 2012 – MATHSAC024

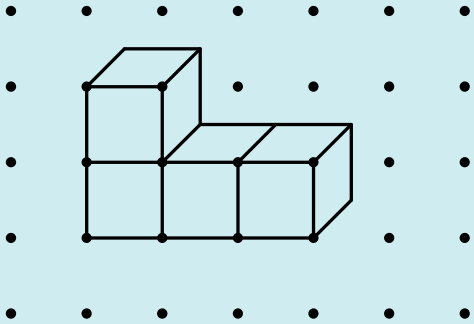
Example

Complete the copy of the shape.



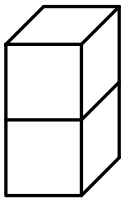


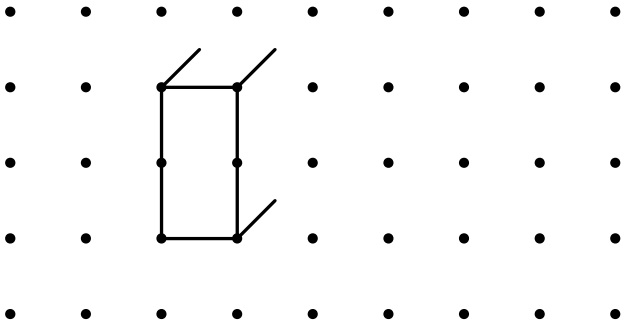
Solution



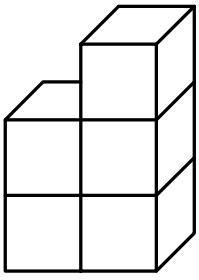
1. Complete the copy of each shape below.

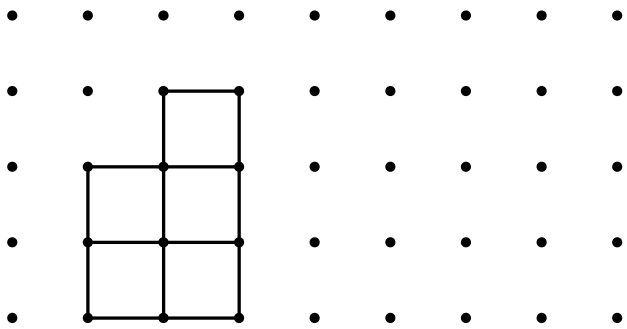
(a)





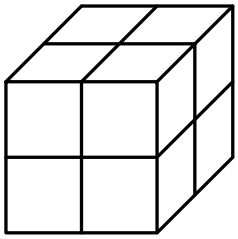
(b)

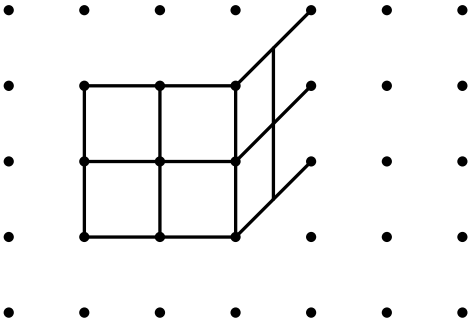




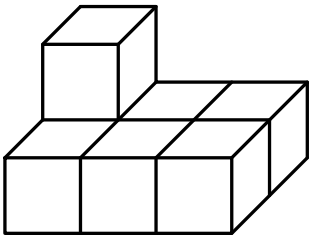
© Department of Education WA 2012 – MATHSAC024

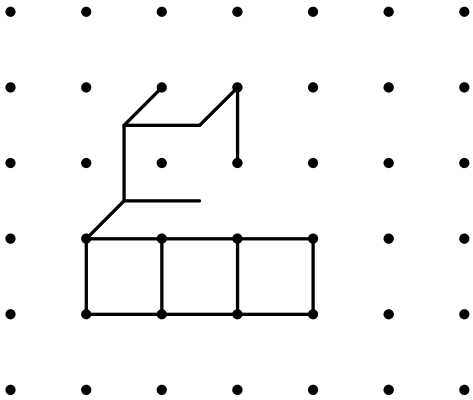
(c)



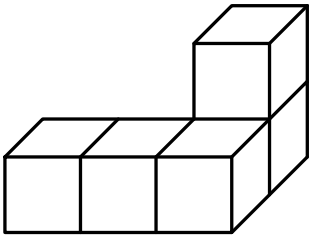


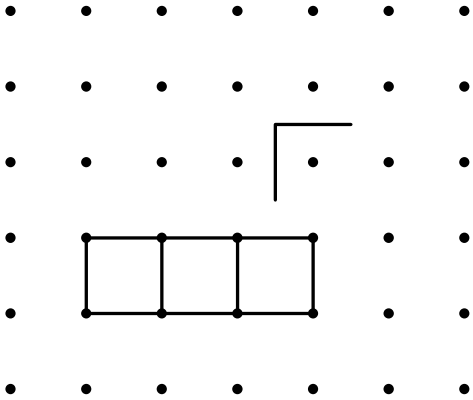
(d)





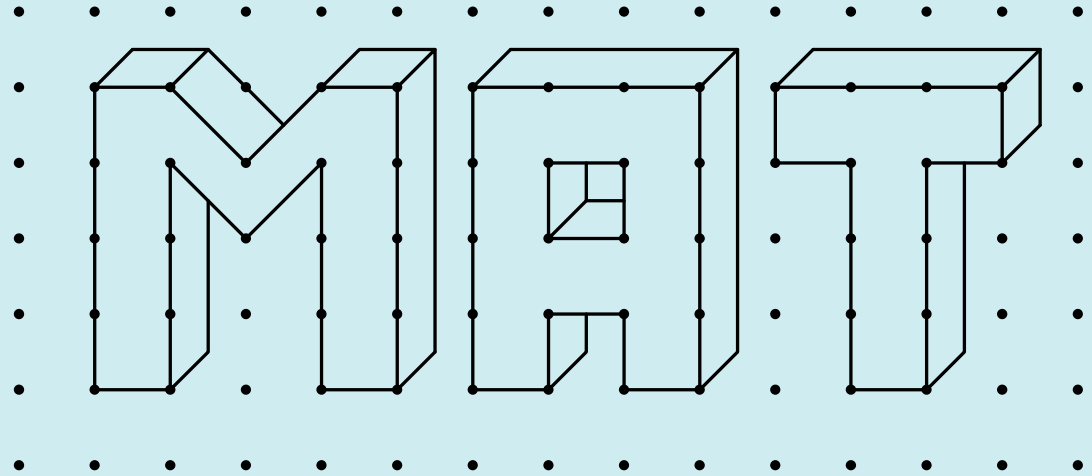
(e)





© Department of Education WA 2012 – MATHSAC024

The following shows how the name ‘MAT’ can be drawn using an **oblique projection**.



2. Draw your name (or initials) using an oblique projection.

3. If you have access to a computer, use a drawing package to draw two-dimensional representations of some simple three-dimensional objects.

Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

3. Isometric projection

When you complete this section you should be able to:

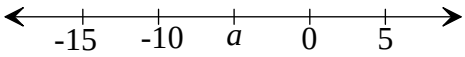
- draw isometric projections.

Key words

- isometric projection

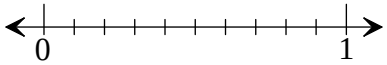
Warm-up 3

1. Circle the prime number. 4, 7, 9, 12, 15
2. $4 \times 4 =$ _____
3. What is the missing number?



$a =$ _____

4. Locate $\frac{1}{4}$ on the number line.

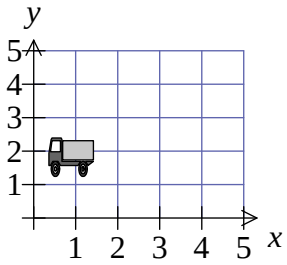


5. Find a quarter of 40. _____
6. Estimate the difference by first rounding to whole numbers.

$8.9 - 2.9 \approx$ _____

7.
$$\begin{array}{r} 1.1 \\ \times \quad 6 \\ \hline \end{array}$$

8. Write $\frac{1}{8}$ as a percentage. _____
9. Complete: $\frac{1}{10}, \frac{4}{10}, \frac{7}{10},$ _____
10.



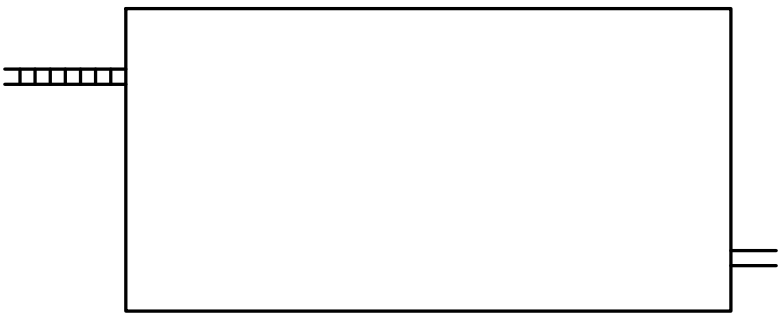
At what point is the truck? _____

© Department of Education WA 2012 – MATHSAC024

Focus problem 3

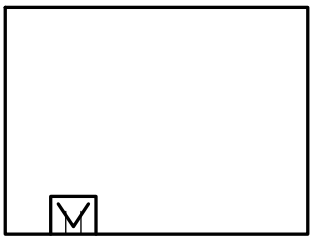
Giles' manager was displeased wish the oblique drawing of the facility. She thought it looked strange so she asked for it to be redrawn using an **isometric projection**.

top plan

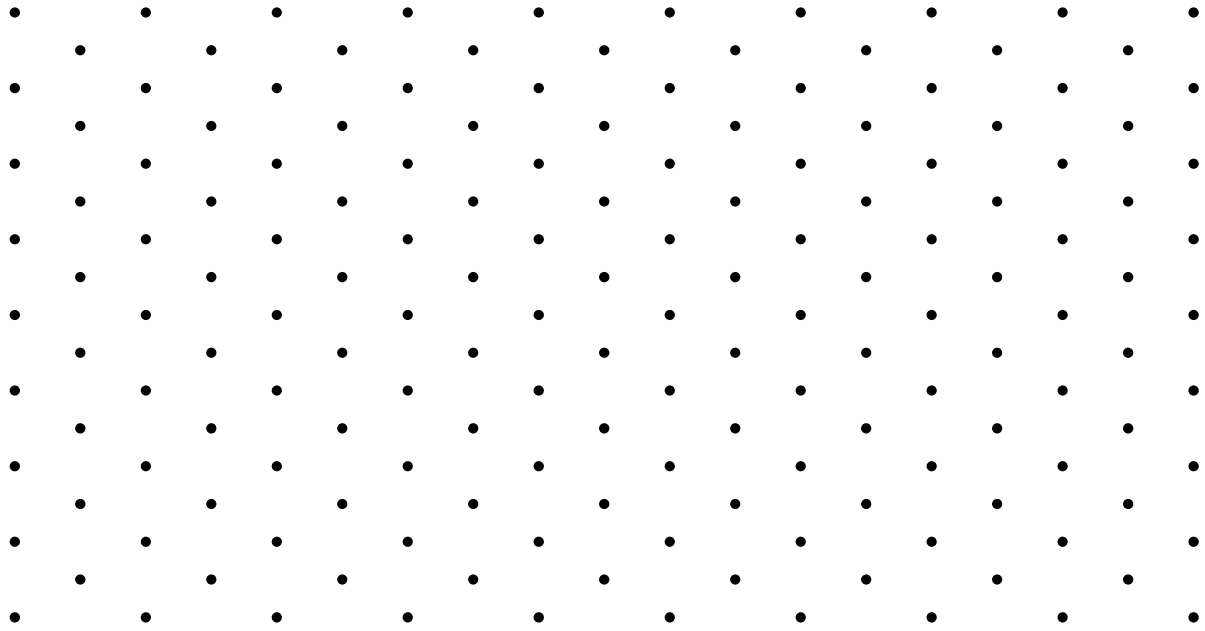


front elevation

right elevation



Draw an isometric projection of the facility below.

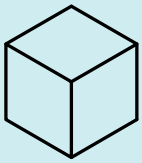


Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

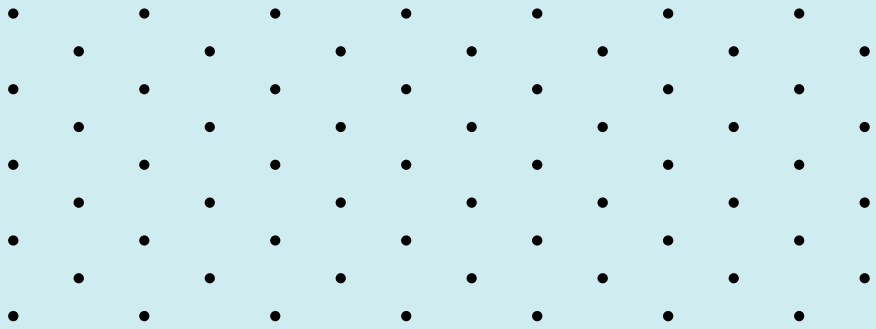
Skills development 3.1

Isometric projection (or corner drawing)

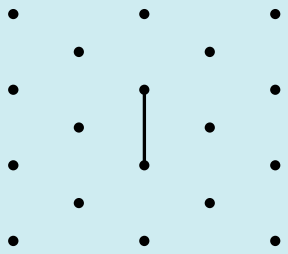


This is a two-dimensional representation of a cube.

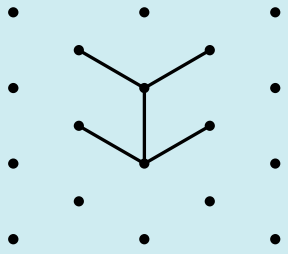
Redraw the cube using the dots and the steps below. You may like to draw a few.



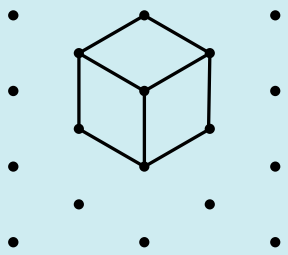
Step 1 – Draw the front edge.



Step 2 – Draw the diagonal lines to the next dot.



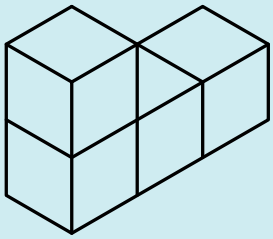
Step 3 – Complete the shape.



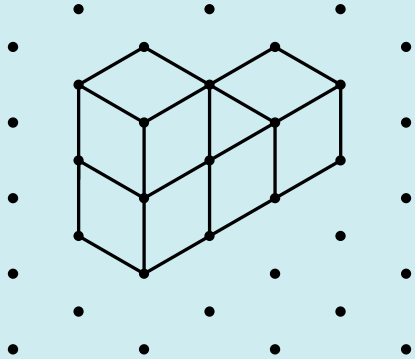
© Department of Education WA 2012 – MATHSAC024

Example

Complete the drawing using an **isometric projection**.

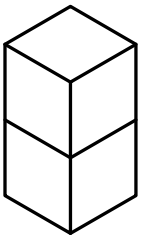


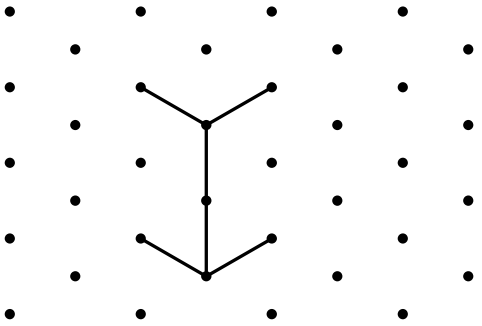
Solution



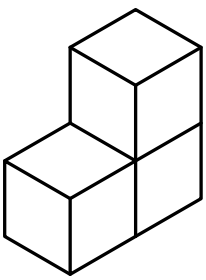
1. Complete the copy of each shape below.

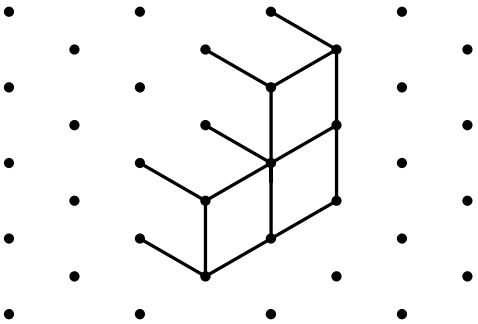
(a)





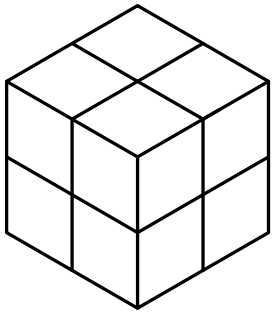
(b)

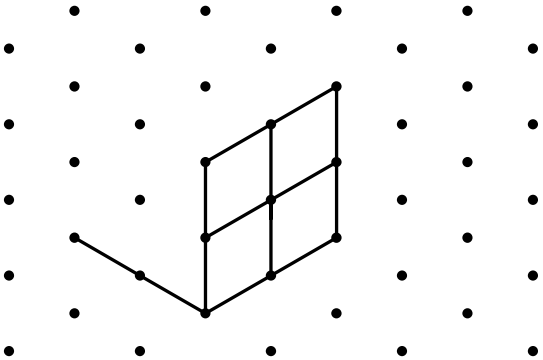




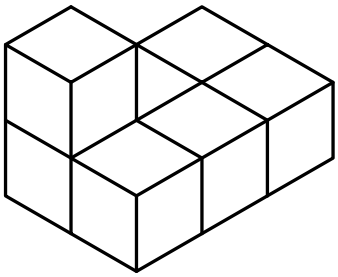
© Department of Education WA 2012 – MATHSAC024

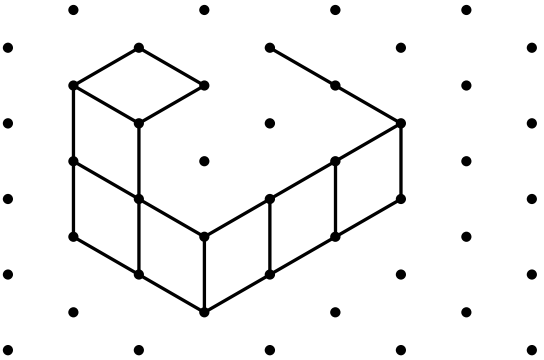
(c)



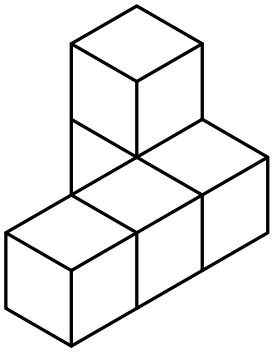


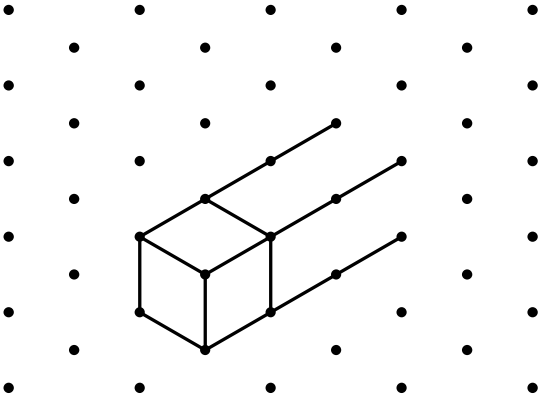
(d)





(e)





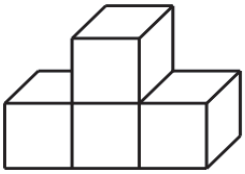
Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

Skills development 3.2

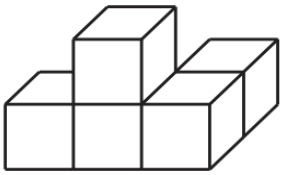
Match each of the following oblique projections with their **isometric projection** equivalent.

A

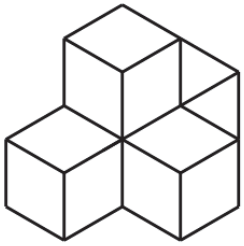


I

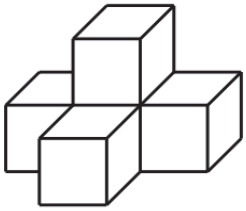
B



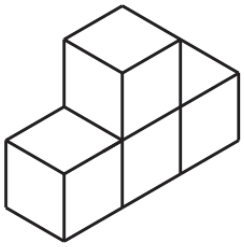
II



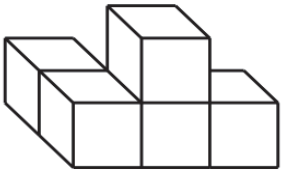
C



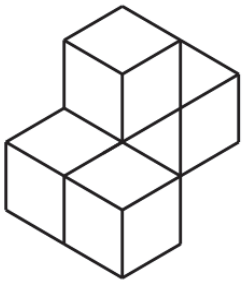
III



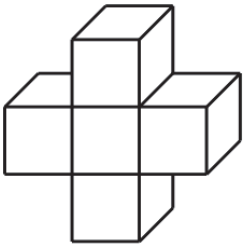
D



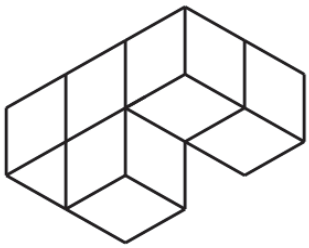
IV



E



V

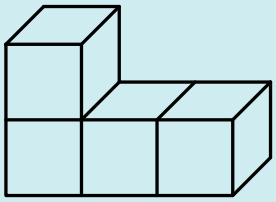


© Department of Education WA 2012 – MATHSAC024

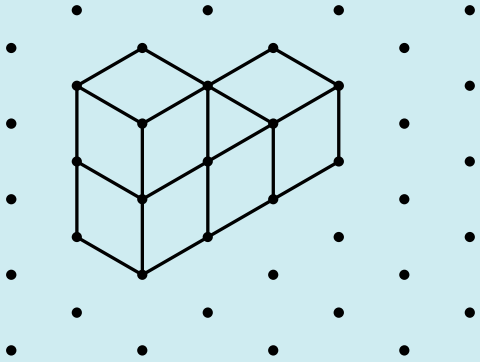
Skills development 3.3

Example

Redraw the following using an **isometric projection**.

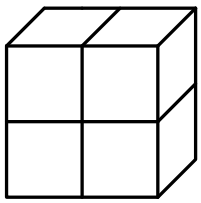


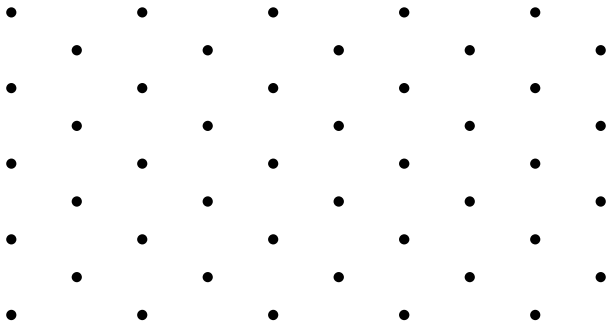
Solution



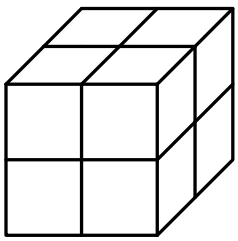
1. Redraw each of the following shapes using an isometric projection.

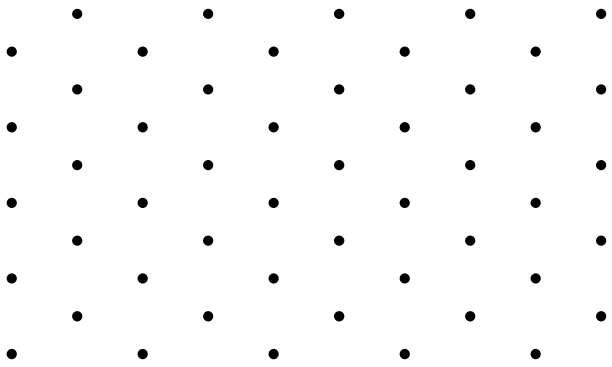
(a)





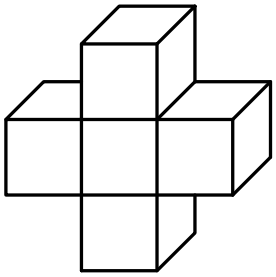
(b)

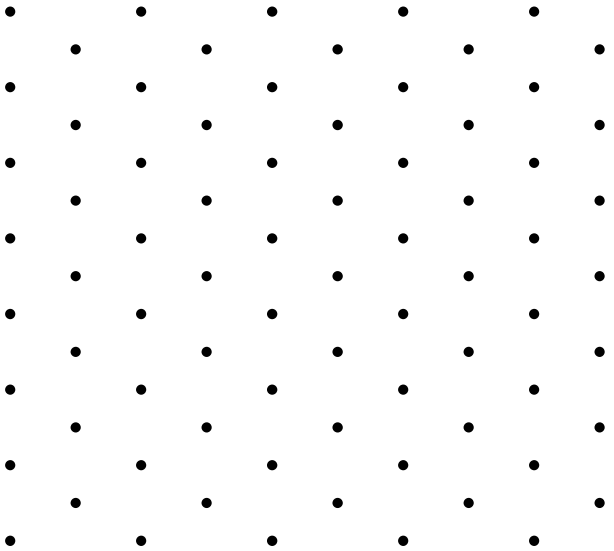




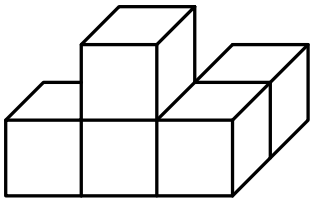
© Department of Education WA 2012 – MATHSAC024

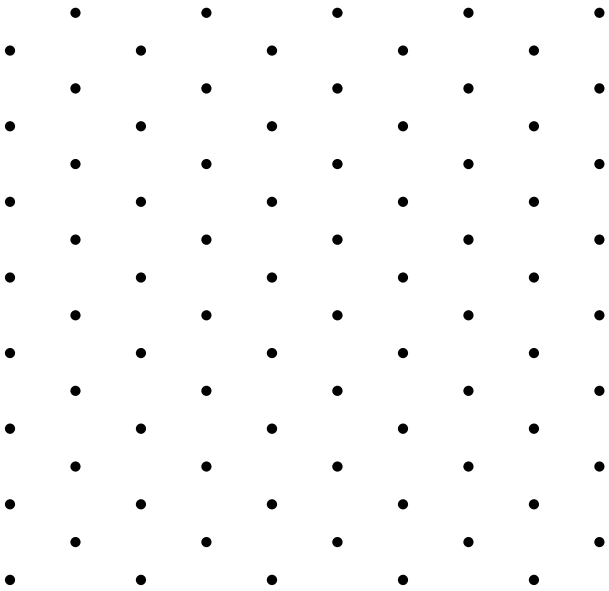
(c)





(d)



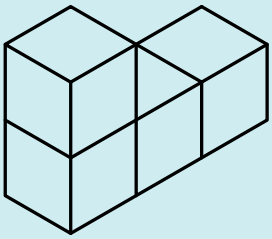


Check your work before continuing.

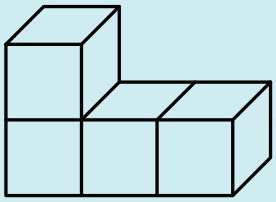
© Department of Education WA 2012 – MATHSAC024

Skills development 3.4

Example
Redraw the following using an oblique projection.

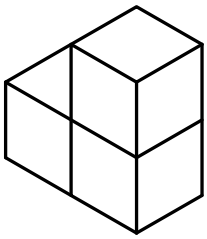


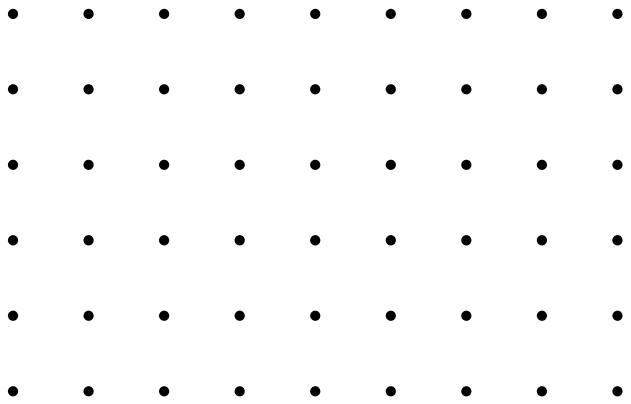
Solution



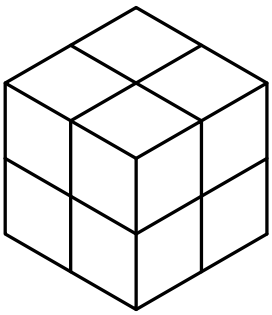
1. Redraw each of the following shapes using an oblique projection.

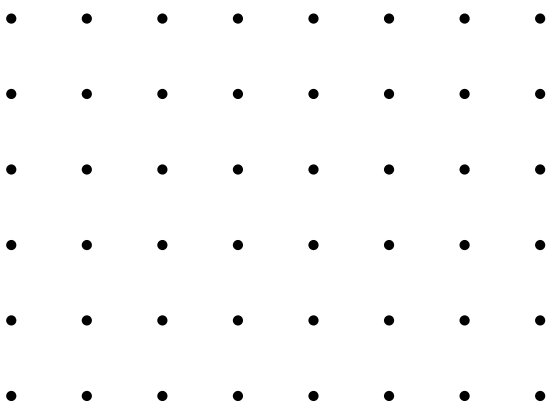
(a)





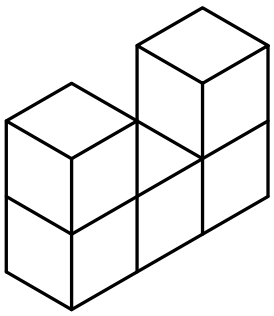
(b)

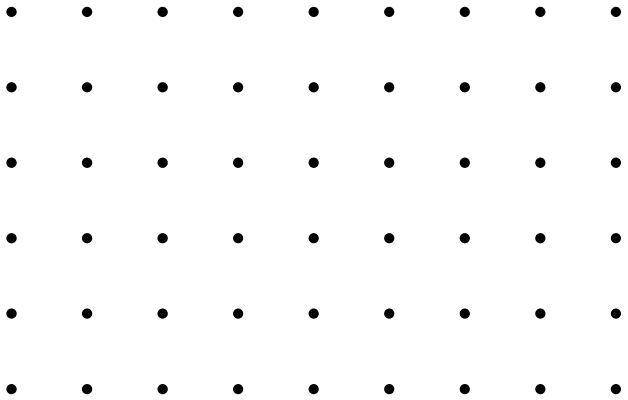




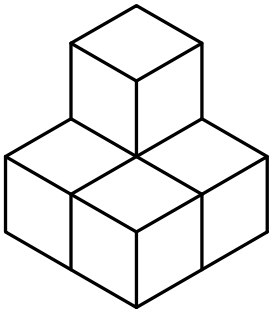
© Department of Education WA 2012 – MATHSAC024

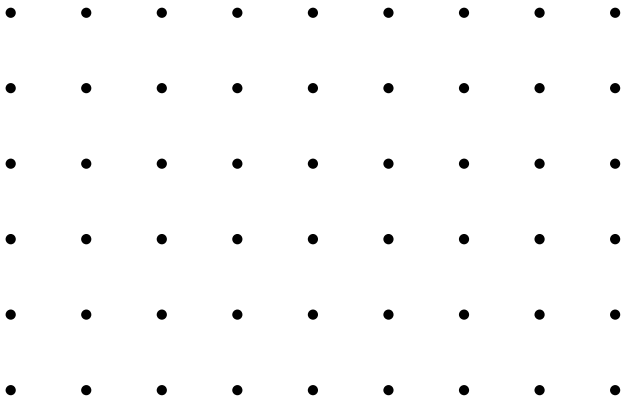
(c)



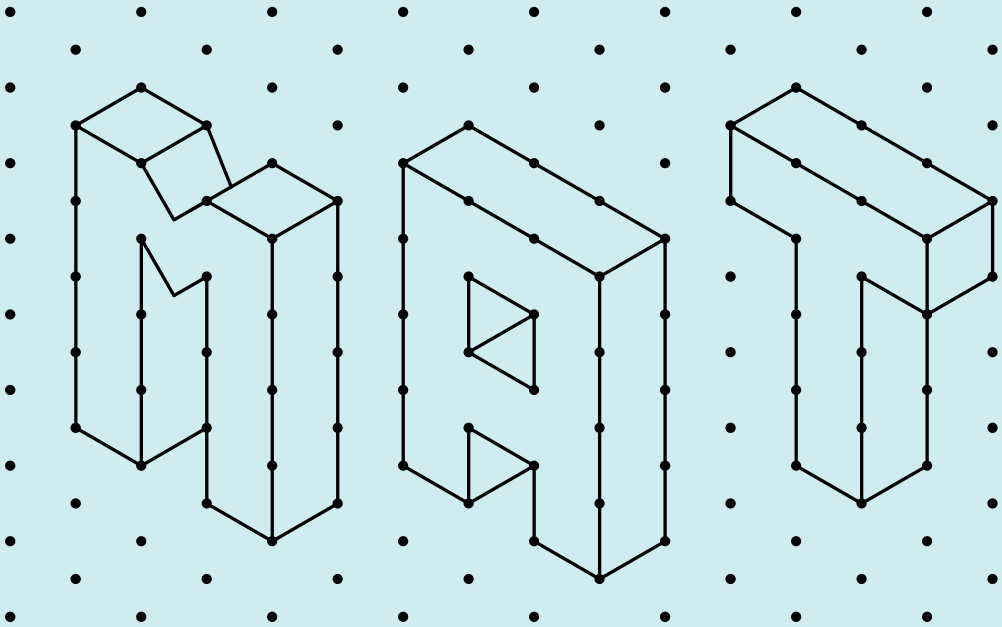


(d)



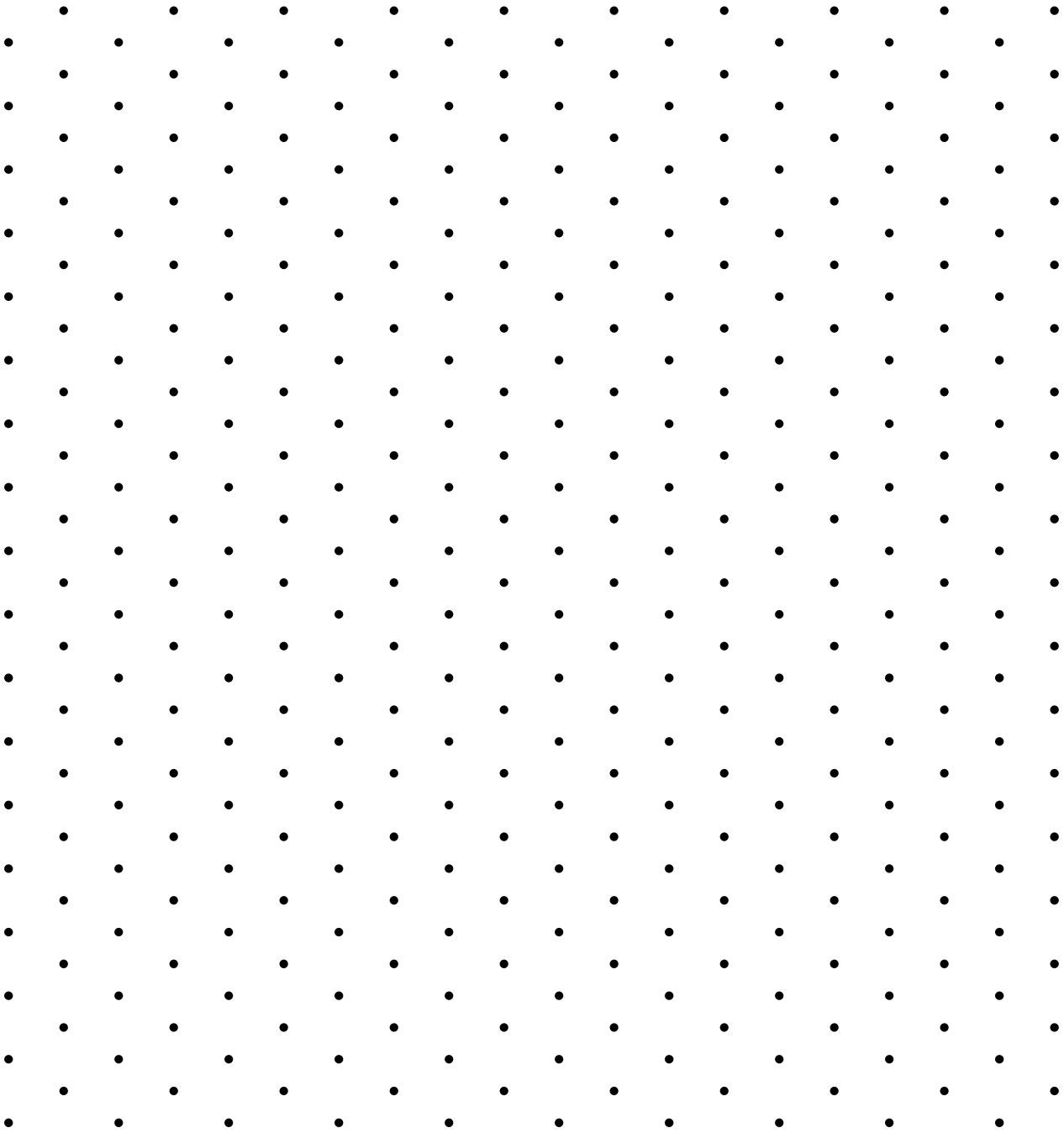


The following shows how the name ‘MAT’ can be drawn using an **isometric projection**.



© Department of Education WA 2012 – MATHSAC024

2. Draw your name (or initials) using an **isometric projection**.



Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

© Department of Education WA 2012 – MATHSAC024

4. Volume

When you complete this section you should be able to:

- determine the volume of objects made from cubes.

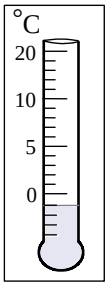
Keywords

- volume
- cubic units

Warm-up 4

1. A composite number has _____ or more factors.

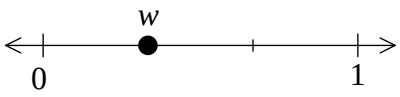
2. $64 \div 8 =$ _____



3. The temperature was minus 1 degrees but it went up 8 degrees.

   What is the new temperature? _____

4. Express the value of w as a fraction.



$w =$

5. $\frac{1}{3} \times 24 =$ _____

6. $0.0009 \times 100 =$ _____

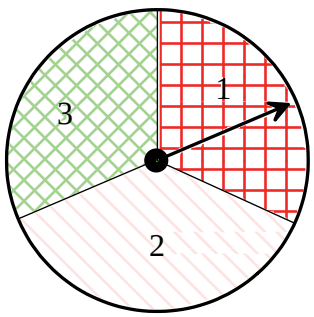
7. $4.172 \div 4 =$ _____

8. Write 30% as fraction.

9. Complete: 82, 76, 70, _____

10. Determine the probability that the spinner will land on a 1.

    Express your answer as a percentage.

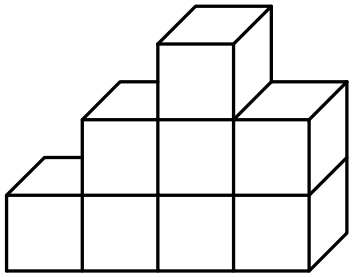


_____

© Department of Education WA 2012 – MATHSAC024

Focus problem 4

Mike is working for a supermarket and a pallet of boxes of cereal arrived. He unpacked the pallet, placing the boxes against the wall, as shown below.



Unfortunately, he did not check the number of boxes off the invoice and now will have to recount them.

How many boxes of cereal are there?

Check your work before continuing.

Cereals

The word ‘cereal’ describes grasses, such as wheat and barley. Some 12 000 years ago, these grasses were domesticated. The grains from these crops provide a rich source of vitamins, minerals and carbohydrates. However, when refined, these grains lose most of their benefits.

As an example, some breakfast cereals are in fact unhealthy. With the addition of salt, sugar and other preservatives, some breakfast cereals resemble a cocktail of chemicals, rather than ‘real’ food.

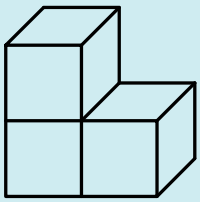
Breakfast is the most important meal of the day, so do not waste the opportunity to feed your body with ‘good’ food.

© Department of Education WA 2012 – MATHSAC024

Skills development 4

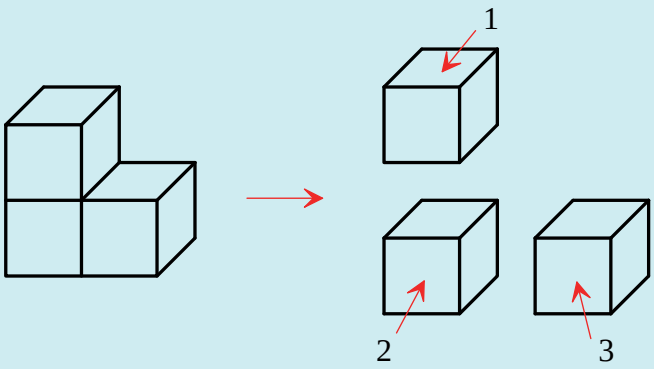
Example

Determine the **volume**.



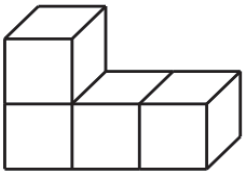
Solution

The volume is 3 cubes (or **cubic units**).

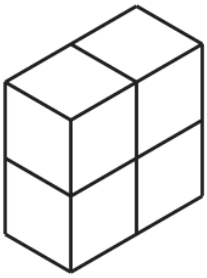


1. Determine the volume for each of the following.

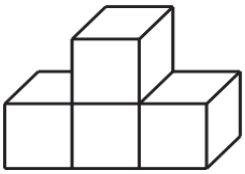
(a)



(b)

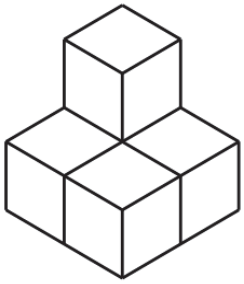


(c)

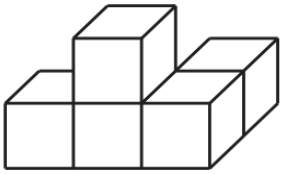


© Department of Education WA 2012 – MATHSAC024

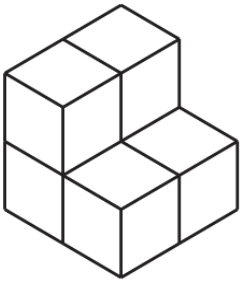
(d)



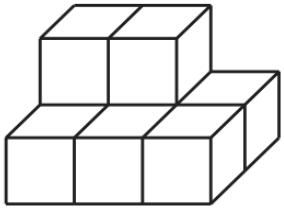
(e)



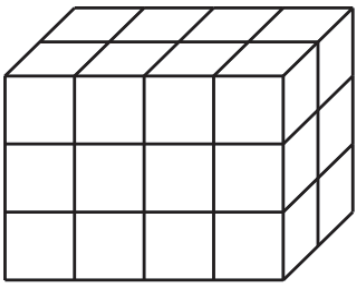
(f)



(g)



(h)



© Department of Education WA 2012 – MATHSAC024

2. Draw an object with a **volume** of 15 cubes.

Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

© Department of Education WA 2012 – MATHSAC024

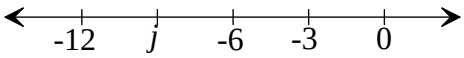
5. Volume of rectangular prisms

When you complete this section you should be able to:

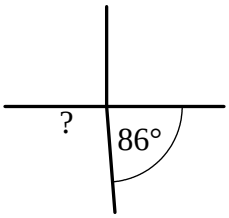
- determine the volume of rectangular prisms.

Warm-up 5

1. Circle the prime factors of 15. 1, 3, 5, 15.
2. 153 + 26 = _____
3. What is the missing number?



   j = _____
4. $\frac{3}{2} + \frac{2}{2} =$ _____
5. Find two-fifths of 35. _____
6. 4206 m = _____ km
7. 4 + 16 ÷ 4 = _____
8. Write $3\frac{1}{2}$ as decimal. _____
9. Complete: 10.4, 9.9, 9.4, _____
10. Determine the size of the missing angle.

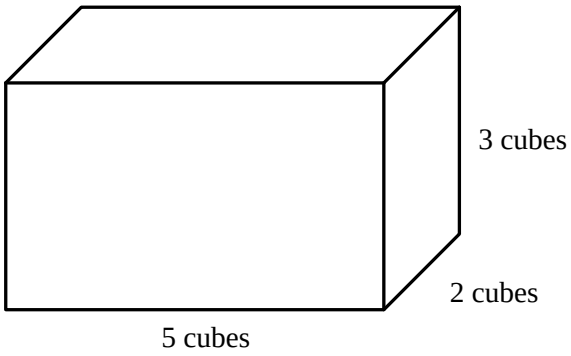


_____

© Department of Education WA 2012 – MATHSAC024

Focus problem 5

Sasha had a box filled with cubes as shown.



Sasha wanted to know how many cubes she had altogether.

See if you can help Sasha determine the number of cubes.

Check your work before continuing.

Aquatic cube?

Created for the 2008 Olympic Games, Beijing's National Aquatic Centre was nicknamed the 'Water Cube'. However, the building was in the shape of a rectangular prism and not a cube.

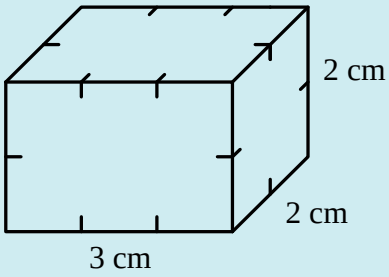
Although a cube is a special type of rectangular prism, it is composed of six square faces, whereas the 'Water Cube' was in fact comprised of oblong rectangles.

© Department of Education WA 2012 – MATHSAC024

Skills development 5.1

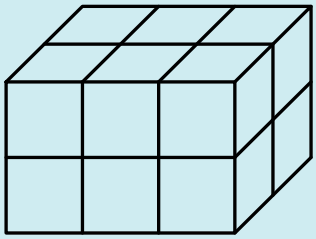
Example

Determine the volume of the following shape.



Solution

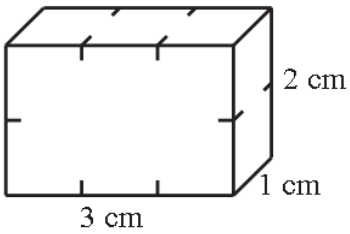
One way to find the volume is to divide the shape up into cubic units and count them.



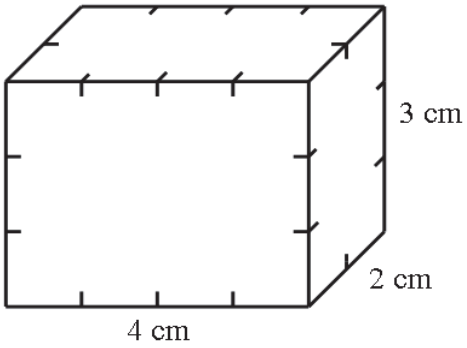
∴The volume of the shape is 12 cubic units, or for this example, 12 cubic centimetres (cm^3).

1. Find the volume of the following.

(a)

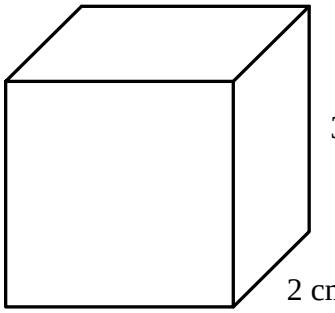


(b)

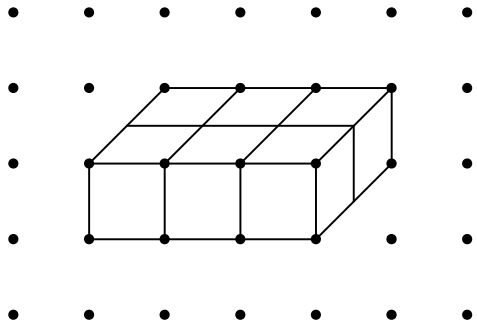


© Department of Education WA 2012 – MATHSAC024

(c)



2. The following rectangular prism has a length of 3 cm and a width of 2 cm. Its volume is 6 cm^3.



In the space below, draw rectangular prisms with a length of 3 cm and a width of 2 cm, but with the following volumes.

(a) 12 cm^3

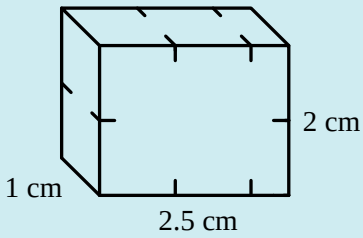
(b) 18 cm^3

© Department of Education WA 2012 – MATHSAC024

Skills development 5.2

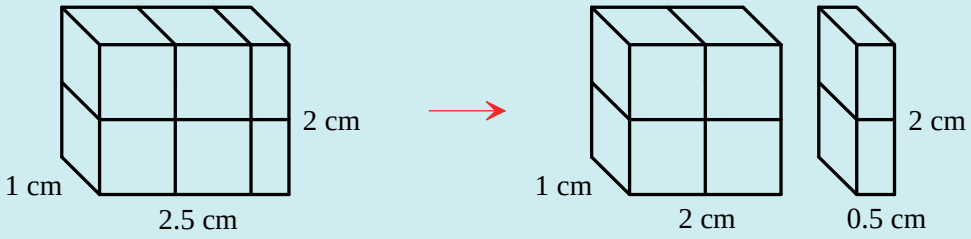
Example

Determine the volume of the following shape.



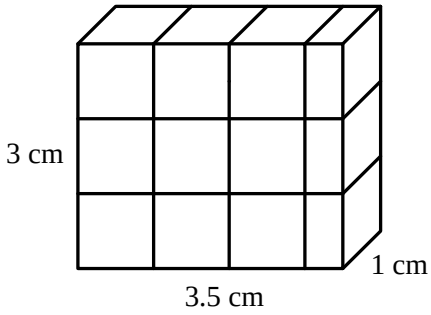
Solution

One way to find the volume is to divide the shape up into cubic units, as before. However, this is slightly different as there are also parts of cubes. Combine the parts of cubes together to make whole cubes and then add them to the count.

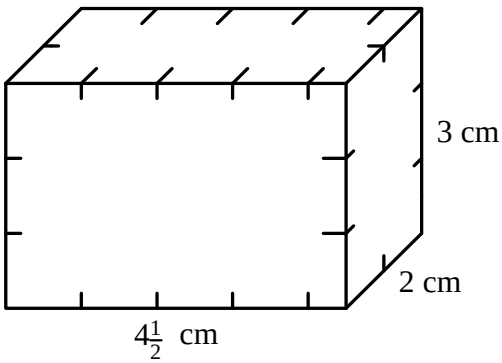


$\therefore$ The volume of the shape is 5 cubic units, or for this example, 5 cubic centimetres (cm^3).

1. (a)

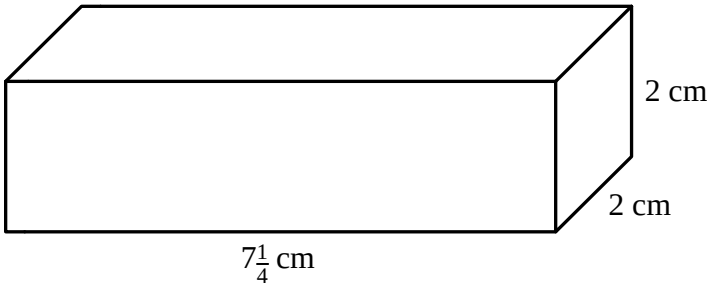


(b)



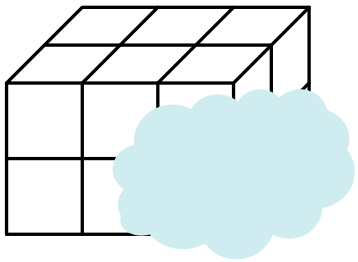
© Department of Education WA 2012 – MATHSAC024

(c)

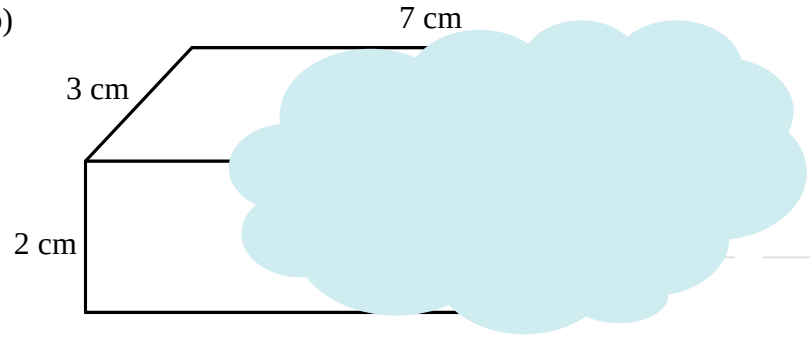


2. Determine the volume for each of the following rectangular prisms. Part of each object is covered, so you may have to come up with another method to find the volume.

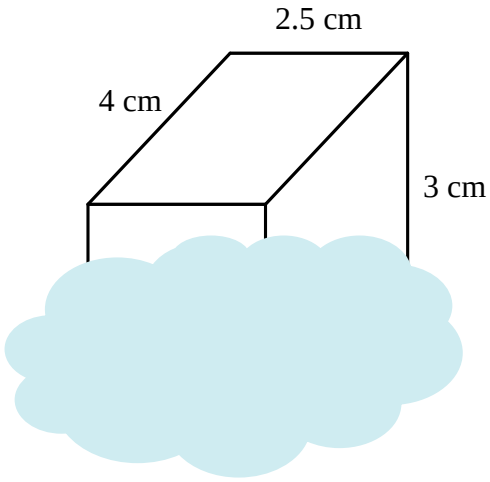
(a)



(b)



(c)



Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

6. Rectangular prism formula

When you complete this section you should be able to:

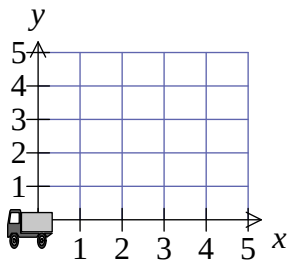
- use the formula for finding the volume of rectangular prisms.

Keywords

- formula

Warm-up 6

1. $6.5 \times 10 =$ _____
2. $45 - 19 =$ _____
3. The temperature is minus 2 degrees.

   How much will it need to increase to get to 1 degree? _____
4. $\frac{5}{10} - \frac{2}{10} =$
5. $\frac{1}{8} \times 40 =$ _____
6. 3200 mg = _____ g
7. $3 \times 6 + 4 =$ _____
8. Write 1 as a percentage. _____
9. $4\frac{6}{9}, 3\frac{4}{9}, 2\frac{2}{9},$ _____
10. 

    The truck is shown at (0, 0).

    If the truck moves 3 units up, where will it then be?

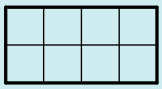
    _____

© Department of Education WA 2012 – MATHSAC024

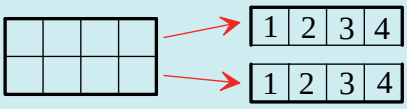
Review 6.1

Example

Determine the area of the rectangle.



Solution



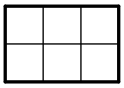
2 rows of 4

$\therefore 2 \times 4 = 8$

$\therefore$ The area of the rectangle is 8 square units.

1. Complete the following to determine the area of each of the rectangles.

(a)

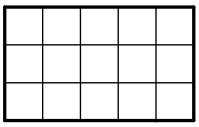


2 rows of _____

$2 \times 3 =$ _____

$\therefore$ The area of the rectangle is _____ square units.

(b)

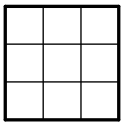


_____ rows of _____

_____ × _____ = _____

$\therefore$ The area of the rectangle is _____ square units.

(c)



_____

_____

$\therefore$ The area of the rectangle is _____ square units.

(d)

_____

_____

$\therefore$ The area of the rectangle is _____ square units.

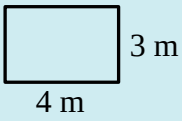
© Department of Education WA 2012 – MATHSAC024

Review 6.2

Example

Use the following **formula** to find the area of the rectangle.

Area (rectangle) = *length* × *width*



Solution

Area (rectangle) = *length* × *width*

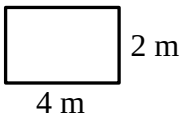
= 4 × 3

= 12

∴ The area of the rectangle is 12 square metres (m^2).

1. Determine the area of each of the following rectangles.

(a)



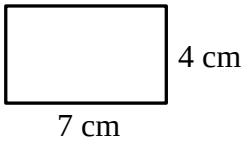
Area (rectangle) = *length* × *width*

= 4 × _____

= _____

∴ The area of the rectangle is _____ m^2.

(b)



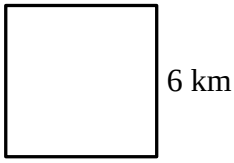
Area (rectangle) = *length* × *width*

= _____ × _____

= _____

∴ The area of the rectangle is _____ cm^2.

(c)

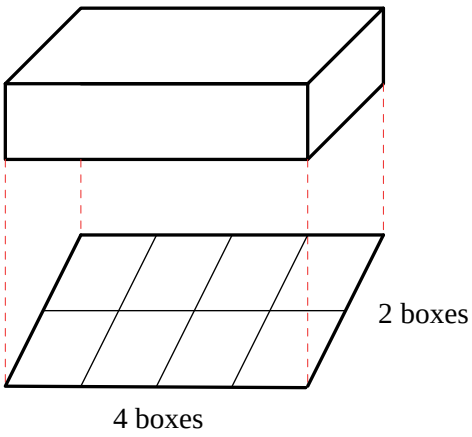


Check your work before continuing.

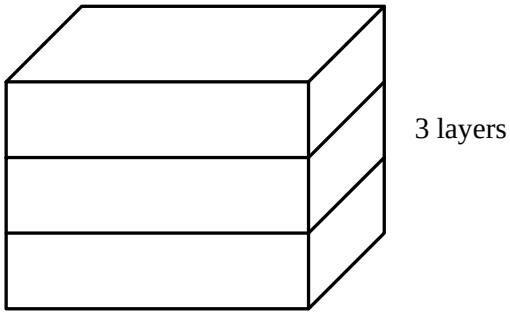
© Department of Education WA 2012 – MATHSAC024

Focus problem 6

Rodney laid out some boxes on the floor in a 4 by 2 grid, as shown below.



He wondered how many boxes he would have if he stacked them three boxes high.



Determine how many boxes Rodney would have in his stack. That is, determine the volume.

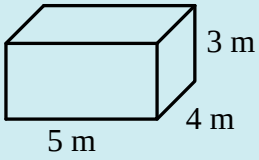
Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

Skills development 6.1

Example

Use the **formula** *Volume = length × width × height* to determine the volume of the rectangular prism.

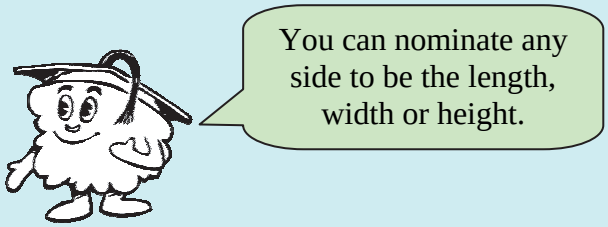


Solution

$V = l \times w \times h$

$= 5 \times 4 \times 3$

$= 60$

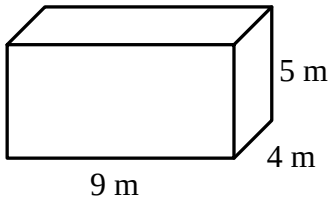


$\therefore$ The volume of the rectangular prism is 60 m^3.

Note that the unit is called 'cubic metres', although it is written as m^3.

1. Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prism.

(a)



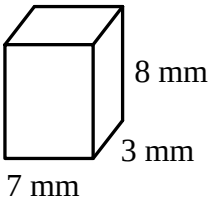
$V = l \times w \times h$

$=$ _____ $\times$ _____ $\times$ _____

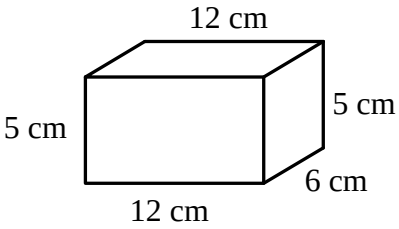
$=$ _____

$\therefore$ The volume is _____ m^3.

(b)



(c)



© Department of Education WA 2012 – MATHSAC024

Skills development 6.2

Example

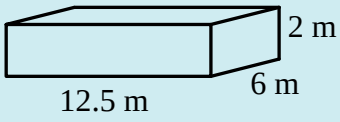
Use the **formula** $V = l \times w \times h$ to determine the volume of the rectangular prism.

Where: V represents volume

l represents length

w represents width

h represents height



Solution

$V = l \times w \times h$

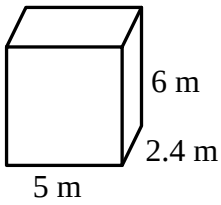
$= 12.5 \times 6 \times 2$

$= 150$

$\therefore$ The volume of the rectangular prism is 150 m^3.

1. Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prism.

(a)



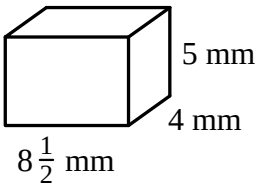
$V = l \times w \times h$

$=$ _____ $\times$ _____ $\times$ _____

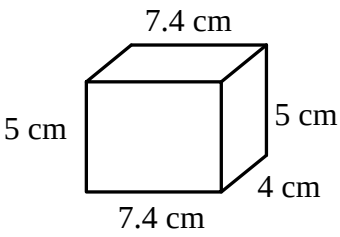
$=$ _____

$\therefore$ The volume is _____ m^3.

(b)



(c)



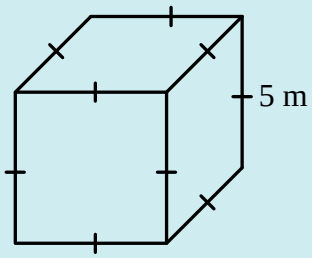
Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

Skills development 6.3

Example

Use the **formula** $V = l \times w \times h$ to determine the volume of the cube.

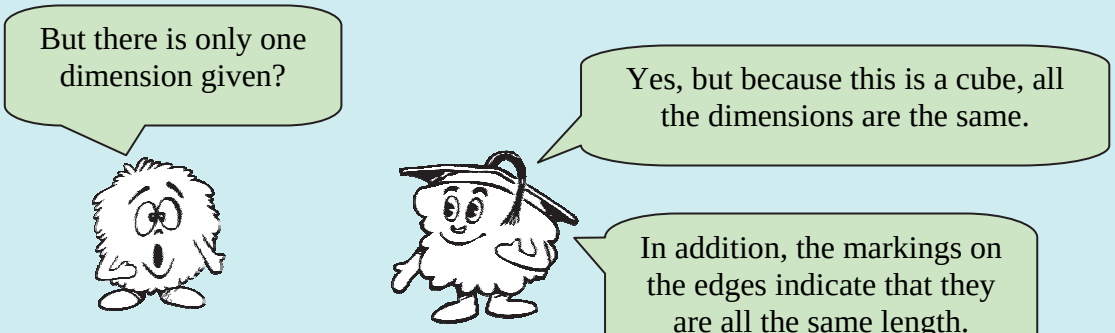


Where: V represents volume

l represents length

w represents width

h represents height



Solution

$V = l \times w \times h$

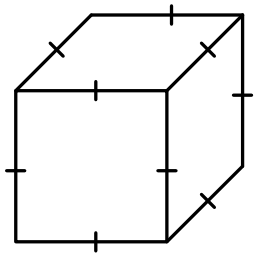
$= 5 \times 5 \times 5$

$= 125$

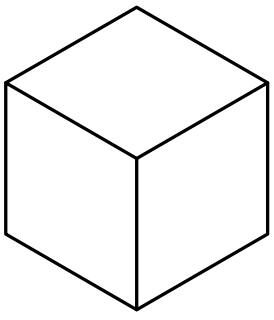
$\therefore$ The volume of the cube is 125 m^3.

1. Use the formula $V = l \times w \times h$ to determine the volume of the cubes below.

(a)



(b)



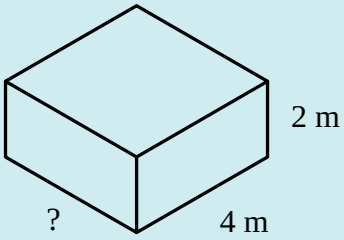
© Department of Education WA 2012 – MATHSAC024

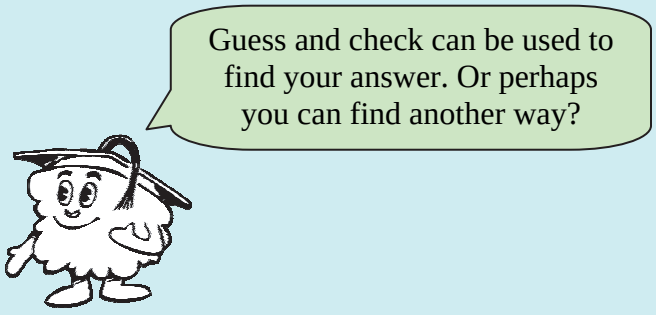
Skills development 6.4

Sometimes you may need to determine one of the dimensions.

Example

Given the volume of a rectangular prism is 24 m^3, the width is 4 m and the height is 2 metres, determine the length.





Solution

$V = l \times w \times h$

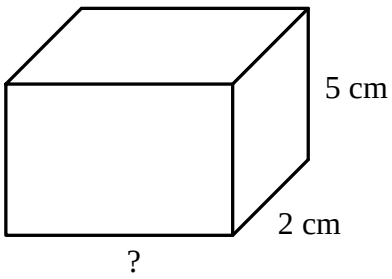
$24 = l \times 4 \times 2$

$\therefore\ l = 3$

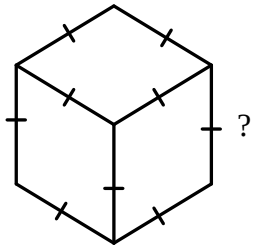
The length of the rectangular prism is 3 m.

1. Given the volumes, determine the missing dimensions for each of the following.

(a) Volume =60 cm^3



(b) Volume =1000 mm^3



Check your work before continuing.

© Department of Education WA 2012 – MATHSAC024

7. Units and the volume formula

When you complete this section you should be able to:

- use the correct units when using the volume formula.

Keywords

- convert

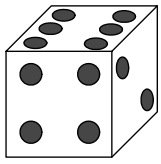
Warm-up 7

1. $63 \div 10 =$ _____
2. $156 \div 3 =$ _____
3. The temperature is 1 degree.

   How much will it need to decrease to get to minus 4 degrees? _____
4. $\frac{1}{2} + \frac{3}{10} =$
5. $\frac{1}{9} \times 45 =$ _____
6. 0.6 kL = _____ L
7. $20 \div (5 \times 2) =$ _____
8. Find 25% of \$120. _____
9. Describe the rule for the following pattern.

   1, 4, 9, 16, 25, …

   _____

   _____
10. 

    A six-sided die is rolled.

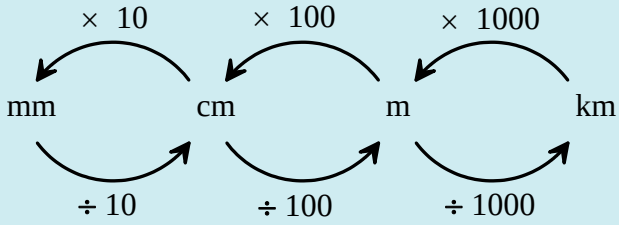
    Express, as a decimal, the probability that it lands on an even number.

    _____

© Department of Education WA 2012 – MATHSAC024

Review 7

When finding areas, you will sometimes need to **convert** units of length. The following diagram may help you to convert some common units of length.



Example

Convert the following units of length.

(a) 2.3 km to m

(b) 3950 mm to m

Solution

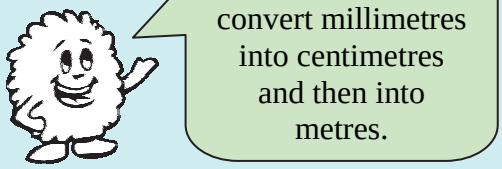
(a) $2.3 \times 1000 = 2300$

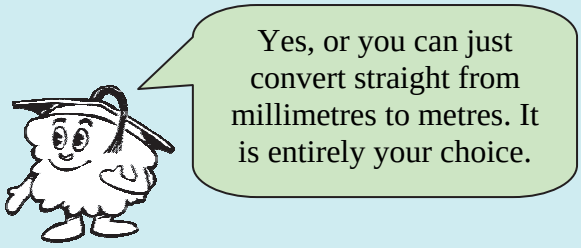
$\therefore 2.3 \text{ km} = 2300 \text{ m}$

(b) $3950 \div 10 = 395$

$395 \div 100 = 3.95$

$\therefore 3950 \text{ mm} = 3.95 \text{ m}$





1. Convert the following units.

   (a) 54 mm = _____ cm

   (b) 23 cm = _____ mm

   (c) 7.8 km = _____ m

   (d) 591 cm = _____ m

   (e) 8.54 m = _____ cm

   (f) 2900 mm = _____ m

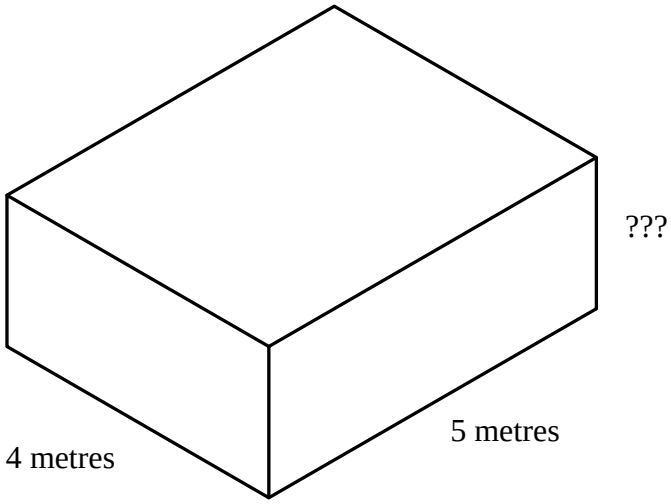
   (g) 6100 m = _____ km

   (h) 1.8 km = _____ cm

© Department of Education WA 2012 – MATHSAC024

Focus problem 7

Carmen wanted to determine the volume of her room. She knew the floor was 4 metres by 5 metres but she did not know the height.



Carmen took her ruler and measured from the floor to the ceiling. She found it to be 210 centimetres.

What is the volume of Carmen's room?

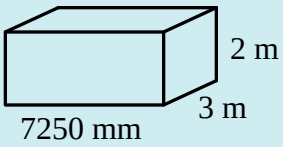
© Department of Education WA 2012 – MATHSAC024

Skills development 7

Before using a volume formula, you must ensure that all units are the same.

Example

Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prism in cubic metres.



Solution

$7250 \div 1000 = 7.25$

$V = l \times w \times h$

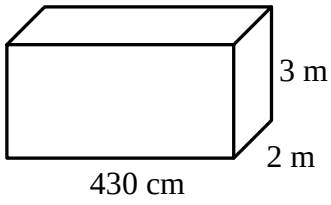
$= 7.25 \times 3 \times 2$

$= 43.5$

$\therefore$ The volume of the rectangular prism is 43.5 m^3.

1. Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prisms that follow.

(a)



$430 \div 100 =$ _____ m

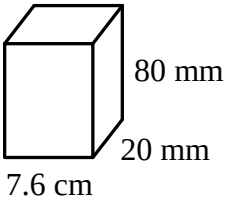
$V = l \times w \times h$

$=$ _____ $\times$ _____ $\times$ _____

$=$ _____

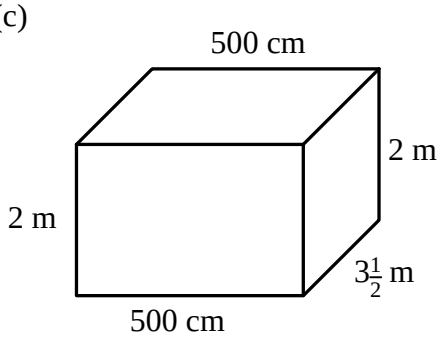
$\therefore$ The volume is _____ m^3.

(b)



© Department of Education WA 2012 – MATHSAC024

(c)



2. Find an object around you that is in the shape of a rectangular prism, for example, a box of tissues or a chest of drawers.

   (a) Sketch your object below.

   (b) Measure your object's dimensions and add them to your sketch above.

   (c) Determine its volume.

© Department of Education WA 2012 – MATHSAC024

3. Mt Augustus is the largest rock in the world, being more than twice the size of Uluru. It stands some 850 metres high, 8 km long and 3 km wide and, although not exactly a rectangular prism, its volume can be approximated using the formula $V = l \times w \times h$.

   Determine the volume of the world's largest rock.

Check your work before continuing.

You may like to ask your teacher for some more questions on finding the volumes of rectangular prisms to further develop your fluency.

The world's biggest rock

Mt Augustus is approximately 500 km east of Carnarvon and 850 km north of Perth. Historically, it was an extremely important source of water for the local Aboriginal people, especially in drought times. It has natural springs at the base of the rock, which attracts a variety of animals, including emus, galahs, swans, kangaroos and Bungarras (goannas).

© Department of Education WA 2012 – MATHSAC024

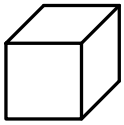
8. Summary

- Plans are a set of two-dimensional drawings used to describe three-dimensional objects.

- These plans may consist of different views, such as the top (roof) plan, front and side elevations.

- Technical plans, such as those used in engineering and architecture will often consist of multiple views. This helps in describing correctly the intended three-dimensional shape.

- Oblique projection (or face-on drawing)



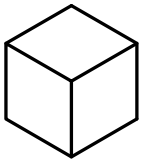
This is a two-dimensional representation of a cube.

Step 1 – Draw the front face.

Step 2 – Draw the slanted or oblique lines at an angle and halfway to the next dot.

Step 3 – Complete the shape. (Add the other vertical and horizontal lines.)

- Isometric projection (or corner drawing)



This is a two-dimensional representation of a cube.

Step 1 – Draw the edge.

Step 2 – Draw the diagonal lines to the next dot.

Step 3 – Complete the shape.

- One way to find the volume of an object is to divide it into cubic units and then count them.

  or $V = l \times w \times h$

- The volume of a rectangular prism can be found using the formula
  Volume = length × width × height.

© Department of Education WA 2012 – MATHSAC024

© Department of Education WA 2012 – MATHSAC024

9. Review tasks

The following tasks will assist you to consolidate your learning and understanding of the concepts introduced in this resource, and assist you to prepare for assessments.

Task A

Name: _____ Suggested time: 40 minutes

Actual time taken: _____

Instructions

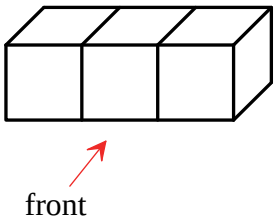
Complete this work on your own.

You may use a calculator, but show how you got your answer.

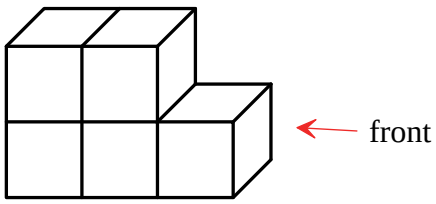
Attempt every question. Take as long as you need and record the time in the space provided above after you have finished.

1. Draw the top plan, front and side elevations for each of the following.

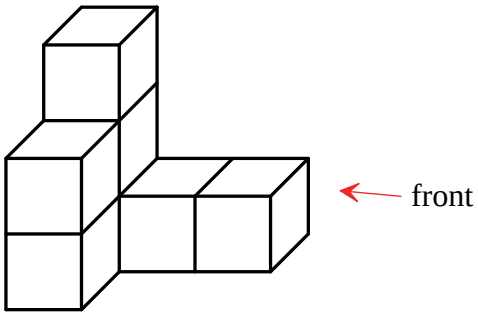
(a)



(b)



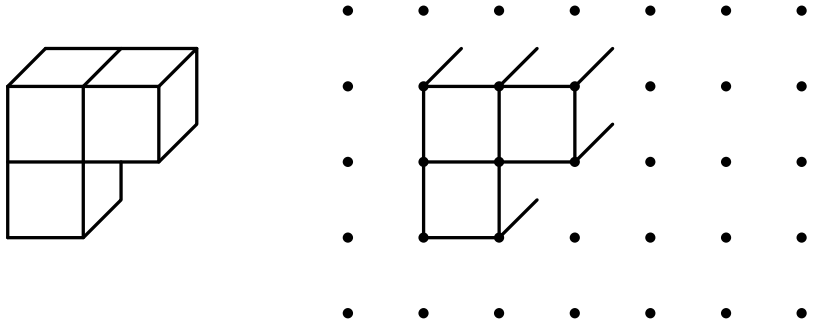
(c)



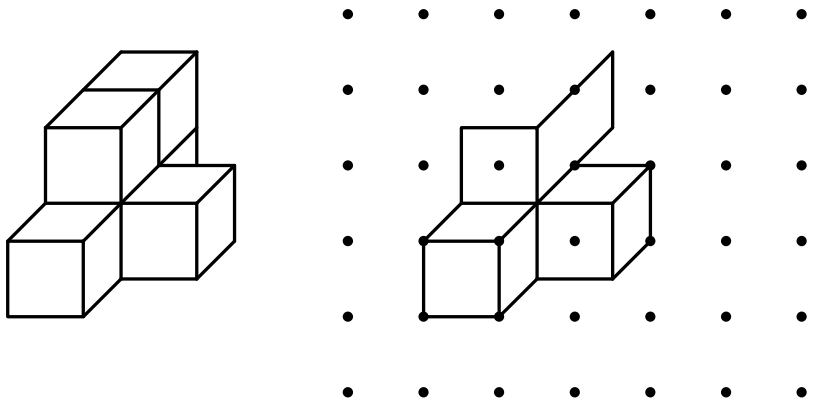
© Department of Education WA 2012 – MATHSAC024

2. Complete the copy of each shape below.

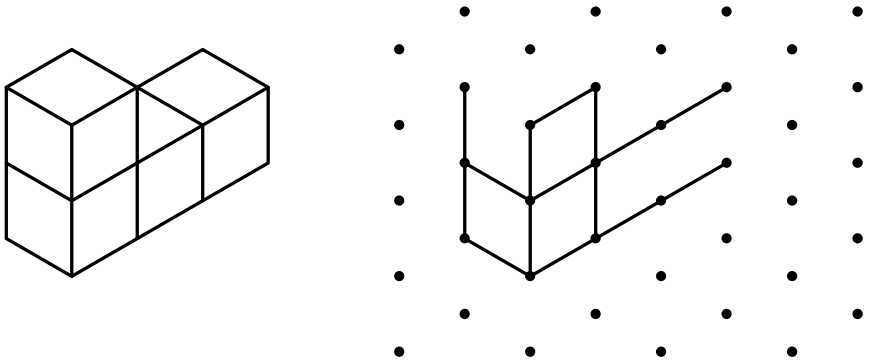
(a)



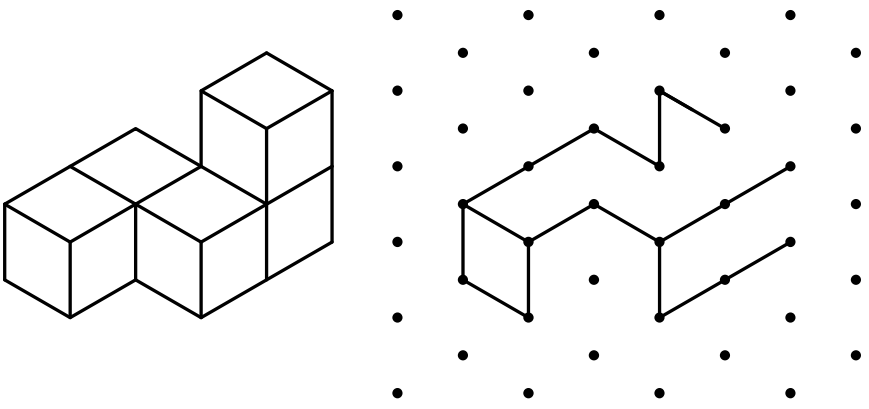
(b)



(c)



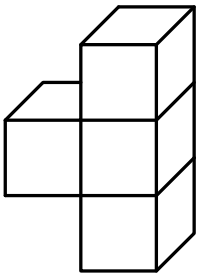
(d)

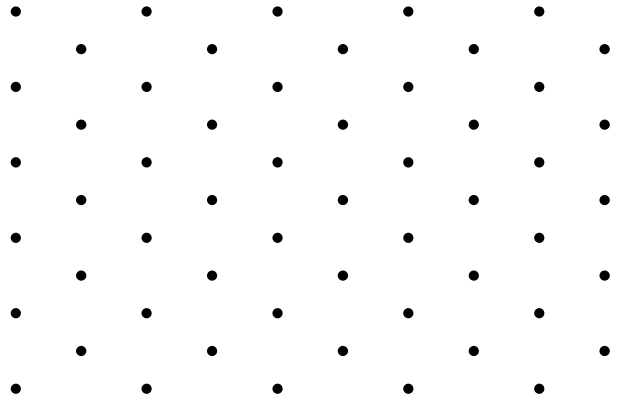


© Department of Education WA 2012 – MATHSAC024

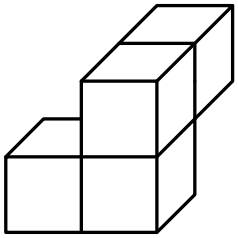
3. Redraw each of the following shapes using an isometric projection.

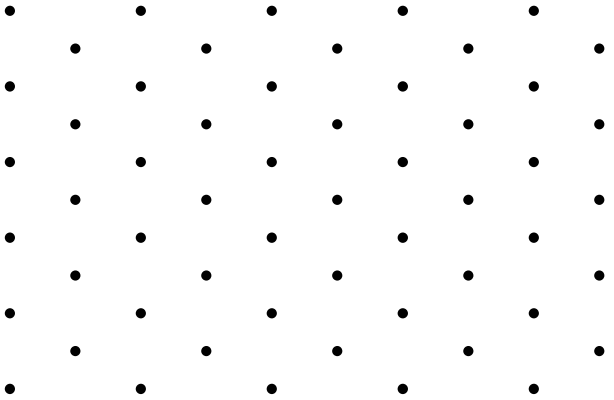
(a)





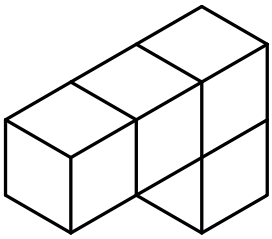
(b)

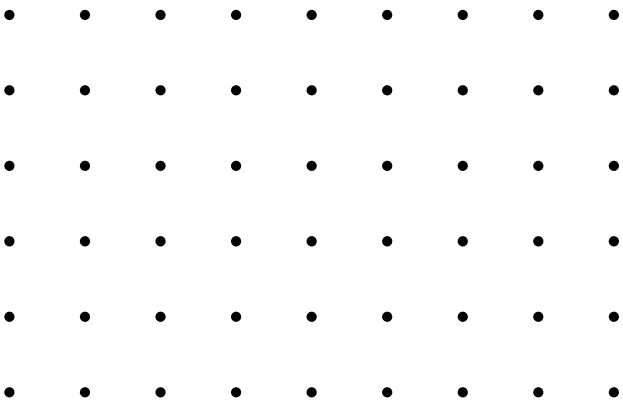




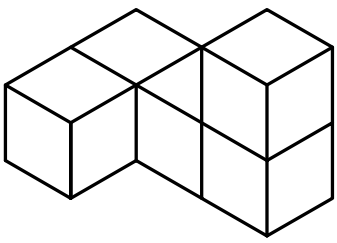
4. Redraw each of the following shapes using an oblique projection.

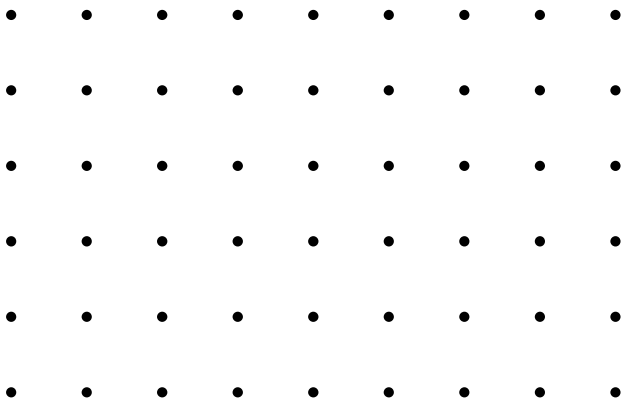
(a)





(b)

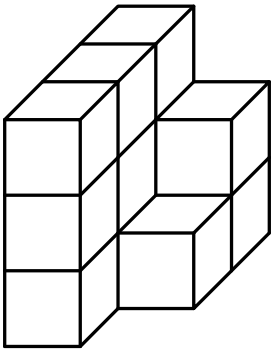




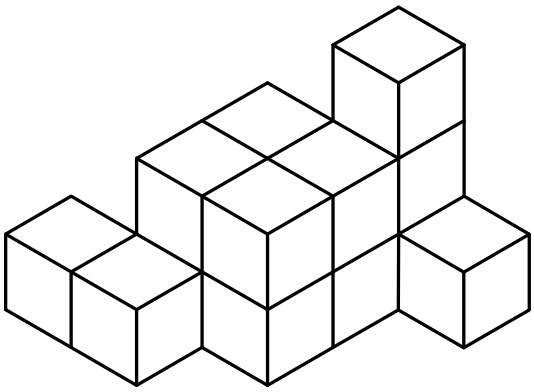
© Department of Education WA 2012 – MATHSAC024

5. Determine the volume for each of the following.

(a)

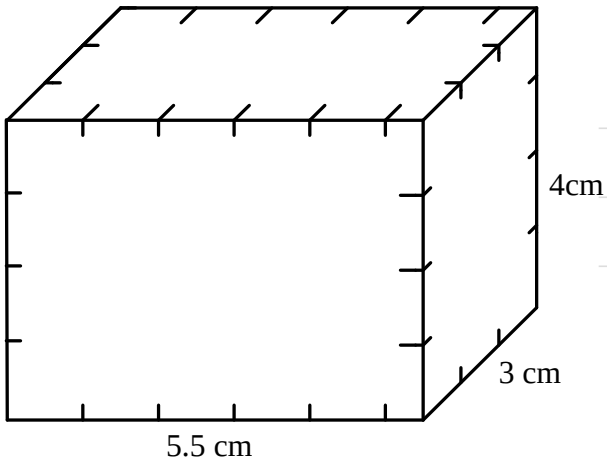


(b)

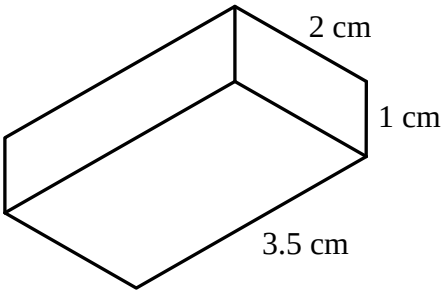


6. Determine the volume for each of the following.

(a)



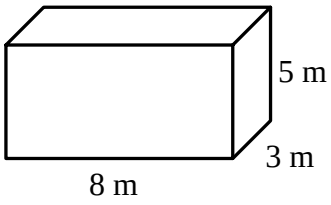
(b)



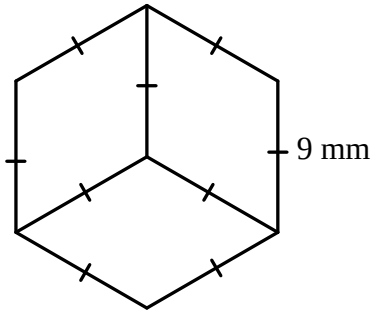
© Department of Education WA 2012 – MATHSAC024

7. Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prisms that follow.

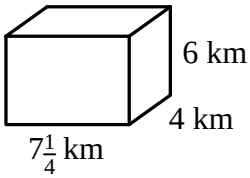
(a)



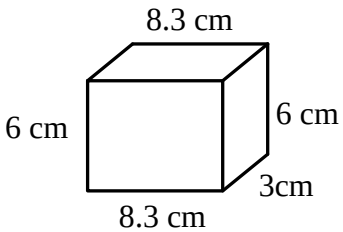
(b)



(c)



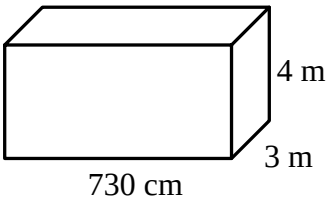
(d)



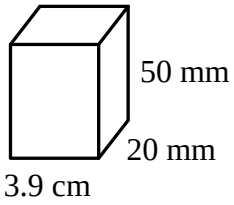
© Department of Education WA 2012 – MATHSAC024

8. Use the formula $V = l \times w \times h$ to determine the volume of the rectangular prisms that follow.

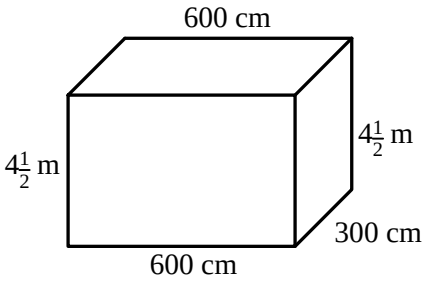
(a)



(b)



(c)



© Department of Education WA 2012 – MATHSAC024

Task B

Name: _____ Suggested time: 30 minutes

Actual time taken: _____

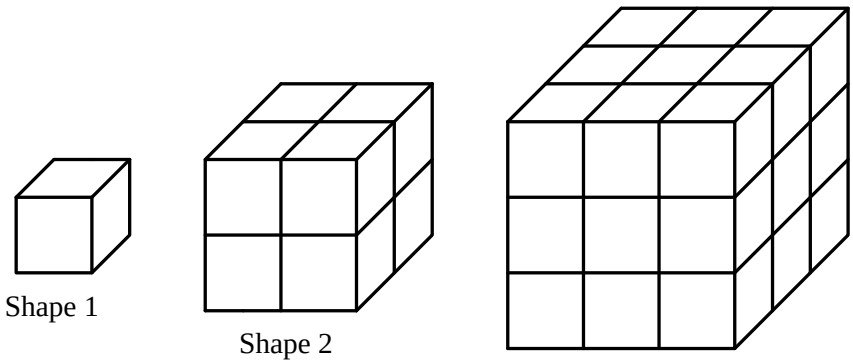
Instructions

Complete this work on your own.

You may use a calculator, but show how you got your answer.

Attempt every question. Take as long as you need and record the time in the space provided below after you have finished.

The following shows a pattern where each of the three dimensions are increasing by one unit.



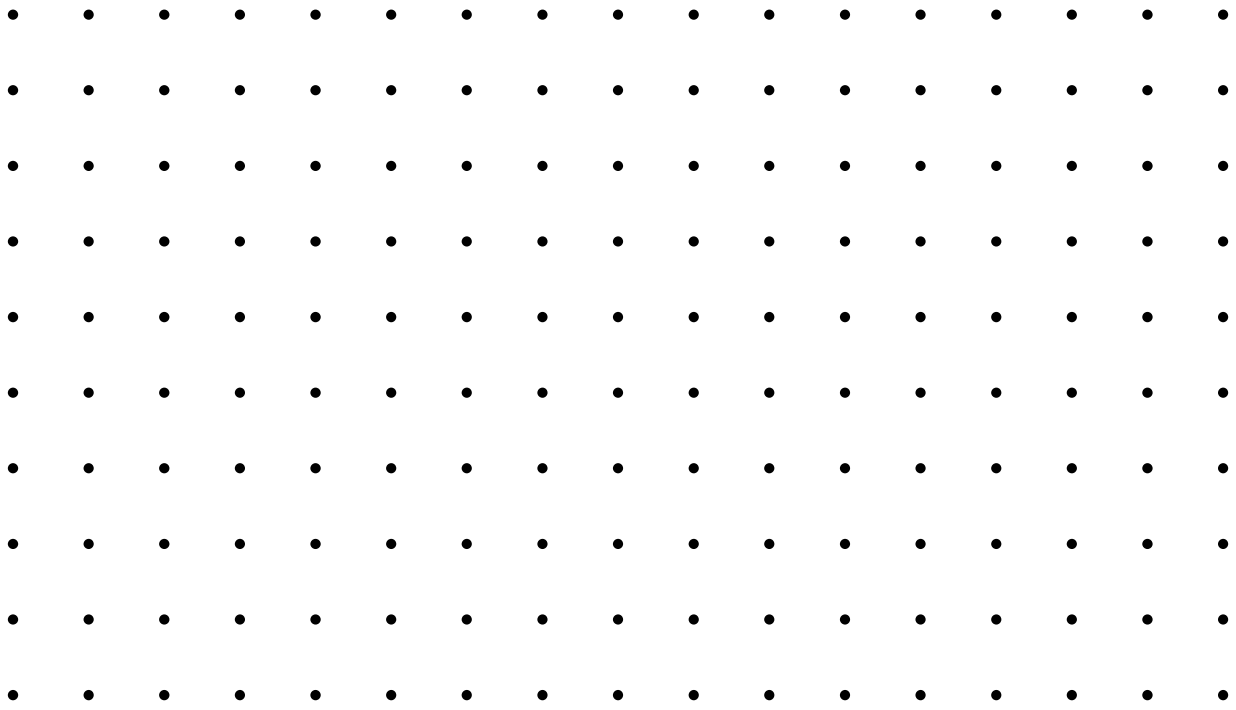
Shape 1 Shape 2 Shape 3

1. Determine the volume of each shape.

© Department of Education WA 2012 – MATHSAC024

2. Predict the volume of the fourth object in the pattern _____

3. Draw the fourth shape that belongs to the pattern, labelling its dimensions with each of their sizes.



4. Determine the volume of the fourth object in the pattern.

5. Determine the volume of the tenth object in the pattern.

6. If you had 2000 cubes, what would be the highest pattern number that you could make?

© Department of Education WA 2012 – MATHSAC024

Self-evaluation task

Please complete the following.

How well did you manage your own learning using this resource?

	Always	Usually	Rarely	Not sure
Each section took approximately 45 minutes to complete.	☐	☐	☐	☐
I needed extra help.	☐	☐	☐	☐
I marked and corrected my work at the end of each section.	☐	☐	☐	☐
I made the journal entries and summaries when asked.	☐	☐	☐	☐
I have kept to my work schedule.	☐	☐	☐	☐

How much mathematics have you learnt using this resource?

	Always	Usually	Rarely	Not sure
Understanding				
I understand the differences between drawing techniques.	☐	☐	☐	☐
Fluency				
I calculated accurately.	☐	☐	☐	☐
I calculated volume of rectangular prisms.	☐	☐	☐	☐
I can recall the rule for the volume of rectangular prisms.	☐	☐	☐	☐
I know to use the rule for the volume of rectangular prisms, including using the correct units.	☐	☐	☐	☐
I accurately drew different views of prisms and solids formed from combinations of prisms.	☐	☐	☐	☐
Problem Solving				
I can formulate and solve problems.	☐	☐	☐	☐
Reasoning				
I can explain why the volume of rectangular prisms rule works.	☐	☐	☐	☐

© Department of Education WA 2012 – MATHSAC024

Write a list of topics for which you need additional assistance.

© Department of Education WA 2012 – MATHSAC024

Solutions

1. Plans

Solutions to Warm-up 1

1. 1, 2, 3, 4 and 6 should be circled.
2. 14
3. $a = (-12)$
4. $\frac{2}{6}$ is the greater fraction.
5. 10
6. 10.3
7. 25.2
8. $\frac{1}{100}$
9. 44
10. Pentagon

Solution to Focus problem 1

What you were asked to do was to draw a house plan.

You should have used a ruler or similar object to draw the straight walls and your plan should be in proportion to the actual house. That is, the biggest room of your house should be the biggest room on the plan.

Show your house plan to an adult for them to check your work.

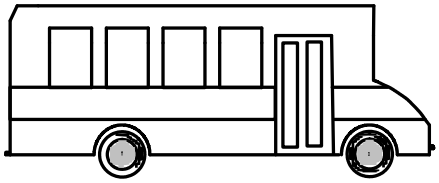
A plan is a two-dimensional drawing used to describe a three-dimensional object or place. The house plan that you were asked to draw was actually a floor plan. That is, it was an aerial view or bird's-eye view of the house without the roof. Architects and other technical drawers will often use other types of views, such as a top (roof) plan and elevations, to describe accurately the object.

Solutions to Skills development 1.1

1. Solutions may vary slightly.

© Department of Education WA 2012 – MATHSAC024

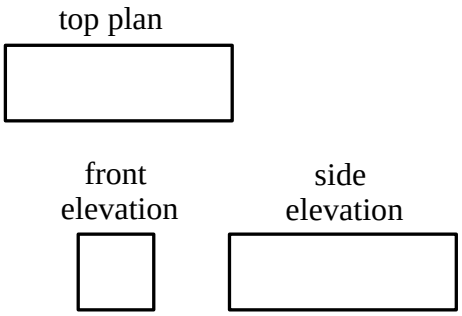
2. Solutions may vary slightly.



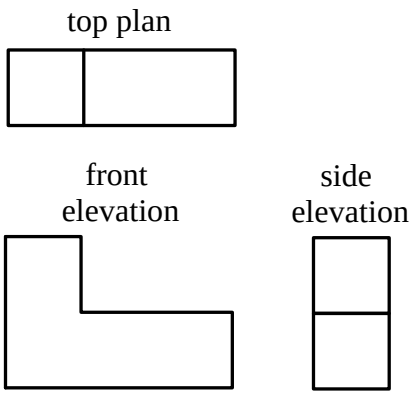
3. Solutions may vary slightly.

Solutions to Skills development 1.2

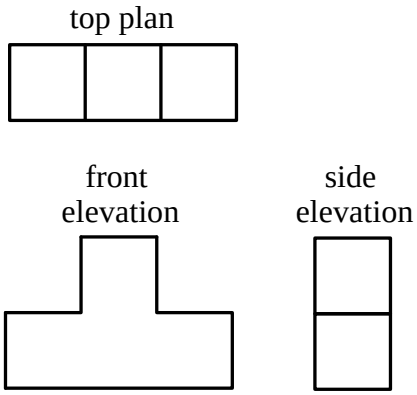
1. (a)



(b)

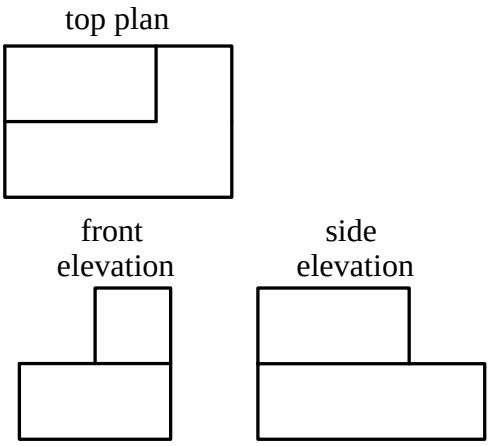


(c)

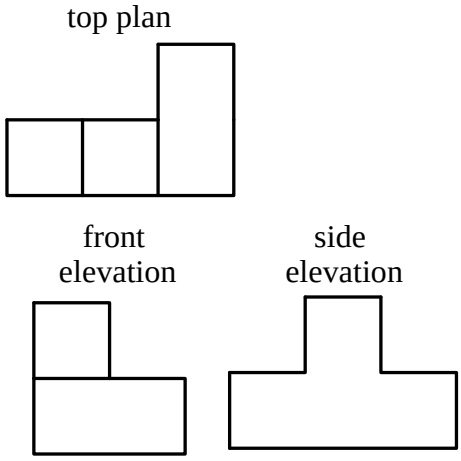


© Department of Education WA 2012 – MATHSAC024

(d)



(e)



2. Solutions will vary. Students should have drawn the top plan, front and side elevations of an object familiar to them.

2. Oblique projection

Solutions to Warm-up 2

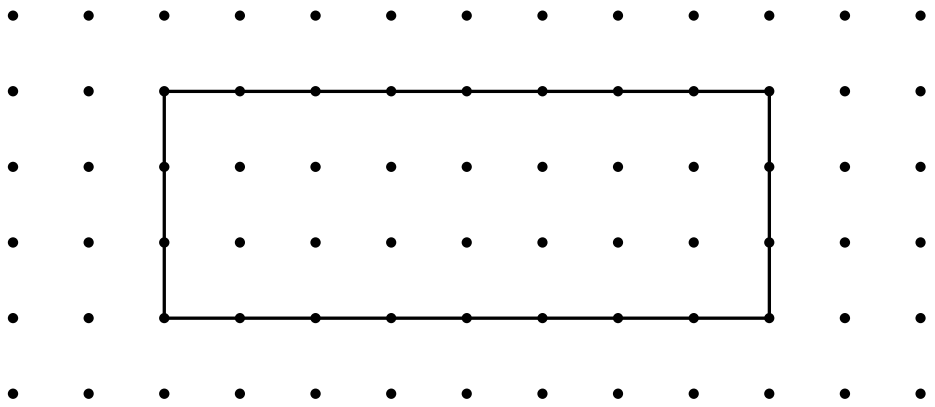
1. The common factor is 3.
2. 7
3. (-3) degrees
4. $\frac{2}{5} \boxed{>} \frac{1}{4}$
5. 100
6. 8
7. 8.2
8. 0.666… (or $0.\dot{6}$)
9. 4.5
10. $180° - 48° = 132°$

© Department of Education WA 2012 – MATHSAC024

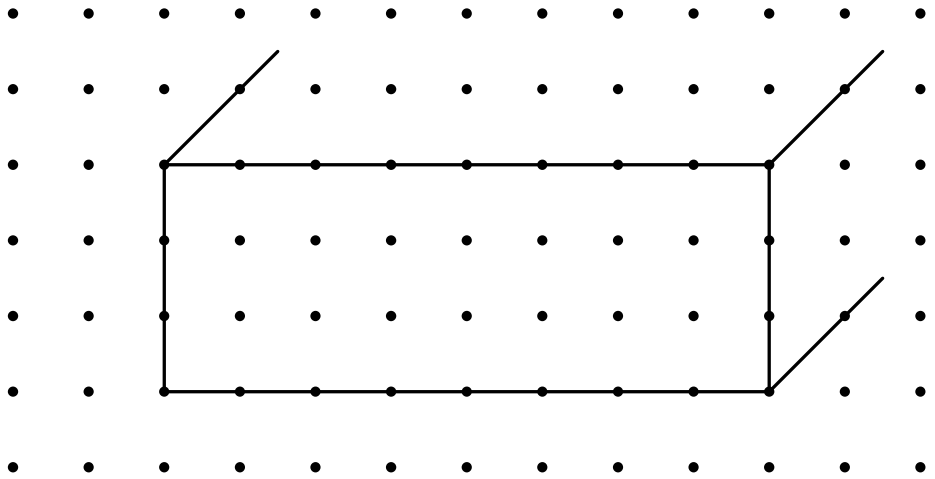
Solution to Focus problem 2

What you were asked to do was to draw a face-on view (or oblique projection) of the facility. An oblique projection is a two-dimensional representation of a three-dimensional object, which uses the front face, and then creates depth by drawing the sides at an angle.

For example, the face, or front of the facility is drawn first.



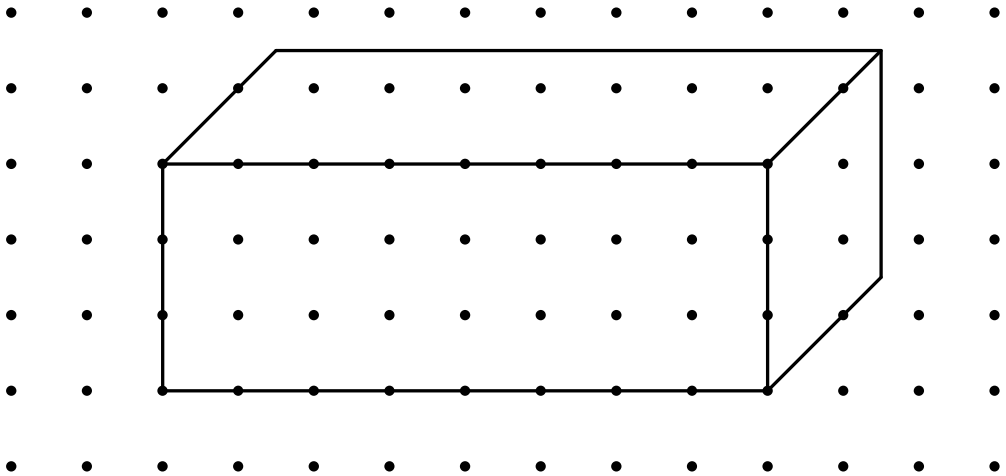
Once the face is drawn, the depth is added. To do this, the sides are drawn on an angle of 45 degrees with the sides drawn to half depth. (Otherwise it would look too deep!)



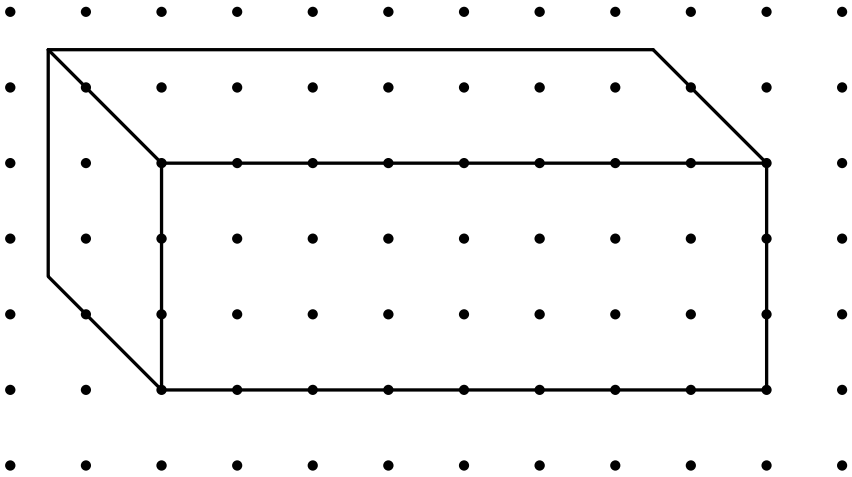
Note how the sides are drawn parallel (that is, they are in the same direction and keep the same distance apart). It does not matter what direction you choose, as long as the sides are all in the same direction (see the object on the next page).

© Department of Education WA 2012 – MATHSAC024

Finally, the other edges are drawn, completing the object.

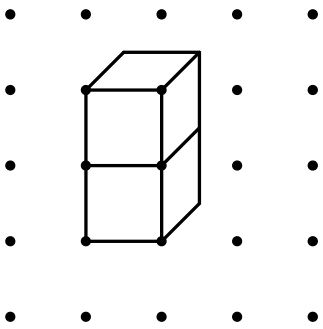


You could also have drawn the facility as shown.



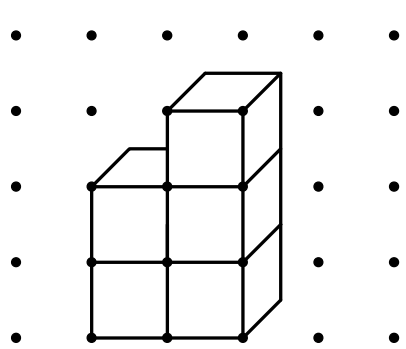
Solutions to Skills development 2

1. (a)

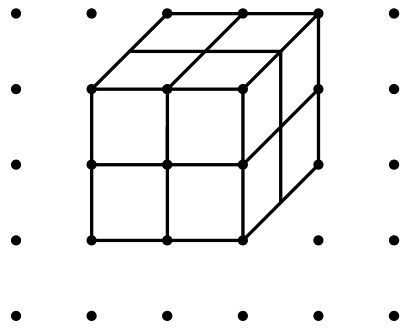


© Department of Education WA 2012 – MATHSAC024

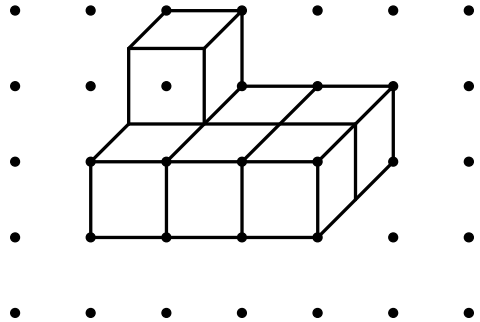
(b)



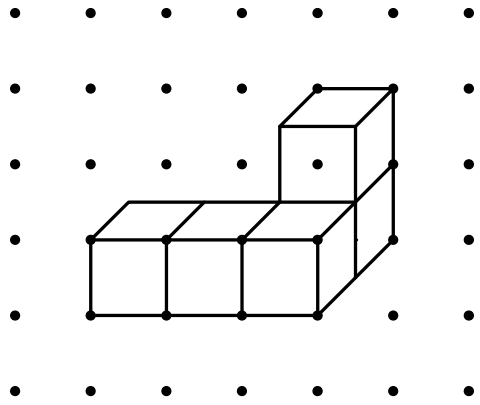
(c)



(d)



(e)

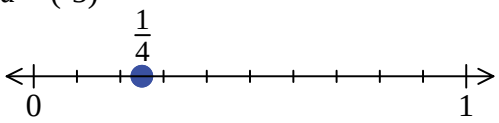


© Department of Education WA 2012 – MATHSAC024

2. Ask an adult to check to see if your name is drawn correctly.
3. No solution is required. You may like to show your drawings to your teacher, another student or another adult.

3. Isometric projection

Solutions to Warm-up 3

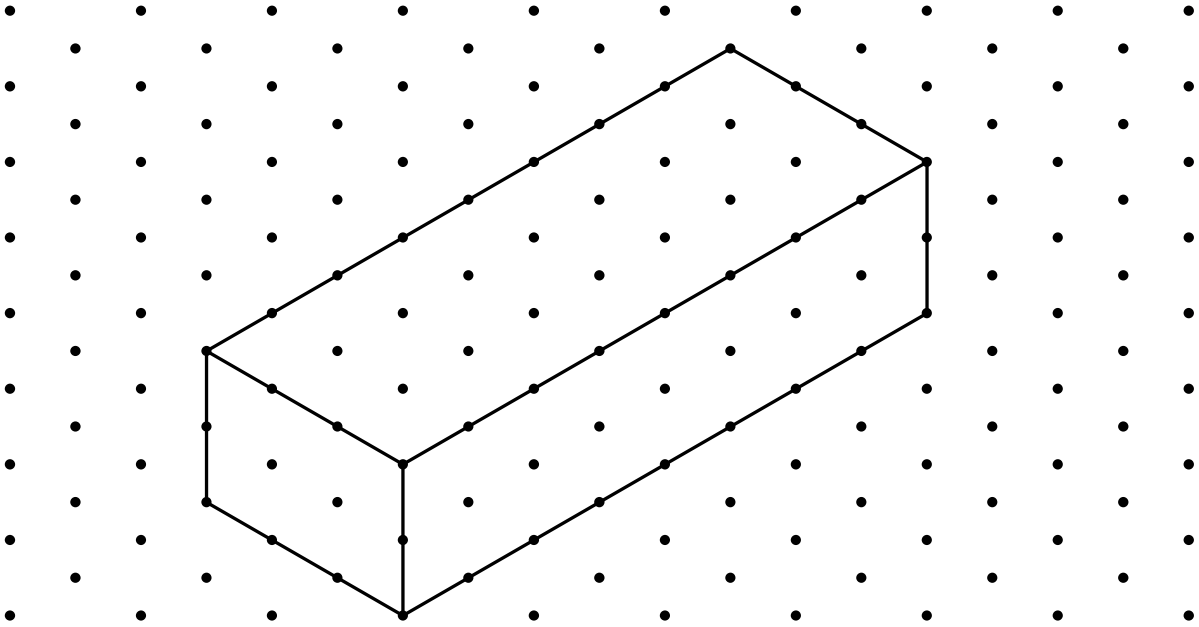
1. The prime number is 7.
2. 16
3. $a = (-5)$
4. 

5. $\frac{1}{4} \times 40 = 10$
6. $9 - 3 = 6$
7. 6.6
8. 12.5%
9. $\frac{10}{10}$ or 1
10. (1, 2)

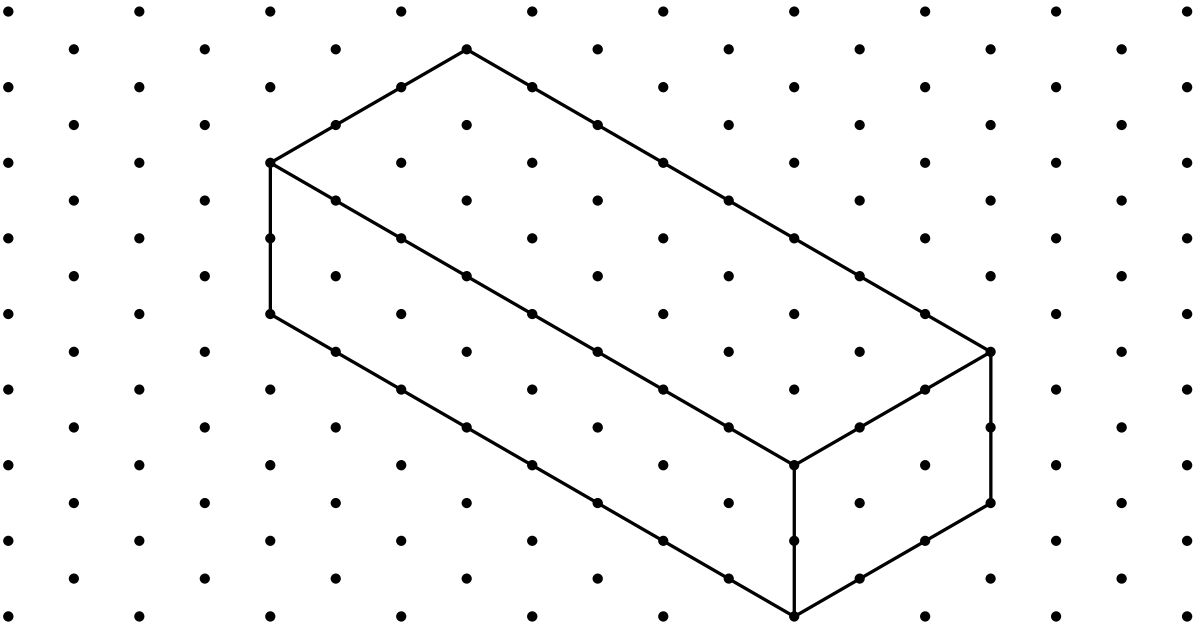
© Department of Education WA 2012 – MATHSAC024

Solution to Focus problem 3

What you were asked to do was to draw the facility using an isometric projection, or edge view. An isometric projection is a two-dimensional representation of a three-dimensional object, which uses equal-scaled dimensions set at an angle of 120 degrees between each.



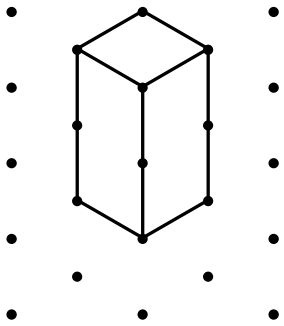
Similar to the oblique projection, the isometric projection can be drawn from different perspectives.



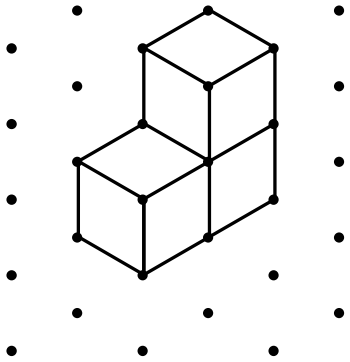
© Department of Education WA 2012 – MATHSAC024

Solutions to Skills development 3.1

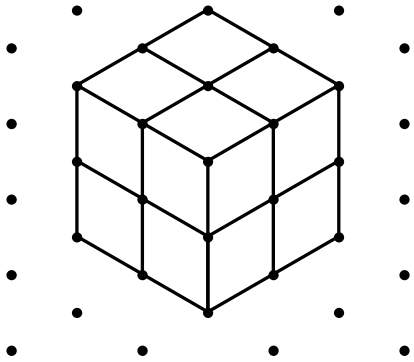
1. (a)



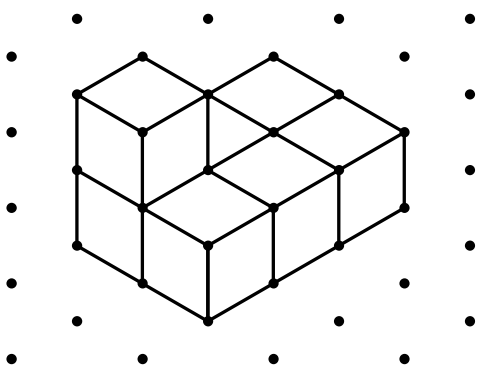
(b)



(c)

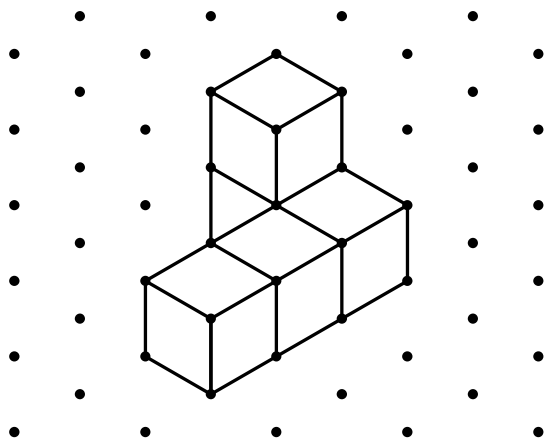


(d)



© Department of Education WA 2012 – MATHSAC024

(e)



Solutions to Skills development 3.2

A-III

B-IV

C-II

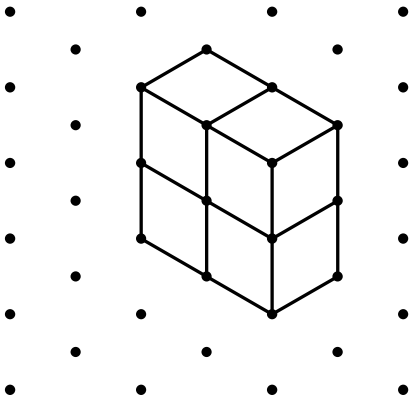
D-V

E-I

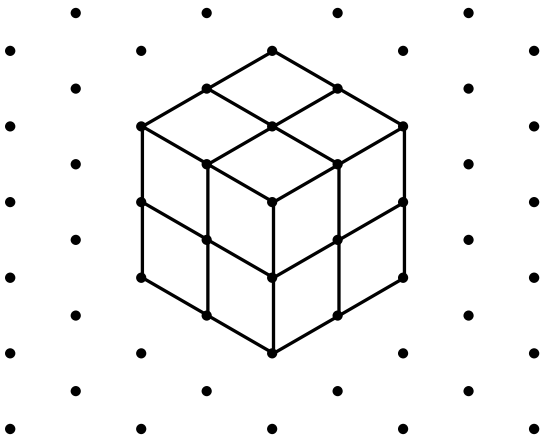
Solutions to Skills development 3.3

1. Solutions may vary slightly.

(a)

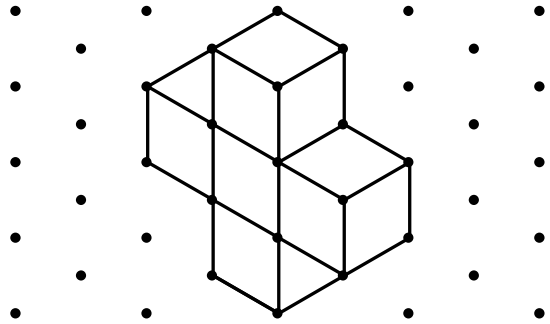


(b)

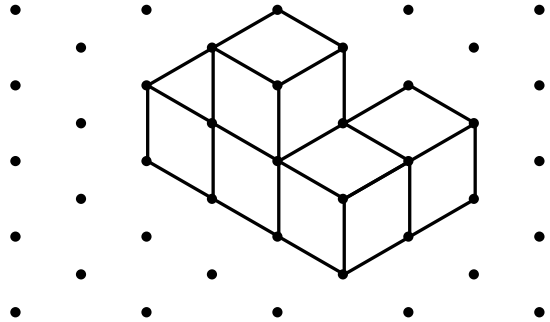


© Department of Education WA 2012 – MATHSAC024

(c)

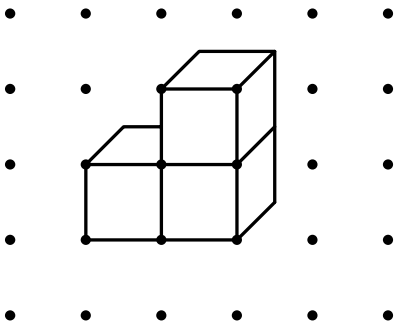


(d)

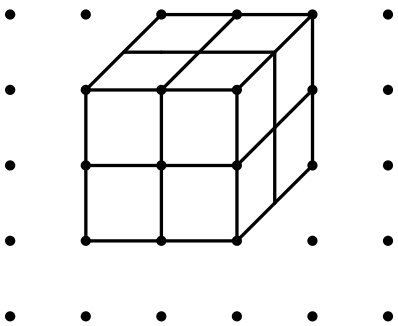


Solutions to Skills development 3.4

1. (a)

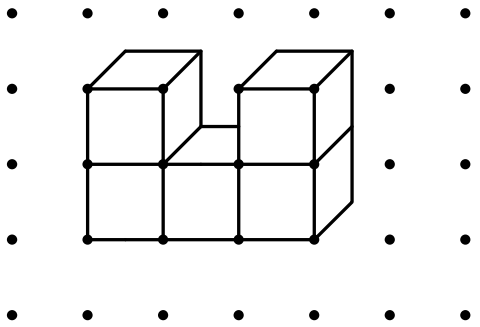


(b)

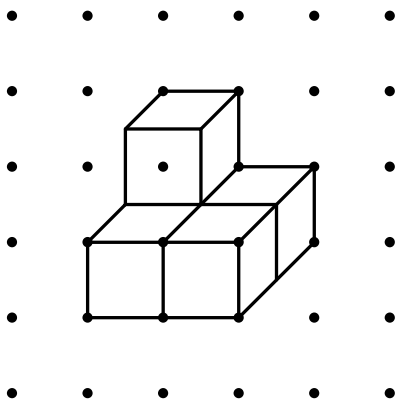


© Department of Education WA 2012 – MATHSAC024

(c)



(d)



2. Ask an adult to check to see if your name is drawn correctly.

4. Volume

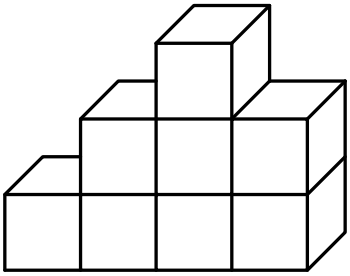
Solutions to Warm-up 4

1. A composite number has three or more factors.
2. 8
3. (-1) + 8 = 7 degrees
4. $w = \frac{1}{3}$
5. 8
6. 0.09
7. 1.043
8. $\frac{30}{100}$ or $\frac{3}{10}$
9. 64
10. $33\frac{1}{3}\%$

© Department of Education WA 2012 – MATHSAC024

Solution to Focus problem 4

What you were asked to do was to find the number of boxes in the shape below, which is also the volume of the shape. Volume is the amount of space that an object occupies. It is measured in cubes.



The volume is eight cubes. Therefore, the solution to the problem would be 8 boxes.

Solutions to Skills development 4

1. (a) 4 cubes
   (b) 4 cubes
   (c) 4 cubes
   (d) 5 cubes
   (e) 5 cubes
   (f) 6 cubes
   (g) 8 cubes
   (h) 24 cubes

2. Solutions may vary. Check your solution with an adult to ensure that you have drawn an object with 15 cubes.

5. Volume of rectangular prisms

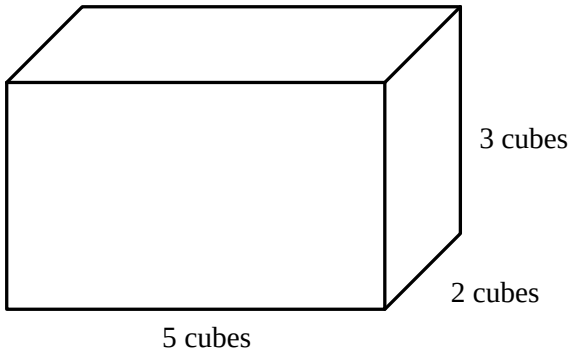
Solutions to Warm-up 5

1. 3 and 5 are the prime factors.
2. 179
3. $j = (-9)$
4. $\frac{5}{2}$ or $2\frac{1}{2}$
5. One-fifth is $35 \div 5 = 7$, so two-fifths are 14.
6. 4.206 km
7. $4 + 4 = 8$
8. 3.5
9. 8.9
10. $180 - 86° = 94°$

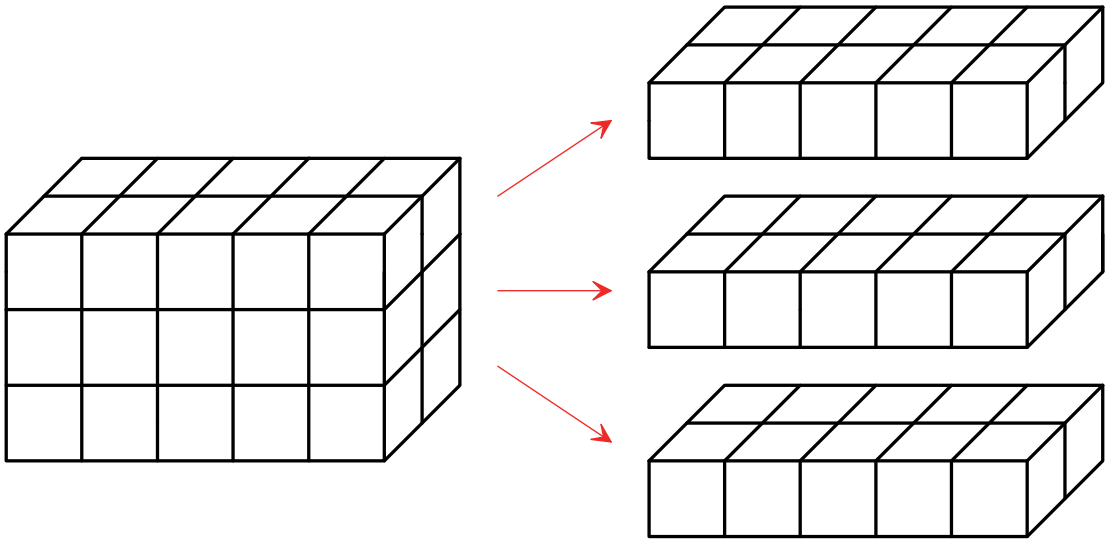
© Department of Education WA 2012 – MATHSAC024

Solution to Focus problem 5

What you were asked to do was to find the volume of the following shape.



One way to do this is to divide the shape up into cubes and then count them.

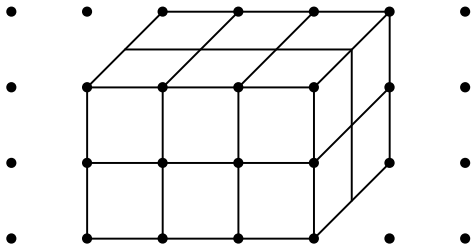


There are ten cubes in the bottom layer (base) and there are three layers. So the solution to the problem is 30 cubes.

Solutions to Skills development 5.1

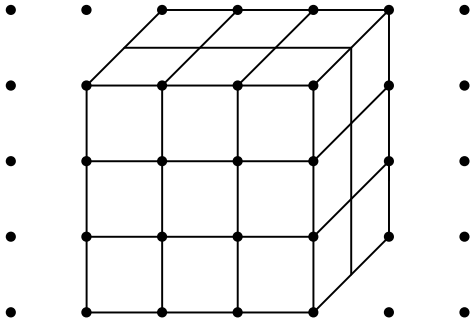
1. (a) 6 cubes
   (b) 24 cubes
   (c) 18 cubes

2. (a)



© Department of Education WA 2012 – MATHSAC024

(b)



Solutions to Skills development 5.2

1. (a) $10\frac{1}{2}$ cubes
   (b) 27 cm^3
   (c) 29 cm^3

2. (a) 12 cubes
   (b) 42 cm^3
   (c) 30 cm^3

You may like to ask your teacher for some more examples to further develop your fluency.

6. Rectangular prism formula

Solutions to Warm-up 6

1. 65
2. 26
3. 3 degrees
4. $\frac{3}{10}$
5. 5
6. 3.2 g
7. 18 + 4 = 22
8. 100%
9. 1
10. (0, 3)

© Department of Education WA 2012 – MATHSAC024

Solutions to Review 6.1

1. Complete the following to determine the area of each of the rectangles.

   (a) 2 rows of 3

   $2 \times 3 = 6$

   $\therefore$ The area of the rectangle is 6 square units.

   (b) 3 rows of 5

   $3 \times 5 = 15$

   $\therefore$ The area of the rectangle is 15 square units.

   (c) 3 rows of 3

   $3 \times 3 = 9$

   $\therefore$ The area of the rectangle is 9 square units.

   (d) 3 rows of 4

   $3 \times 4 = 12$

   $\therefore$ The area of the rectangle is 12 square units.

Solutions to Review 6.2

1. Determine the area of each of the following rectangles.

   (a) $Area_{\text{(rectangle)}} = length \times width$

   $= 4 \times 2$

   $= 8$

   $\therefore$ The area of the rectangle is 8 m^2.

   (b) $Area_{\text{(rectangle)}} = length \times width$

   $= 7 \times 4$

   $= 28$

   $\therefore$ The area of the rectangle is 28 cm^2.

   (c) $Area_{\text{(rectangle)}} = length \times width$

   $= 6 \times 6$

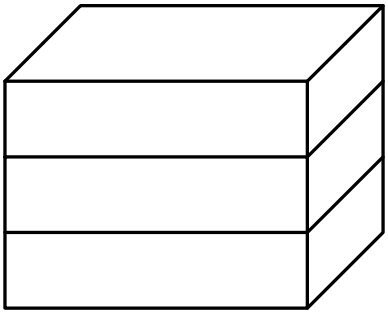
   $= 36$

   $\therefore$ The area of the rectangle is 36 cm^2.

© Department of Education WA 2012 – MATHSAC024

Solution to Focus problem 6

What you were asked to do was to determine the volume of the following.



3 layers

Before you can find this volume, you need to determine the number of boxes in one layer. To do this, you can find the area of the base, which is a rectangle.

$$\begin{aligned} Area_{\text{(rectangle)}} &= length \times width \\ &= 4 \times 2 \\ &= 8 \end{aligned}$$

$\therefore$ Eight cubes fit in one layer.

You can then multiply the area of the base by the number of layers (height).

Note that the formula for determining the volume of a rectangular prism combines these two calculations together.

So
$$\begin{aligned} Volume_{\text{(rectangle prism)}} &= length \times width \times height \\ &= 4 \times 2 \times 3 \\ &= 24 \end{aligned}$$

$\therefore$ The volume is 24 cubes.

That is, Rodney would have 24 boxes if he stacked three layers on top of the 4 by 2 grid.

Solutions to Skills development 6.1

1. (a) $V = l \times w \times h$

$= 9 \times 4 \times 5$

$= 180$

$\therefore$ The volume is 180 m^3.

(b) $V = l \times w \times h$

$= 7 \times 3 \times 8$

$= 168$

$\therefore$ The volume is 168 mm^3.

© Department of Education WA 2012 – MATHSAC024

(c) $V = l \times w \times h$

$= 12 \times 6 \times 5$

$= 360$

$\therefore$ The volume is 360 cm^3.

Solutions to Skills development 6.2

1. (a) $V = l \times w \times h$

$= 5 \times 2.4 \times 6$

$= 72$

$\therefore$ The volume is 72 m^3.

(b) $V = l \times w \times h$

$= 8\frac{1}{2} \times 4 \times 5$

$= 170$

$\therefore$ The volume is 170 mm^3.

(c) $V = l \times w \times h$

$= 7.4 \times 4 \times 5$

$= 148$

$\therefore$ The volume is 148 cm^3.

Solutions to Skills development 6.3

1. (a) $V = l \times w \times h$

$= 7 \times 7 \times 7$

$= 343$

$\therefore$ The volume is 72 cm^3.

(b) $V = l \times w \times h$

$= 8.5 \times 8.5 \times 8.5$

$= 614.125$

$\therefore$ The volume is 614 mm^3 (rounded to the nearest mm).

Solutions to Skills development 6.4

1. (a) $V = l \times w \times h$

$60 = l \times 2 \times 5$

$\therefore l = 6$

$\therefore$ The length of the rectangular prism is 6 cm.

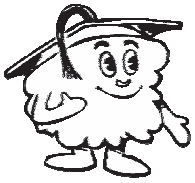
© Department of Education WA 2012 – MATHSAC024

(b) $V = l \times w \times h$

$1000 = l \times l \times l$

$\therefore l = 10$

$\therefore$ The length of the rectangular prism is 10 mm.



You may like to ask your teacher for some more examples to further develop your fluency.

7. Units and the volume formula

Solutions to Warm-up 7

1. 6.3
2. 52
3. 5 degrees
4. $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$ or $\frac{4}{5}$
5. 5
6. 600 L
7. $20 \div 10 = 2$
8. $30
9. They are square numbers.
   $1 \times 1 = 1$
   $2 \times 2 = 4$
   $3 \times 3 = 9$
   ...
10. 0.5

Solutions to Review 7

1. (a) 54 mm = 5.4 cm
   (b) 23 cm = 230 mm
   (c) 7.8 km = 7800 m
   (d) 591 cm = 5.91 m
   (e) 8.54 m = 854 cm
   (f) 2900 mm = 2.9 m
   (g) 6100 m = 6.1 km
   (h) 1.8 km = 180 000 cm

© Department of Education WA 2012 – MATHSAC024

Solution to Focus problem 7

What you were asked to do was to find the volume of Carmen's room. However, before you can use the volume formula, you need to ensure that all the dimensions are in the same units.

$210 \div 100 = 2.1$

$\therefore$ 210 cm = 2.1 m

$$V = l \times w \times h$$
$$= 4 \times 5 \times 2.1$$
$$= 42$$

$\therefore$ The volume is 42 m^3.

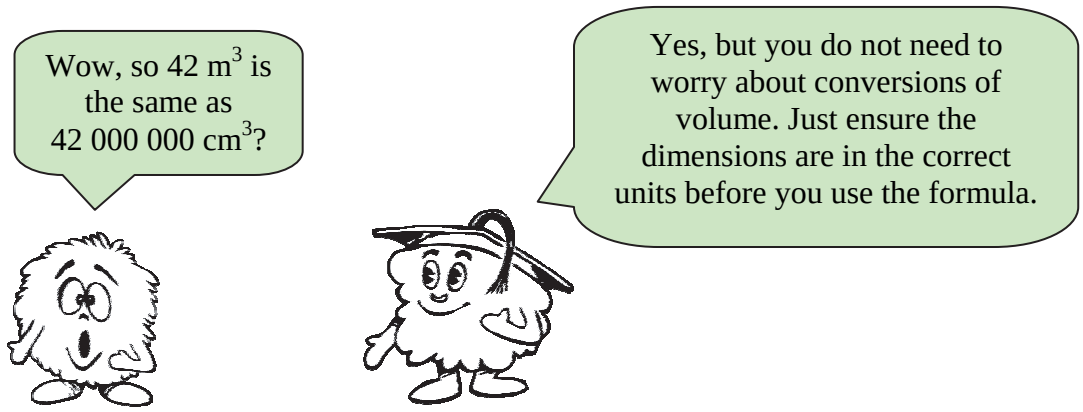
Note that the question did not ask for the volume in a particular unit. You could have determined the volume in cubic centimetres.

$4 \times 100 = 400$

$5 \times 100 = 500$

$$V = l \times w \times h$$
$$= 400 \times 500 \times 210$$
$$= 42\,000\,000$$

$\therefore$ The volume is 42 000 000 cm^3.



Solutions to Skills development 7

1. (a) $430 \div 100 = 4.3$ m

$$V = l \times w \times h$$
$$= 4.3 \times 2 \times 3$$
$$= 25.8$$

$\therefore$ The volume is 25.8 m^3.

© Department of Education WA 2012 – MATHSAC024

(b) $7.6 \times 10 = 76$ mm

$V = l \times w \times h$

$= 76 \times 20 \times 80$

$= 121\ 600$

$\therefore$ The volume is 121 600 mm^3.

(c) $500 \div 100 = 5$ m

$V = l \times w \times h$

$= 5 \times 3\frac{1}{2} \times 2$

$= 35$

$\therefore$ The volume is 35 m^3.

2. Solutions will vary. Ask an adult to check your sketch, dimensions and your volume calculations.

   (a) Your sketch should be drawn using a ruler and it should demonstrate your ability to represent three-dimensional shapes, either using a plan view or an oblique or isometric drawing technique.

   (b) The dimensions should be included on your sketch. Ensure you have used and displayed the correct units.

   (c) The volume should be calculated with all working shown. The correct units should also be shown as part of your answer.

3. $850 \div 1000 = 0.85$ km

$V = l \times w \times h$

$= 8 \times 3 \times 0.85$

$= 20.4$

$\therefore$ The volume of Mt Augustus is 20.4 km^3 (or 20 400 000 000 m^3).

© Department of Education WA 2012 – MATHSAC024

Solutions to Review tasks

Solutions to Task A

1. (a)

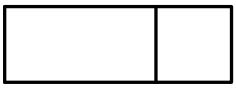
top plan

front elevation

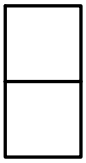
side elevation

(b)

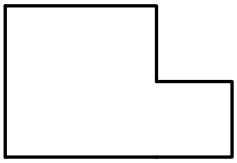
top plan



front elevation

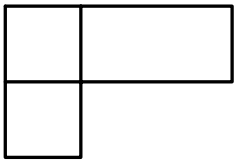


side elevation

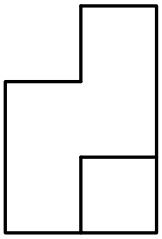


(c)

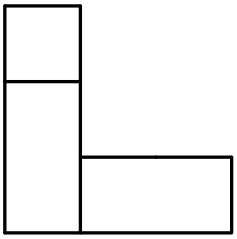
top plan



front elevation

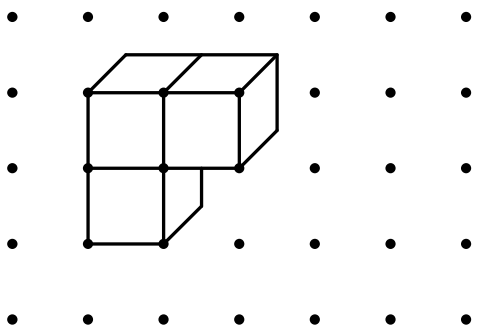


side elevation

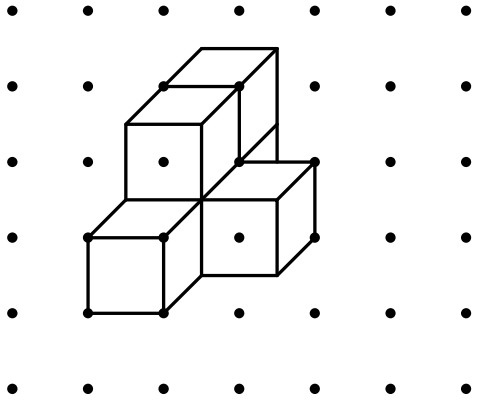


© Department of Education WA 2012 – MATHSAC024

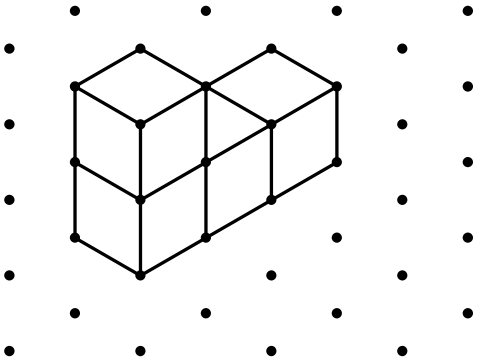
2. (a)



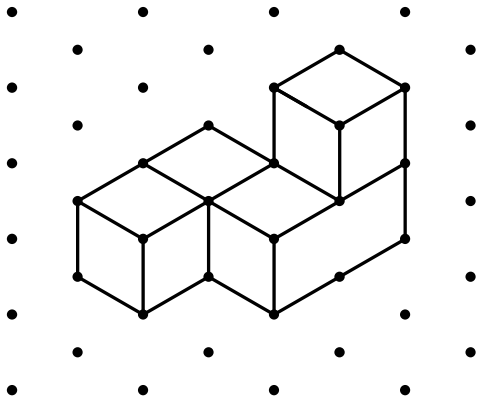
(b)



(c)

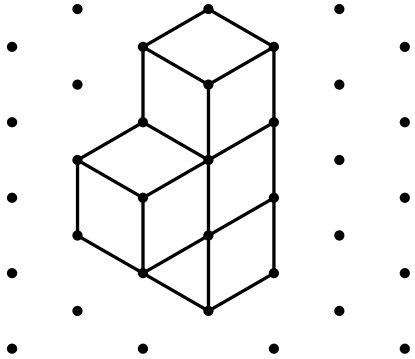


(d)

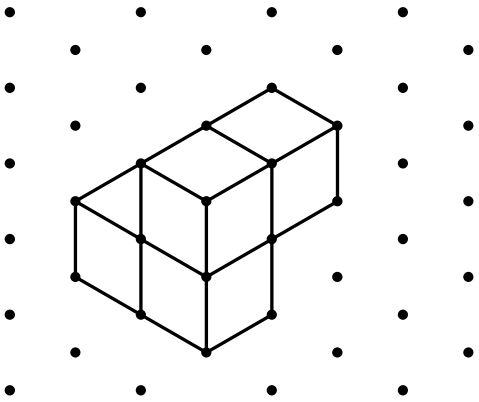


© Department of Education WA 2012 – MATHSAC024

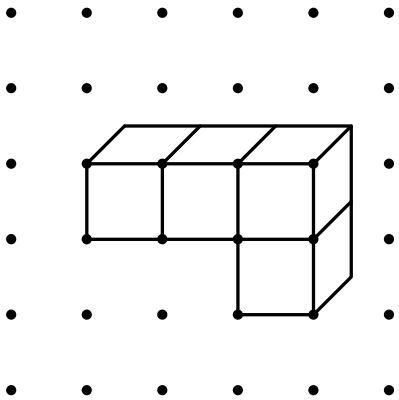
3. (a)



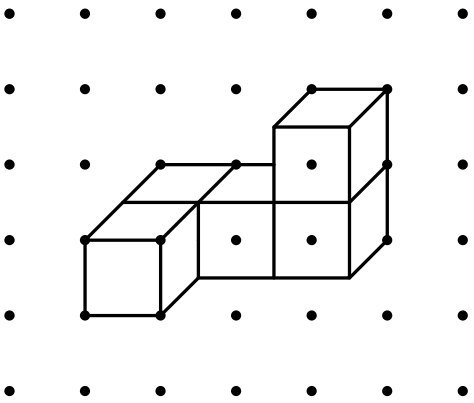
(b)



4. (a)



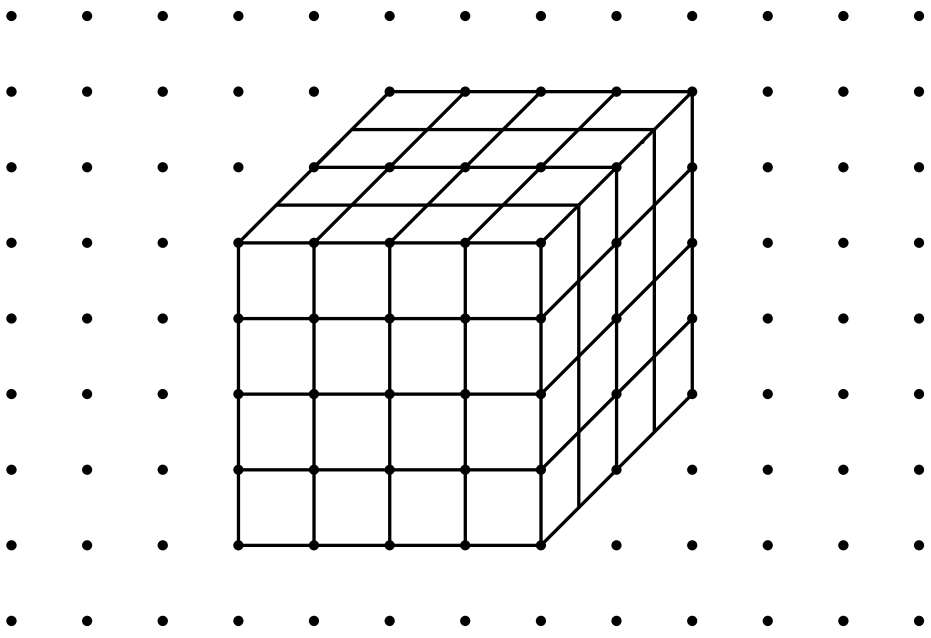
(b)



© Department of Education WA 2012 – MATHSAC024

5. (a) 12 cubes
   (b) 14 cubes
6. (a) 66 cubes (cm^3)
   (b) 7 cubes (cm^3)
7. (a) 120 m^3
   (b) 729 mm^3
   (c) 174 km^3
   (d) 149.4 cm^3
8. (a) 87.6 m^3 or 87 600 000 cm^3
   (b) 39 000 mm^3 or 39 cm^3
   (c) 81 m^3 or 81 000 000 cm^3

Solutions to Task B

1. Shape 1 : 1 cube
   Shape 2 : 8 cubes
   Shape 3 : 27 cubes
2. As this is a prediction, there is no wrong or right answer.
3. 

4. *Volume* $= 4 \times 4 \times 4$
   $= 64$
   $\therefore$ The volume is 64 cubes.

© Department of Education WA 2012 – MATHSAC024

5. *Volume* $= 10 \times 10 \times 10$

   $= 1000$

   $\therefore$ The volume is 1000 cubes.

6. Shape 12 would the largest pattern you could make with the 2000 cubes.

   $12 \times 12 \times 12 = 1728$ (under 2000 cubes)

   $13 \times 13 \times 13 = 2197$ (over 2000 cubes)

© Department of Education WA 2012 – MATHSAC024

Department of
Education

MATHSAC024
3D OBJECTS
REPRESENTATIONS AND VOLUME
ISBN: 9780730744443